Lecture 12

Heterogeneous Firms

7/14

Goals

- Last week, we studied firm entry under homogeneous firms.
- Today, we introduce the idea of **firm heterogeneity**.
- Our primary question is, does firm heterogeneity cause wage inequality?

Entry of Heterogeneous Firms

Melitz, "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 2003.

Heterogeneous Firms

- In the model we studied in the previous lecture, the aggregate output is given by

$$Y = \left(\int_0^n y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- Each firm is indexed by z.
- Firms are uniformly distributed on $[0, n]$.
- While each intermediate good is differentiated, other than that firms are homogeneous.
- Productivity φ is the same for all firms.

Heterogeneous Firms

- Today, we consider **heterogeneous** firms.
  - It is assumed that productivity φ is distributed.
- Aggregate output is now:

$$Y = \left(\int_{z \in Z} y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- Z is the set of all firms (= varieties).
  - While product variety is uniformly distributed, we no longer consider the interval $[0, n]$ because each firm is also indexed not only by z, but also by φ.

Intermediate Goods

- Let $p(z)$ denote the input price of variety z.
- The input demand minimizes total expenditure:

$$\min_{y(z)} \int_{z\in Z} p(z)y(z)dz$$
$$s.t.$$
$$Y = \left(\int_{z\in Z} y(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate Goods

- The Lagrangian is

$$\int_{z\in Z} p(z)y(z)dz + \lambda\left[Y - \left(\int_{z\in Z} y(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}\right]$$

- FOC for variety z is

$$p(z) = \lambda\frac{\sigma}{\sigma-1}\left(\int_{z\in Z} y(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}y(z)^{\frac{\sigma-1}{\sigma}-1}$$

- FOC for variety j is

$$p(j) = \lambda\frac{\sigma}{\sigma-1}\left(\int_{z\in Z} y(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}y(j)^{\frac{\sigma-1}{\sigma}-1}$$

Intermediate Goods

- Taking the ratio:

$$\frac{p(z)}{p(j)} = \frac{y(z)^{\frac{\sigma-1}{\sigma}-1}}{y(j)^{\frac{\sigma-1}{\sigma}-1}} = \left(\frac{y(z)}{y(j)}\right)^{\frac{-1}{\sigma}}$$

- Thus,

$$y(j) = \left(\frac{p(z)}{p(j)}\right)^{\sigma} y(z) = p(z)^{\sigma} p(j)^{-\sigma} y(z)$$

- Thus,

$$y(j)^{\frac{\sigma-1}{\sigma}} = p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}}$$

Intermediate Goods

- Substitute it into the production function:

$$Y = \left(\int_{z\in Z} y(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\int_{z\in Z} \left[p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}}\right] dj\right)^{\frac{\sigma}{\sigma-1}}$$

$$= p(z)^{\sigma} y(z) \left(\int_{z\in Z} [p(j)^{1-\sigma}] dj\right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate Goods

- We obtain

$$y(z) = Yp(z)^{-\sigma}\left(\int_{z\in Z}[p(j)^{1-\sigma}]dj\right)^{\frac{\sigma}{1-\sigma}}$$

- This is the input demand function for variety z.
- We can simplify it further…

Intermediate Goods

- Now let us use $y(j) = p(z)^{\sigma} p(j)^{-\sigma} y(z)$ to rewrite the expenditures:

$$\int_{z \in Z} p(j) y(j) dj = y(z) p(z)^{\sigma} \int_{z \in Z} p(j)^{1-\sigma} dj$$

- Perfect competition in the final-goods market implies zero profit: $PY - \int_{z \in Z} p(j) y(j) dj = 0$
- Thus,

$$PY = y(z) p(z)^{\sigma} \int_{z \in Z} p(j)^{1-\sigma} dj$$

Intermediate Goods

- Solve it for $y(z)$ as

$$y(z) = PY\left[\int_{z\in Z} p(j)^{1-\sigma}dj\right]^{-1} p(z)^{-\sigma}$$

- Substitute it into the production function:

$$Y = PY\left[\int_{z\in Z} p(j)^{1-\sigma}dj\right]^{-1} \left(\int_{z\in Z} p(z)^{1-\sigma}dz\right)^{\frac{\sigma}{\sigma-1}}$$

- It simplifies to

$$1 = P\left[\int_{z\in Z} p(j)^{1-\sigma}dj\right]^{\frac{1}{\sigma-1}}$$

Intermediate Goods

- Finally,

$$P = \left[\int_{z \in Z} p(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$$

- RHS is the **price index** of the intermediate goods.

Intermediate Goods

- The input demand is therefore

$$y(z) = Yp(z)^{-\sigma}\left(\int_{z\in Z}[p(j)^{1-\sigma}]dj\right)^{\frac{\sigma}{1-\sigma}}$$
$$= Yp(z)^{-\sigma}P^{\sigma}$$

- Thus,

$$\frac{y(z)}{Y} = \left[\frac{p(z)}{P}\right]^{-\sigma}$$

- Relative demand for input z is decreasing in the relative price of the input.

Intermediate Goods

- Production requires labor input $\ell(z)$:
$$y(z) = \max\{0, \varphi[\ell(z) - f]\}$$
- φ is **distributed** across the firms.
- The labor units needed to produce $y(z)$ is
$$\ell(z) = f + \frac{y(z)}{\varphi}$$
- Labor market is perfectly competitive.
  - w: competitive wage rate.
  - All workers receive the same wage rate.

Intermediate Goods

- The profit is

$$\pi(z) = p(z)y(z) - w\ell(z)$$

- Input demand function:

$$\frac{y(z)}{Y} = \left[\frac{p(z)}{P}\right]^{-\sigma} \Leftrightarrow y(z) = YP^{\sigma}p(z)^{-\sigma}$$

- Production technology:

$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Markup Pricing

- Profit:

$$\pi(z) = p(z)y(z) - w\ell(z)$$
$$= p(z)y(z) - w\left[f + \frac{y(z)}{\varphi}\right]$$
$$= YP^{\sigma}p(z)^{1-\sigma} - wf - \frac{w}{\varphi}YP^{\sigma}p(z)^{-\sigma}$$

- FOC with respect to $p(z)$:

$$p(z) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi} = \frac{1}{\rho}\frac{w}{\varphi} = markup \times \frac{w}{MPL}$$

σ and ρ

- Aggregate output using σ:

$$Y = \left(\int_{z \in Z} y(z)^{\frac{\sigma - 1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma - 1}}$$

- Aggregate output using ρ:

$$Y = \left(\int_{z \in Z} y(z)^{\rho} dz \right)^{\frac{1}{\rho}}$$

- It is nice to know that

$$markup = \frac{1}{\rho}$$

Revenue

- Notice that

$$p(z) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi} = \frac{1}{\rho}\frac{w}{\varphi} = p(\varphi)$$

- **In what follows, we index each firm by $\boldsymbol{\varphi}$.**
- The firm's revenue is

$$r(\varphi) = p(\varphi)y(\varphi)$$

$$= \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}y(\varphi) = \frac{1}{\rho}\frac{w}{\varphi}y(\varphi)$$

Profit

- The profit is

$$\pi(\varphi) = p(\varphi)y(\varphi) - w\left[f + \frac{y(\varphi)}{\varphi}\right]$$
$$= \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}y(\varphi) - \frac{w}{\varphi}y(\varphi) - wf$$
$$= \frac{1}{\sigma - 1}\frac{w}{\varphi}y(\varphi) - wf$$
$$= \frac{r(\varphi)}{\sigma} - wf$$

Productivity and Output

- From the input demand,

$$\frac{y(\varphi)}{Y} = \left[\frac{p(\varphi)}{P}\right]^{-\sigma}$$

- With $p(z) = \frac{1}{\rho}\frac{w}{\varphi}$,

$$\begin{aligned} y(\varphi) &= p(\varphi)^{-\sigma} Y P^{\sigma} \\ &= \left(\frac{1}{\rho}\frac{w}{\varphi}\right)^{-\sigma} \quad Y P^{\sigma} \\ &= w^{-\sigma}(\rho\varphi)^{\sigma} Y P^{\sigma} \\ &= w^{-\sigma}(P\rho\varphi)^{\sigma} Y \end{aligned}$$

Productivity and Output

- From $y(\varphi) = w^{-\sigma}(P\rho\varphi)^{\sigma}Y$, for firms with φ_1 and φ_2, we obtain

$$\frac{y(\varphi_1)}{y(\varphi_2)} = \frac{w^{-\sigma}(P\rho\varphi_1)^{\sigma}Y}{w^{-\sigma}(P\rho\varphi_2)^{\sigma}Y} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$$

- Thus,

$$\frac{y(\varphi_1)}{y(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$$

Productivity and Output

- Alternatively, from FOC (p. 8),

$$\frac{y(j)}{y(z)} = \left(\frac{p(z)}{p(j)}\right)^{\sigma}$$

- Suppose firm-j's productivity is φ_1 and firm-z's productivity is φ_2. Then,

$$\frac{y(\varphi_1)}{y(\varphi_2)} = \left(\frac{p(\varphi_2)}{p(\varphi_1)}\right)^{\sigma} = \left(\frac{\frac{1}{\rho}\frac{w}{\varphi_2}}{\frac{1}{\rho}\frac{w}{\varphi_1}}\right)^{\sigma} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$$

Productivity and Revenue

- From the input demand,

$$\frac{y(\varphi)}{Y} = \left[\frac{p(\varphi)}{P}\right]^{-\sigma}$$

- Thus, the revenue is

$$\begin{aligned} r(\varphi) = p(\varphi)y(\varphi) &= p(\varphi)^{1-\sigma} Y P^{\sigma} \\ &= \left(\frac{1}{\rho}\frac{w}{\varphi}\right)^{1-\sigma} Y P^{\sigma} \\ &= w^{1-\sigma}(\rho\varphi)^{\sigma-1} P^{\sigma-1} P Y \\ &= w^{1-\sigma}(P\rho\varphi)^{\sigma-1} P Y \end{aligned}$$

Productivity and Revenue

- From $r(\varphi) = w^{1-\sigma}(P\rho\varphi)^{\sigma-1}PY$, for firms with φ_1 and φ_2, we obtain

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \frac{w^{1-\sigma}(P\rho\varphi_1)^{\sigma-1}Y}{w^{1-\sigma}(P\rho\varphi_2)^{\sigma-1}Y} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

- Thus,

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

Aggregation

- Suppose that the **equilibrium** productivity distribution of firms is given by $\mu(\varphi)$. Then,

$$P = \left[\int_0^\infty p(\varphi)^{1-\sigma} n\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

- n is the mass of firms.
- Substitute $p(\varphi) = \frac{1}{\rho}\frac{w}{\varphi}$ into the price index:

$$P = \left[\int_0^\infty \left(\frac{1}{\rho}\frac{w}{\varphi}\right)^{1-\sigma} n\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

Aggregation

- Thus,

$$P = \left[n\rho^{\sigma-1} w^{1-\sigma} \int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi \right]^{\frac{1}{1-\sigma}}$$

$$= n^{\frac{1}{1-\sigma}} \rho^{-1} w \underbrace{\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi \right]^{\frac{1}{\sigma-1}}}_{\tilde{\varphi}^{-1}}$$

$$= n^{\frac{1}{1-\sigma}} \frac{w}{\rho\tilde{\varphi}}$$

$$= n^{\frac{1}{1-\sigma}} p(\tilde{\varphi})$$

Aggregation

- (7) in the paper is therefore

$$\tilde{\varphi} = \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$$

- This is a weighted average of the firm productivity levels for a distribution of operating firms.
- This is called the **average productivity**.
  - $\mu(\varphi)$ is to be determined.
- All aggregate variables can be expressed in terms of the average productivity.

Aggregation

- From the input demand on page 16,

$$y(z) = YP^{\sigma} p(z)^{-\sigma}$$

- Evaluate it at the aggregate productivity:

$$\begin{aligned} y(\tilde{\varphi}) &= YP^{\sigma} p(\tilde{\varphi})^{-\sigma} \\ &= Y \left[n^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) \right]^{\sigma} p(\tilde{\varphi})^{-\sigma} \\ &= n^{\frac{\sigma}{1-\sigma}} Y \end{aligned}$$

- Thus,

$$Y = n^{\frac{\sigma}{\sigma-1}} y(\tilde{\varphi})$$

Firm Entry and Exit

- There is a large pool of potential entrants.
- Each entrant pays a fixed entry cost $F > 0$ and finds out its productivity φ.
  - φ is drawn from a cumulative distribution function $G(\varphi)$ with density $g(\varphi)$ on $(0, \infty)$.
  - The firm's productivity is φ forever.
  - The firm receives $\pi(\varphi)$ in every period.
  - With probability δ, the firm is forced to exit.
    - Purely exogeneous shock.
- **Entry occurs before you learn about your type.**

Productivity Distribution

- Remember Q2 of HW1.
- Consider a **Pareto distribution** with parameters $s > 1$ and $\varphi_{\min} > 0$ on $[\varphi_{\min}, \infty)$.
  - s is called the shape parameter.
- Cumulative distribution function:

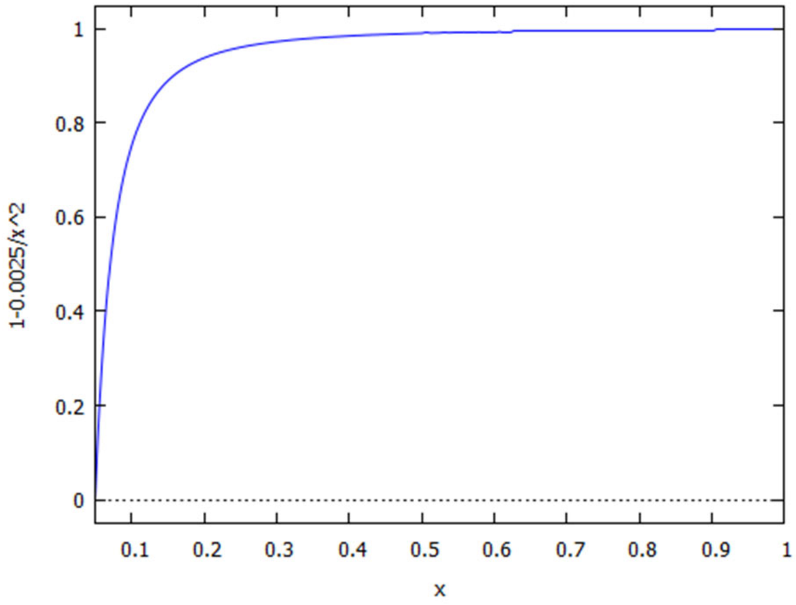
$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^s$$

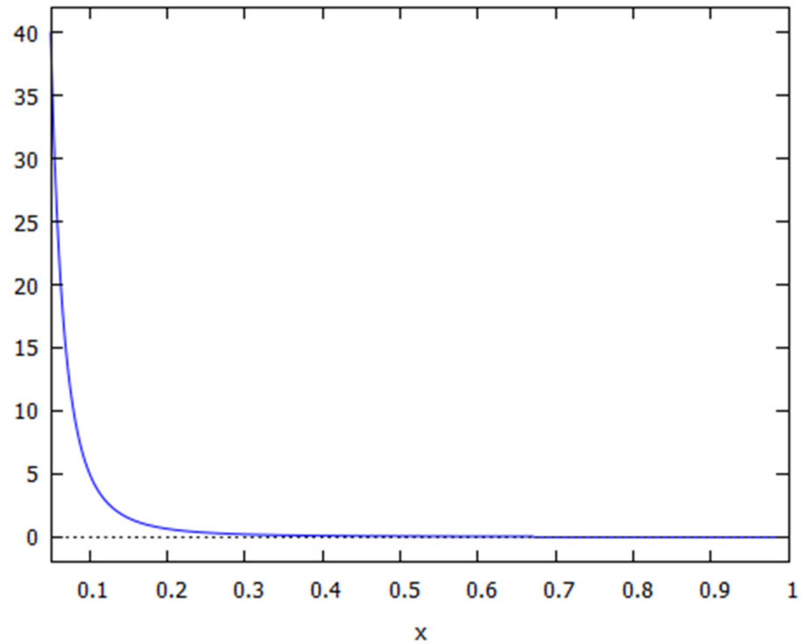
- Its density is

$$G'(\varphi) = g(\varphi) = s(\varphi_{\min})^s \varphi^{-s-1}$$

Productivity Distribution

$G(\varphi): k = 2, \varphi_{\min} = 0.05$ $g(\varphi): k = 2, \varphi_{\min} = 0.05$





Productivity Distribution

- To draw a cdf in Maxima, execute

$$\mathrm{plot2d([1-(0.05/x)^{\wedge}2],[x,0.05,1]);}$$

  - In this example, $s = 2$ and $\varphi_{\min} = 0.05$.
- Pareto distribution is often used to mimic the size distribution of firms and productivity distribution.
- An important property is that it is highly likely to draw a low-productivity outcome.

Cutoff Productivity

- φ^* : lowest productivity level of producing firms.
- Then, the exit cutoff is determined by
$$\pi(\varphi^*) = 0$$
- Given this cutoff, the productivity distribution of operating firms is
$$\mu(\varphi) = \frac{g(\varphi)}{1 - G(\varphi^*)}$$
on support $[\varphi^*, \infty)$.
- The probability of successful entry is $1 - G(\varphi^*)$

Firm Entry and Exit

- Each productivity type φ is realized only after entry (i.e., only after paying the entry fixed cost F).
- Each firm then determines whether to operate (with survival probability $1-\delta$) or to exit.
- The value of operation is

$$v(\varphi) = \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t \pi(\varphi)\right\}$$

$$= \max\{0, \delta^{-1}\pi(\varphi)\}$$

Firm Entry and Exit

- The value of entry v_e is

$$v_e = -F + \int_0^\infty v(\varphi) dG(\varphi)$$

$$= -F + \int_0^\infty \max\{0, \delta^{-1}\pi(\varphi)\} dG(\varphi)$$

$$= -F + \delta^{-1} \int_{\varphi^*}^\infty \pi(\varphi) g(\varphi) d\varphi$$

$$= -F + \delta^{-1}[1 - G(\varphi^*)] \int_{\varphi^*}^\infty \pi(\varphi)\mu(\varphi) d\varphi$$

Firm Entry and Exit

- Free entry of firms implies

$$v_e = 0 \Leftrightarrow \frac{\delta F}{1 - G(\varphi^*)} = \int_{\varphi^*}^{\infty} \pi(\varphi)\mu(\varphi)d\varphi \equiv \bar{\pi}$$

- The RHS is the average (=expected) profit conditional on successful entry.
- LHS of the free entry condition is:

$$\bar{\pi} = \frac{\delta F}{1 - G(\varphi^*)}$$

- This is (FE) in Melitz (2003).

Average Profit

- We now explore RHS of the free entry condition:

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi)\mu(\varphi)d\varphi$$

- The innovation in Melitz (2003) is to introduce the idea of the **average productivity** to summarize all aggregate variables as functions of the cutoff productivity.

Average Profit

- With the cutoff, the average productivity is

$$\tilde{\varphi} = \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$$

$$= \left[\int_{\varphi^*}^\infty \varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi^*)}d\varphi\right]^{\frac{1}{\sigma-1}}$$

$$= \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^\infty \varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \equiv \tilde{\varphi}(\varphi^*)$$

- This is (9) in Melitz (2003).

Average Profit

- Remember

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

- The relationship between the cutoff productivity and firm-φ is

$$\frac{r(\varphi)}{r(\varphi^*)} = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1}$$

$$\Leftrightarrow r(\varphi) = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} r(\varphi^*)$$

Average Profit

- From the expression on page 20, $\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf$.
- Firm at the cutoff earns zero profit:

$$\pi(\varphi^*) = 0 \Leftrightarrow \frac{r(\varphi^*)}{\sigma} = wf$$

- Thus, the profit for firm-φ is

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf = \frac{r(\varphi^*)}{\sigma}\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - wf$$

$$= wf\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

Average Profit

- Thus,

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi)\mu(\varphi)d\varphi$$

$$= \int_{\varphi^*}^{\infty} wf\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right]\mu(\varphi)d\varphi$$

$$= wf\left\{\left(\frac{1}{\varphi^*}\right)^{\sigma-1} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi - 1\right\}$$

$$= wf\left\{\left(\frac{1}{\varphi^*}\right)^{\sigma-1} \frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}g(\varphi)d\varphi - 1\right\}$$

Average Profit

- Finally, we obtain:

$$\bar{\pi} = wf\left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

- This is (ZCP) in Melitz (2003).
  - Melitz assumes $w = 1$.
- This condition is often called the **zero cutoff profit** condition in the literature. To me, this is only the RHS of the free entry condition.

Equilibrium

- ZCP:

$$\bar{\pi} = wf\left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

- FE:

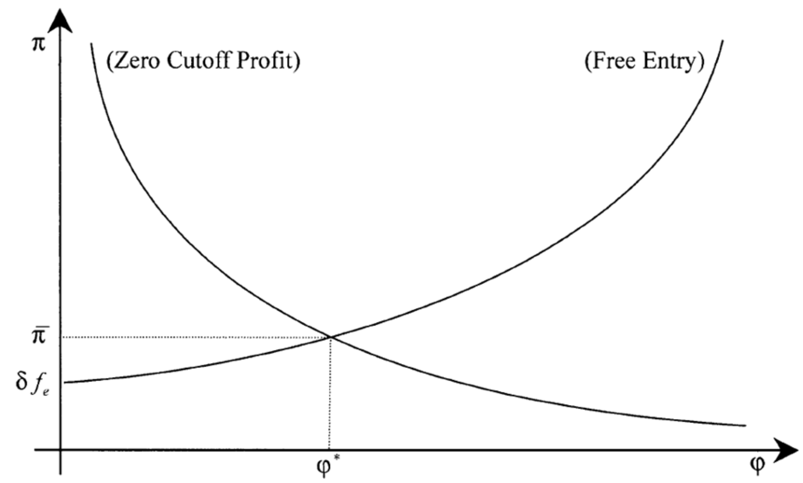
$$\bar{\pi} = \frac{\delta F}{1 - G(\varphi^*)}$$

- Average productivity:

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)}\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$$

Equilibrium

- ZCP implies a negative relationship between φ^* and $\bar{\pi}$:
$$\bar{\pi} = wf\left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$
- FE implies a positive relationship between φ^* and $\bar{\pi}$:
$$\bar{\pi} = \frac{\delta F}{1 - G(\varphi^*)}$$



Pareto Distribution Example

- With $G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^s$, (FE) is

$$\bar{\pi} = \frac{\delta F}{1 - G(\varphi^*)} = \left(\frac{\varphi^*}{\varphi_{\min}}\right)^s \delta F$$

- Evidently,

$$\frac{\partial \bar{\pi}}{\partial \varphi^*} > 0$$

Pareto Distribution Example

- With $G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^s$ and $g(\varphi) = s(\varphi_{\min})^s \varphi^{-s-1}$, we obtain:

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$$

$$= \left[s(\varphi^*)^s \int_{\varphi^*}^{\infty} \varphi^{\sigma-s-2} d\varphi\right]^{\frac{1}{\sigma-1}}$$

- Thus,

$$\tilde{\varphi}(\varphi^*)^{\sigma-1} = s(\varphi^*)^s \int_{\varphi^*}^{\infty} \varphi^{\sigma-s-2} d\varphi$$

Pareto Distribution Example

- Assume that $s > \sigma - 1$
- Then,

$$\int_{\varphi^*}^{\infty} \varphi^{\sigma-s-2} d\varphi = \left[\frac{1}{\sigma - s - 1}\varphi^{\sigma-s-1}\right]_{\varphi^*}^{\infty}$$

$$= \frac{1}{s - \sigma + 1}(\varphi^*)^{\sigma-1-s}$$

- Thus,

$$\tilde{\varphi}(\varphi^*)^{\sigma-1} = \frac{s}{s - \sigma + 1}(\varphi^*)^{\sigma-1}$$

Pareto Distribution Example

- Thus,

$$\frac{\tilde{\varphi}(\varphi^*)^{\sigma-1}}{(\varphi^*)^{\sigma-1}} = \frac{s}{s-\sigma+1}$$

- ZCP is therefore

$$\bar{\pi} = wf\left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

$$= wf\left(\frac{s}{s-\sigma+1} - 1\right) = \frac{wf(\sigma-1)}{s-\sigma+1}$$

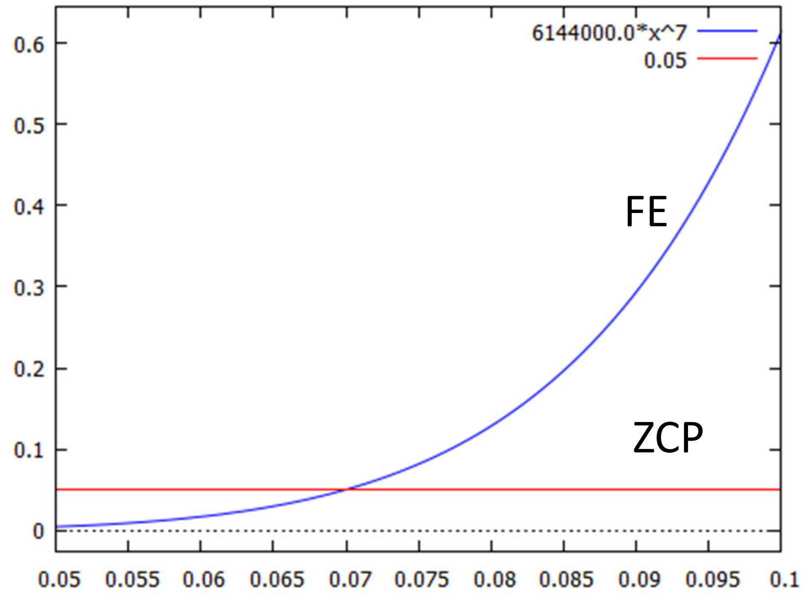
- This is due to the property of Pareto distribution.

Parameterization

- $\sigma = 6$ (the implies markup is $\frac{1}{\rho} = 1.2$)
- $\delta = 0.008$ (from Ebell and Haefke)
- $F = 0.6$ (from Ebell and Haefke)
- $\varphi_{\min} = 0.05$
- $s = 7$ (satisfies $s > \sigma - 1$)
- $w = 1$ (from Melitz)
- $f = 0.02$

Parameterization

- ZCP:
$$\bar{\pi} = \frac{wf(\sigma - 1)}{s - \sigma + 1} = 0.05$$
- FE:
$$\bar{\pi} = \left(\frac{\varphi^*}{\varphi_{\min}}\right)^s \delta F$$
$$= \left(\frac{\varphi^*}{0.05}\right)^7 0.008 \times 0.6$$



Reading Articles Like Professionals

- Remember that I am not following all equations in Melitz as presented.
  - Of course, for the first time, I read the paper line by line with paper and pencil.
- I accept and borrow all assumptions in the article, but I solve the model myself as if the model is mine.
  - When the math looks too general, I specify functional forms as I like (and parameter values) to draw diagrams.
- This is **replication**.
- Replication is an important academic skill.

Entry of Heterogeneous Firms and Job Creation

Felbermayr and Prat, “Product Market Regulation, Firm Selection, and Unemployment” *Journal of the European Economic Association*, 2011.

The Basic Idea

- We now introduce the DMP component into the Melitz model.
- In other words, we introduce firm heterogeneity in the model of Ebell and Haefke.
- Using the model, we shall ask whether firm heterogeneity can explain wage inequality.

No Scale Effects

- Aggregate output in Melitz:

$$Y = \left(\int_{z \in Z} y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- Aggregate output in Felbermayr and Prat is

$$Y = \left(n^{-\frac{1}{\sigma}} \int_{z \in Z} y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- The price index is thus

$$P = \left[\frac{1}{n} \int_{z \in Z} p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$$

Homogeneous Firms

- The value of a firm:

$$J(\ell) = \max_{v} \left\{ py - w(\ell)\ell - cv + \frac{1}{1+r} J(\ell_{+1}) \right\}$$

$$s.t.$$

$$\frac{y(z)}{Y} = n^{-1} \left[\frac{p(z)}{P} \right]^{-\sigma}$$

$$y = \varphi \ell$$

$$\ell_{+1} = (1-\lambda)\ell + q(\theta)v$$

Heterogeneous Firms

- The Bellman equation:

$$J(\ell,\varphi) = \max_{v}\left\{py - w(\ell)\ell - cv - f + \frac{1-\delta}{1+r}J(\ell_{+1},\varphi)\right\}$$

$s.t.$

$$\frac{y(z)}{Y} = n^{-1}\left[\frac{p(z)}{P}\right]^{-\sigma}, y = \varphi\ell, \ell_{+1} = (1-\lambda)\ell + q(\theta)v$$

- The value function depends on φ.
- All variables such as ℓ and v depend on φ.
- $f > 0$: fixed cost of operation (Necessary for ZCP).
- $0 < \delta < 1$: sudden death probability.

Exogenous Exit of Firms

- When heterogeneous firm enter the market in each period, the shape of the productivity distribution of operating firms may **evolve over time**.
- All firms are assumed to face the same sudden death probability $\delta > 0$.
- This will trim the upper tail of the distribution to help maintain the shape of the productivity distribution.

Exogenous Exit of Firms

- With the sudden death probability, the discount factor should be modified to include the survival probability:

$$\frac{1-\delta}{1+r} = \frac{1}{1+\frac{r+\delta}{1-\delta}} = \frac{1}{1+\tilde{r}}$$

- The effective discount rate $\tilde{r}$ increases as δ increases.
- In what follows, we use $\tilde{r}$.
- The unemployed discounts future using r.

Homogeneous Firms

- Re member the model of the previous lecture.
- The job-creation condition:

$$w(\ell) = \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} - \frac{(r + \lambda)c}{q(\theta)}$$

- The wage equation:

$$w(\ell) = \beta \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} + (1 - \beta)b + \beta c\theta$$

- With homogeneous firms,

$$\Phi = P n^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} \varphi^{\frac{\sigma - 1}{\sigma}} = \varphi \ell^{\frac{1}{\sigma}}$$

Homogeneous Firms

- The job-creation condition:

$$w = \frac{\sigma - 1}{\sigma - \beta}\varphi - \frac{(r + \lambda)c}{q(\theta)}$$

- The wage equation:

$$w = \beta\frac{\sigma - 1}{\sigma - \beta}\varphi + (1 - \beta)b + \beta c\theta$$

- Worker flows:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

$$u = 1 - n\ell$$

Heterogeneous Firms

- Now consider heterogeneous firms.
- The job-creation condition:

$$w(\ell, \varphi) = \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} - \frac{(\tilde{r} + \lambda)c}{q(\theta)}$$

- The wage equation:

$$w(\ell, \varphi) = \beta \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} + (1 - \beta)b + \beta c \theta$$

- $\Phi = Pn^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} \varphi^{\frac{\sigma - 1}{\sigma}}$ implies that these equations depend on φ. Do they imply wage inequality?

Heterogeneous Firms

- Eliminate Φ from these equations to obtain
$$(1-\beta)w(\ell,\varphi) = \beta\frac{(\tilde{r}+\lambda)c}{q(\theta)} + (1-\beta)b + \beta c\theta$$
- This implies that $w(\ell,\varphi)$ is **independent** of φ.
- In other words, **firm heterogeneity itself cannot cause wage inequality**.
- The wage rate is also independent of ℓ.
- We shall reduce the notation as
$$w(\ell,\varphi) = w$$

Heterogeneous Firms

- Now, eliminate w from the job-creation condition and the wage equation to obtain
$$(1-\beta)\frac{\sigma-1}{\sigma-\beta}\Phi\ell^{\frac{-1}{\sigma}}=\frac{(\tilde{r}+\lambda)c}{q(\theta)}+(1-\beta)b+\beta c\theta$$
- $\Phi=Pn^{-\frac{1}{\sigma}}Y^{\frac{1}{\sigma}}\varphi^{\frac{\sigma-1}{\sigma}}$ implies
$$(1-\beta)\frac{\sigma-1}{\sigma-\beta}Pn^{-\frac{1}{\sigma}}Y^{\frac{1}{\sigma}}\varphi^{\frac{\sigma-1}{\sigma}}\ell^{\frac{-1}{\sigma}}$$
$$=\frac{(\tilde{r}+\lambda)c}{q(\theta)}+(1-\beta)b+\beta c\theta$$

Heterogeneous Firms

- Then,

$$[y(\varphi)]^{\frac{-1}{\sigma}} = \frac{\frac{(\tilde{r}+\lambda)c}{q(\theta)} + (1-\beta)b + \beta c\theta}{(1-\beta)\frac{\sigma-1}{\sigma-\beta}Pn^{-\frac{1}{\sigma}}Y^{\frac{1}{\sigma}}\varphi}$$
$$= \varphi^{-1}\Omega(\theta)$$

- From the inverse demand function,

$$p(\varphi) = P\left[\frac{ny(\varphi)}{Y}\right]^{-\frac{1}{\sigma}} = \varphi^{-1}\Omega(\theta)P\left[\frac{n}{Y}\right]^{-\frac{1}{\sigma}}$$

Heterogeneous Firms

- Substitute $\Omega(\theta)$ back into the equation to obtain

$$p(\varphi) = \frac{\sigma - \beta}{\sigma - 1}\varphi^{-1}\frac{\frac{(\tilde{r}+\lambda)c}{q(\theta)} + (1-\beta)b + \beta c\theta}{1-\beta}$$

$$= \frac{\sigma}{\sigma - 1}\frac{\sigma - \beta}{\sigma}\varphi^{-1}\left[w + \frac{(\tilde{r}+\lambda)c}{q(\theta)}\right]$$

- In Melitz, the price is

$$p(\varphi) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi} = \frac{1}{\rho}\frac{w}{\varphi}$$

- The two equations are the same when:
  - $c = 0$: labor market frictions disappear.
  - $\beta = 0$: worker's bargaining power is zero

Aggregation

- The price index satisfies

$$P = \left[\frac{1}{n}\int_{z\in Z} p(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}$$
$$= \left[\frac{1}{n}\int_{0}^{\infty} p(\varphi)^{1-\sigma} n\mu(\varphi) d\varphi\right]^{\frac{1}{1-\sigma}}$$

- n is the mass of all operating firms.
- $p(\varphi) = \frac{\sigma}{\sigma-1}\frac{\sigma-\beta}{\sigma}\varphi^{-1}\left[w + \frac{(\tilde{r}+\lambda)c}{q(\theta)}\right] = \varphi^{-1}\omega(\theta)$, where

$$\omega(\varphi) = \frac{\sigma}{\sigma - 1}\frac{\sigma - \beta}{\sigma}\left[w + \frac{(\tilde{r} + \lambda)c}{q(\theta)}\right]$$

Aggregation

- Substitute $p(\varphi) = \varphi^{-1}\omega(\theta)$ into the price index:

$$P = \left[\frac{1}{n}\int_0^\infty p(\varphi)^{1-\sigma} n\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

$$= \left[\frac{1}{n}\int_0^\infty [\varphi^{\sigma-1}\omega(\theta)^{1-\sigma} n]\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

$$= \left[\omega(\theta)^{1-\sigma}\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

$$= \omega(\theta)\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

Aggregation

- Consider

$$P = \omega(\theta)\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

- As in Melitz, let us define the **average productivity**:

$$\tilde{\varphi} = \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$$

- Then, we obtain

$$P = \tilde{\varphi}^{-1}\omega(\theta) = p(\tilde{\varphi})$$

- In Melitz, $P = n^{\frac{1}{1-\sigma}}p(\tilde{\varphi})$ holds because of scale effects.

Aggregation

- $p(\varphi) = \varphi^{-1}\omega(\theta)$ and $p(\tilde{\varphi}) = \tilde{\varphi}^{-1}\omega(\theta)$ imply

$$\frac{p(\varphi)}{p(\tilde{\varphi})} = \frac{\varphi^{-1}}{\tilde{\varphi}^{-1}}$$
$$\Leftrightarrow$$
$$p(\varphi)\varphi = p(\tilde{\varphi})\tilde{\varphi}$$

Aggregation

- Substitute $\omega(\theta)$ into $P = \tilde{\varphi}^{-1}\omega(\theta)$ to obtain
$$P = \tilde{\varphi}^{-1}\frac{\sigma-\beta}{\sigma-1}\left[w + \frac{(\tilde{r}+\lambda)c}{q(\theta)}\right]$$
$\Leftrightarrow$
$$\frac{\sigma-1}{\sigma-\beta}P\tilde{\varphi} = w + \frac{(\tilde{r}+\lambda)c}{q(\theta)}$$
$\Leftrightarrow$
$$(1-\beta)\frac{\sigma-1}{\sigma-\beta}P\tilde{\varphi} = \frac{(\tilde{r}+\lambda)c}{q(\theta)} + (1-\beta)b + \beta c\theta$$
- As in Ebell and Haefke, $P = 1$.

Aggregation

- Thus, the job creation condition (with the wage equation) becomes
$$(1-\beta)\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}=\frac{(\tilde{r}+\lambda)c}{q(\theta)}+(1-\beta)b+\beta c\theta$$
- Equilibrium tightness θ depends on the average productivity.
- In Ebell and Haefke (p91 of the previous lecture),
$$(1-\beta)\frac{\sigma-1}{\sigma-\beta}\varphi=\frac{(r+\lambda)c}{q(\theta)}+\beta c\theta+(1-\beta)b$$

Aggregation

- Similarly, the job-creation condition (using w) is

$$w = \frac{\sigma - 1}{\sigma - \beta}\tilde{\varphi} - \frac{(\tilde{r} + \lambda)c}{q(\theta)}$$

- In Ebell and Haefke (p.80 of the previous lecture), it is

$$w = \frac{\sigma - 1}{\sigma - \beta}\varphi - \frac{(r + \lambda)c}{q(\theta)}$$

Aggregation

- The wage rate:

$$(1-\beta)w = \beta\frac{(\tilde{r}+\lambda)c}{q(\theta)} + (1-\beta)b + \beta c\theta$$

- Use the job creation condition to rewrite it as

$$w = \beta\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi} + (1-\beta)b + \beta c\theta$$

- In Ebell and Haefke (p.80 of the previous lecture),

$$w = \beta\frac{\sigma-1}{\sigma-\beta}\varphi + (1-\beta)b + \beta c\theta$$

Aggregation

- Let us compare the wage equations:

$$w(\ell,\varphi)=\beta\frac{\sigma-1}{\sigma-\beta}\Phi\ell^{\frac{-1}{\sigma}}+(1-\beta)b+\beta c\theta$$

$$w=\beta\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}+(1-\beta)b+\beta c\theta$$

- Because $py=\Phi\ell^{\frac{\sigma-1}{\sigma}}$, these equations imply

$$\frac{R(\varphi)}{\ell(\varphi)}=\Phi\ell^{\frac{-1}{\sigma}}=\tilde{\varphi}$$

- Thus, the revenue per worker is the same for all firms.

Aggregation

- Remember

$$\Phi = Pn^{-\frac{1}{\sigma}}Y^{\frac{1}{\sigma}}\varphi^{\frac{\sigma-1}{\sigma}}$$

- Thus, the revenue per employee is

$$\frac{R(\varphi)}{\ell(\varphi)} = \Phi\ell^{\frac{-1}{\sigma}} = Pn^{-\frac{1}{\sigma}}Y^{\frac{1}{\sigma}}\varphi^{\frac{\sigma-1}{\sigma}}\ell^{\frac{-1}{\sigma}} = \tilde{\varphi}$$

- Thus, for firms with φ_1 and φ_2,

$$\frac{\ell(\varphi_1)}{\ell(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

Aggregation

- From the input demand function,

$$\frac{y(\varphi)}{Y} = n^{-1}\left[\frac{p(\varphi)}{P}\right]^{-\sigma}$$

$$\Leftrightarrow \frac{\varphi\ell(\varphi)}{Y} = n^{-1}\left[\frac{\varphi^{-1}\omega(\theta)}{P}\right]^{-\sigma}$$

- Thus, for firms with φ_1 and φ_2,

$$\frac{\ell(\varphi_1)}{\ell(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

Firm Entry

- As in Melitz, each entrant must pay $F > 0$ to find out its productivity φ.
- The value of entry is $J(0, \varphi)$.
- After learning about φ, firms with $\varphi < \varphi^*$ exit.
- Free entry implies

$$-F + \int_0^\infty J(0, \varphi)dG(\varphi) = 0$$

- The exit cutoff φ^* satisfies

$$J(0, \varphi^*) = 0$$

Firm Entry

- Free entry implies

$$0 = -F + \int_0^\infty J(0,\varphi)dG(\varphi)$$
$$= -F + \int_0^\infty J(0,\varphi)dG(\varphi)$$
$$= -F + \int_{\varphi^*}^\infty J(0,\varphi)g(\varphi)d\varphi$$
$$= -F + [1 - G(\varphi^*)] \int_{\varphi^*}^\infty J(0,\varphi)\mu(\varphi)d\varphi$$

Firm Entry

- Thus,

$$\frac{F}{1-G(\varphi^*)} = \int_{\varphi^*}^{\infty} J(0,\varphi)\mu(\varphi)d\varphi$$

$$\Leftrightarrow$$

$$\frac{\tilde{r}F}{1-G(\varphi^*)} = \int_{\varphi^*}^{\infty} \tilde{r}J(0,\varphi)\mu(\varphi)d\varphi = \Pi$$

- We need to find $J(0,\varphi)$.
    - More specifically, we wish to rewrite more explicitly as a function of φ.

Firm Entry

- With homogeneous firms, the value of entry is

$$J(0) = -c\frac{\ell_{+1}}{q(\theta)} + \frac{1}{1+r}J(\ell_{+1})$$

- With heterogeneous firms,

$$J(0,\varphi) = -c\frac{\ell_{+1}}{q(\theta)} - f + \frac{1}{1+\tilde{r}}J(\ell_{+1},\varphi)$$

Firm Entry

- In any steady state,

$$J(0,\varphi) = -c\frac{\ell(\varphi)}{q(\theta)} - f + \frac{1}{1+\tilde{r}}J(\ell,\varphi)$$

- From the Bellman equation,

$$J(\ell,\varphi) = py - w(\ell)\ell - cv - f + \frac{1}{1+\tilde{r}}J(\ell,\varphi)$$

$$\ell = (1-\lambda)\ell + q(\theta)v$$

- Eliminate v to obtain

$$\frac{\tilde{r}}{1+\tilde{r}}J(\ell,\varphi) = py - w(\ell)\ell - \frac{c\lambda}{q(\theta)}\ell - f$$

Firm Entry

- In any steady state,

$$\tilde{r}J(0,\varphi) = -\frac{(\tilde{r}+\lambda)c}{q(\theta)}\ell(\varphi) - \tilde{r}f + R(\varphi) - w\ell(\varphi) - f$$

$$= \left[\tilde{\varphi} - w - \frac{(\tilde{r}+\lambda)c}{q(\theta)}\right]\ell(\varphi) - (1+\tilde{r})f$$

$$= \frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi) - (1+\tilde{r})f$$

- Observe:

$$w = \frac{\sigma-1}{\sigma-\beta}\tilde{\varphi} - \frac{(\tilde{r}+\lambda)c}{q(\theta)}$$

Firm Entry

- Thus, the value of successful entry is

$$\tilde{r}J(0,\varphi) = \frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi) - (1+\tilde{r})f$$

- Now, for firms with φ and φ^*,

$$\frac{\ell(\varphi)}{\ell(\varphi^*)} = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1}$$

- Thus,

$$\tilde{r}J(0,\varphi) = \frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi^*)\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - (1+\tilde{r})f$$

Firm Entry

- The exit cutoff φ^* satisfies

$$J(0, \varphi^*) = 0$$

- Thus,

$$\tilde{r}J(0, \varphi^*) = \frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi^*) - (1+\tilde{r})f = 0$$

$$\Leftrightarrow$$

$$\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi^*) = (1+\tilde{r})f$$

Firm Entry

- Thus,

$$\Pi = \int_{\varphi^*}^{\infty} \tilde{r} J(0,\varphi)\mu(\varphi)d\varphi$$

$$= \int_{\varphi^*}^{\infty} \left[\frac{1-\beta}{\sigma-\beta} \tilde{\varphi}\ell(\varphi^*) \left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - (1+\tilde{r})f \right] \mu(\varphi)d\varphi$$

$$= \frac{1-\beta}{\sigma-\beta} \tilde{\varphi}\ell(\varphi^*) \left(\frac{1}{\varphi^*} \right)^{\sigma-1} \underbrace{\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi}_{\tilde{\varphi}^{\sigma-1}} - (1+\tilde{r})f$$

$$= \frac{1-\beta}{\sigma-\beta} \tilde{\varphi}\ell(\varphi^*) \left(\frac{\tilde{\varphi}}{\varphi^*} \right)^{\sigma-1} - (1+\tilde{r})f$$

Firm Entry

- Now,

$$\Pi = \frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi^*)\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - (1+\tilde{r})f$$

$$\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\ell(\varphi^*) = (1+\tilde{r})f$$

- They jointly imply

$$\Pi = (1+\tilde{r})f\left[\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

Firm Entry

- With the cutoff, the average productivity is

$$\tilde{\varphi} = \left[\int_0^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$$

$$= \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi^*)}d\varphi\right]^{\frac{1}{\sigma-1}}$$

$$= \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \equiv \tilde{\varphi}(\varphi^*)$$

Steady State Equilibrium

- ZCP:

$$\Pi = (1 + \tilde{r})f\left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

- FE:

$$\Pi = \frac{\tilde{r}F}{1 - G(\varphi^*)}$$

- Average Productivity:

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)}\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$$

Equilibrium (Melitz)

- ZCP:

$$\bar{\pi} = wf\left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

- FE:

$$\bar{\pi} = \frac{\delta F}{1 - G(\varphi^*)}$$

- Average productivity:

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)}\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$$

Steady State Equilibrium

- ZCP and FE jointly determine φ^* and hence $\tilde{\varphi}$.
  - Note that they are independent of the labor market variables.
  - Labor market conditions do not influence firm entry.
- Given the equilibrium level of $\tilde{\varphi}$, the job-creation condition and the wage rate are:

$$w = \frac{\sigma - 1}{\sigma - \beta}\tilde{\varphi} - \frac{(\tilde{r} + \lambda)c}{q(\theta)}$$

$$w = \beta\frac{\sigma - 1}{\sigma - \beta}\tilde{\varphi} + (1 - \beta)b + \beta c\theta$$

Further Readings

- Felbermayr and Prat, “Product Market Regulation, Firm Selection, and Unemployment” *Journal of the European Economic Association*, 2011.
- Tanaka, “Technological Progress, Firm Selection, and Unemployment,” *Economics Bulletin*, 2018.
  - The Felbermayr-Prat model + technological progress.
  - Part of his doctoral dissertation submitted to Nagoya University.

Reading Assignment

- Helpman, Itskhoki, and Redding, “Inequality and Unemployment in a Global Economy” *Econometrica*, 2010.