

Lecture 11

Firm Entry

7/7

Perfect Competition

- The profit is

$$\Pi = AK^{\alpha}L^{1-\alpha} - rK - wL$$

- We substitute the FOCs into the above

$$MPK = \alpha AK^{\alpha-1}L^{1-\alpha} = r$$

$$MPL = (1 - \alpha)AK^{\alpha}L^{-\alpha} = w$$

- We can show that

$$\Pi = 0$$

- Under perfect competition (with constant-returns-to-scale production function), firms always earn zero profit in equilibrium.

Implications for Firm Entry

- What does zero profit imply?
- For any potential entrant and any incumbent, it is **indifferent** between entry and exit.
- As a result, there is no mechanism determining the number of firms.
 - In other words, we cannot study how firm entry promotes job creation.

Goals

- Last week, we studied a version of the Dixit-Stiglitz model of monopolistic competition.
- Today, we extend the model to include the determination of the number of firms.

Monopolistic Competition with Firm Entry

Matsuyama, Kiminori. “Complementarities and cumulative processes in models of monopolistic competition.” *Journal of Economic Literature* (1995), Section 3A (The Basic Model)

Without Entry

- Consider the model in the previous lecture.
- Let $y(z)$ denote the input of variety z .
- Production function:

$$Y = \left(\int_0^1 y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 1$ is the elasticity of substitution between any two varieties.
- The number of monopolists is normalized by 1.

With Entry

- Let us now consider entry.
- Let n denote the **number of input variety**.
- Let $y(z)$ denote the input of variety z .

- Production function:

$$Y = \left(\int_0^n y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 1$ is the elasticity of substitution between any two varieties.

Intermediate Goods

- Let $p(z)$ denote the input price of variety z .
- The input demand minimizes total expenditure:

$$\begin{aligned} & \min_{y(z)} \int_0^n p(z)y(z)dz \\ & s. t. \\ & Y = \left(\int_0^n y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Intermediate Goods

- The Lagrangian is

$$\int_0^n p(z)y(z)dz + \lambda \left[Y - \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right]$$

- FOC for variety z is

$$p(z) = \lambda \frac{\sigma}{\sigma-1} \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} y(z)^{\frac{\sigma-1}{\sigma}-1}$$

- FOC for variety j is

$$p(j) = \lambda \frac{\sigma}{\sigma-1} \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} y(j)^{\frac{\sigma-1}{\sigma}-1}$$

Intermediate Goods

- Taking the ratio:

$$\frac{p(z)}{p(j)} = \frac{y(z)^{\frac{\sigma-1}{\sigma}-1}}{y(j)^{\frac{\sigma-1}{\sigma}-1}} = \left(\frac{y(z)}{y(j)}\right)^{\frac{-1}{\sigma}}$$

- Thus,

$$y(j) = \left(\frac{p(z)}{p(j)}\right)^{\sigma} y(z) = p(z)^{\sigma} p(j)^{-\sigma} y(z)$$

- Thus,

$$y(j)^{\frac{\sigma-1}{\sigma}} = p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}}$$

Intermediate Goods

- Substitute it into the production function:

$$\begin{aligned} Y &= \left(\int_0^n y(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_0^n \left[p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}} \right] dj \right)^{\frac{\sigma}{\sigma-1}} \\ &= p(z)^{\sigma} y(z) \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Intermediate Goods

- We obtain

$$y(z) = Yp(z)^{-\sigma} \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{1-\sigma}}$$

- This is the input demand function for variety z .
- We can simplify it further...

Intermediate Goods

- Now let us use $y(j) = p(z)^\sigma p(j)^{-\sigma} y(z)$ to rewrite the expenditures:

$$\int_0^n p(j)y(j)dj = y(z)p(z)^\sigma \int_0^n p(j)^{1-\sigma} dj$$

- Perfect competition in the final-goods market implies zero profit: $PY - \int_0^n p(j)y(j)dj = 0$

- Thus,

$$PY = y(z)p(z)^\sigma \int_0^n p(j)^{1-\sigma} dj$$

Intermediate Goods

- Solve it for $y(z)$ as

$$y(z) = PY \left[\int_0^n p(j)^{1-\sigma} dj \right]^{-1} p(z)^{-\sigma}$$

- Substitute it into the production function:

$$Y = PY \left[\int_0^n p(j)^{1-\sigma} dj \right]^{-1} \left(\int_0^n p(z)^{1-\sigma} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- It simplifies to

$$1 = P \left[\int_0^n p(j)^{1-\sigma} dj \right]^{\frac{1}{\sigma-1}}$$

Intermediate Goods

- Finally,

$$P = \left[\int_0^n p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}$$

- RHS is the **price index** of the intermediate goods.

Intermediate Goods

- The input demand is therefore

$$\begin{aligned} y(z) &= Yp(z)^{-\sigma} \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{1-\sigma}} \\ &= Yp(z)^{-\sigma} P^{\sigma} \end{aligned}$$

- Thus,

$$\frac{y(z)}{Y} = \left[\frac{p(z)}{P} \right]^{-\sigma}$$

- Relative demand for input z is decreasing in the relative price of the input.

Intermediate Goods

- Production requires labor input $\ell(z)$:
$$y(z) = \max\{0, \varphi[\ell(z) - f]\}$$
- φ is the marginal product of labor (MPL).
- f is the **overhead cost** of production.
 - e.g.) I spend so much time on course preparation!
- The labor units needed to produce $y(z)$ is

$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Intermediate Goods

- Let w denote the wage rate. The profit is
$$\pi(z) = p(z)y(z) - w\ell(z)$$
- We assume perfectly competitive labor market.
- Input demand function:

$$\frac{y(z)}{Y} = \left[\frac{p(z)}{P} \right]^{-\sigma} \Leftrightarrow y(z) = Y P^{\sigma} p(z)^{-\sigma}$$

- Production technology:

$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Markup Pricing

- Profit:

$$\begin{aligned}\pi(z) &= p(z)y(z) - w\ell(z) \\ &= p(z)y(z) - w \left[f + \frac{y(z)}{\frac{\varphi}{w}} \right] \\ &= YP^\sigma p(z)^{1-\sigma} - wf - \frac{w}{\varphi} YP^\sigma p(z)^{-\sigma}\end{aligned}$$

- FOC with respect to $p(z)$:

$$\begin{aligned}p(z) &= \frac{\frac{\sigma}{\sigma-1} w}{\varphi} \\ &= \textit{markup} \times \frac{w}{MPL}\end{aligned}$$

Price Index

- Thus,

$$\begin{aligned} P &= \left[\int_0^n \left[\frac{\sigma}{\sigma-1} \frac{w}{\varphi} \right]^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \\ &= \left[n \left[\frac{\sigma}{\sigma-1} \frac{w}{\varphi} \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1} \frac{w}{\varphi} n^{\frac{1}{1-\sigma}} \end{aligned}$$

- P is decreasing in n (because $\sigma > 1$).

Aggregate Output

- Because $p(z)$ is the same for all intermediate-good firms, the input $y(z)$ is the same for all z .
- Thus, the aggregate production satisfies

$$Y = \left(\int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} = y n^{\frac{\sigma}{\sigma-1}}$$

- Because ny units of intermediate goods are used, the aggregate productivity is

$$\frac{Y}{ny} = n^{\frac{1}{\sigma-1}}$$

Revenue

- Input demand:

$$\begin{aligned}y(z) &= Y P^\sigma p(z)^{-\sigma} \\&= Y \left[\frac{\sigma}{\sigma - 1} \frac{w}{\varphi} n^{\frac{1}{1-\sigma}} \right]^\sigma \left[\frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \right]^{-\sigma} \\&= Y n^{\frac{\sigma}{1-\sigma}}\end{aligned}$$

- The firm's revenue is

$$r(z) = p(z)y(z) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} y(z)$$

Profit

- The profit is

$$\begin{aligned}\pi(z) &= p(z)y(z) - w \left[f + \frac{y(z)}{\varphi} \right] \\ &= \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} y(z) - \frac{w}{\varphi} y(z) - wf \\ &= \frac{1}{\sigma - 1} \frac{w}{\varphi} y(z) - wf \\ &= \frac{r(z)}{\sigma} - wf\end{aligned}$$

Closing the Model

- Households supply L units of labor.
- There are n firms, and each employs ℓ workers.
- Thus,

$$L = n\ell = n \left[f + \frac{y}{\varphi} \right]$$

- Thus,

$$\frac{y}{\varphi} = \frac{L}{n} - f$$

Firm Entry

- The profit is

$$\begin{aligned}\pi(z) &= \frac{1}{\sigma - 1} \frac{w}{\varphi} y(z) - wf \\ &= \frac{1}{\sigma - 1} w \left[\frac{L}{n} - f \right] - wf\end{aligned}$$

- Thus, $\pi(z) \geq 0$ if and only if

$$\frac{1}{\sigma - 1} \left[\frac{L}{n} - f \right] \geq f \Leftrightarrow \frac{\mu}{1 + \mu n} \frac{L}{n} \geq f$$

- This is the condition for firm entry.

Equilibrium Number of Firms

- The free entry condition:

$$\begin{aligned}\pi(z) = 0 &\Leftrightarrow \frac{\mu}{1 + \mu n} \frac{L}{n} = f \\ &\Leftrightarrow n = \frac{\mu}{1 + \mu f} \frac{L}{f}\end{aligned}$$

- Thus, the equilibrium number of firms is
 - Increasing in the markup rate μ .
 - Decreasing in the overhead fixed cost f .
- $L = n\ell$ implies that an increasing in n decreases ℓ , which decreases y and increases p .

Externality

- The aggregate production:

$$Y = \left(\int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} = y n^{\frac{\sigma}{\sigma-1}}$$

- Because ny units of intermediate goods are used, the aggregate productivity is

$$\frac{Y}{ny} = n^{\frac{1}{\sigma-1}}$$

- Firm entry causes **externality**, or **scale effects**.
- Models of international trade tend to have it.

Eliminating the Scale Effects

- In many labor market applications, we work with models without the scale effects.
- To do so, we usually **discount** the production function by the scale component:

$$Y = n^{\frac{-1}{\sigma-1}} \left(\int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} = \left(n^{-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \left(\int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$
$$= \left(n^{-\frac{1}{\sigma}} \int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

Eliminating the Scale Effects

The specific assumptions are as follows.

I.A. Workers

There are L workers/consumers, indexed by j . In each period, worker j has a utility function given by

$$(1) \quad V_j = \left[m^{-1/\sigma} \sum_{i=1}^m C_{ij}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)},$$

2.1.1. Monopolistic competition in the goods market

Households are both consumers and workers. As consumers they are risk neutral in the aggregate consumption good. Agents have Dixit-Stiglitz preferences over a continuum of differentiated goods. We use Blanchard and Giavazzi (2003)'s formulation, which allows us to connect demand elasticity σ to the number of firms n , while also allowing us to focus on the direct effects of increased competition on the demand elasticity facing firms.⁴ Goods demand each period is derived from the household's optimization problem:

$$\max \left(n^{-\frac{1}{\sigma}} \int c_{i,j}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

Eliminating the Scale Effects

- Let $p(z)$ denote the input price of variety z .
- The input demand minimizes total expenditure:

$$\min_{y(z)} \int_0^n p(z)y(z)dz$$

s. t.

$$Y = \frac{1}{N} \left(\int_0^n y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- For now, Let us define the scale component as

$$n^{\frac{1}{\sigma-1}} = N$$

Eliminating the Scale Effects

- The Lagrangian is

$$\int_0^n p(z)y(z)dz + \lambda \left[Y - \frac{1}{N} \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right]$$

- FOC for variety z is

$$p(z) = \lambda \frac{1}{N} \frac{\sigma}{\sigma-1} \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} y(z)^{\frac{\sigma-1}{\sigma}-1}$$

- FOC for variety j is

$$p(j) = \lambda \frac{1}{N} \frac{\sigma}{\sigma-1} \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} y(j)^{\frac{\sigma-1}{\sigma}-1}$$

Eliminating the Scale Effects

- Taking the ratio:

$$\frac{p(z)}{p(j)} = \frac{y(z)^{\frac{\sigma-1}{\sigma}-1}}{y(j)^{\frac{\sigma-1}{\sigma}-1}} = \left(\frac{y(z)}{y(j)}\right)^{\frac{-1}{\sigma}}$$

- Thus,

$$y(j) = \left(\frac{p(z)}{p(j)}\right)^{\sigma} y(z) = p(z)^{\sigma} p(j)^{-\sigma} y(z)$$

- Thus,

$$y(j)^{\frac{\sigma-1}{\sigma}} = p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}}$$

Eliminating the Scale Effects

- Substitute it into the production function:

$$\begin{aligned} Y &= \frac{1}{N} \left(\int_0^n y(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \\ &= \frac{1}{N} \left(\int_0^n \left[p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}} \right] dj \right)^{\frac{\sigma}{\sigma-1}} \\ &= \frac{1}{N} p(z)^{\sigma} y(z) \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Eliminating the Scale Effects

- We obtain

$$y(z) = YNp(z)^{-\sigma} \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{1-\sigma}}$$

- This is the input demand function for variety z .
- We can simplify it further...

Eliminating the Scale Effects

- Now let us use $y(j) = p(z)^\sigma p(j)^{-\sigma} y(z)$ to rewrite the expenditures:

$$\int_0^n p(j)y(j)dj = y(z)p(z)^\sigma \int_0^n p(j)^{1-\sigma} dj$$

- Perfect competition in the final-goods market implies zero profit: $PY - \int_0^n p(j)y(j)dj = 0$

- Thus,

$$PY = y(z)p(z)^\sigma \int_0^n p(j)^{1-\sigma} dj$$

Eliminating the Scale Effects

- Solve it for $y(z)$ as

$$y(z) = PY \left[\int_0^n p(j)^{1-\sigma} dj \right]^{-1} p(z)^{-\sigma}$$

- Substitute it into the production function:

$$Y = \frac{1}{N} PY \left[\int_0^n p(j)^{1-\sigma} dj \right]^{-1} \left(\int_0^n p(z)^{1-\sigma} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- It simplifies to

$$N = P \left[\int_0^n p(j)^{1-\sigma} dj \right]^{\frac{1}{\sigma-1}}$$

Eliminating the Scale Effects

- Thus,

$$P = N \left[\int_0^n p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}$$

- Remember that $N = n^{\frac{1}{\sigma-1}}$. Thus,

$$\begin{aligned} P &= n^{\frac{1}{\sigma-1}} \left[\int_0^n p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \\ &= \left[\frac{1}{n} \int_0^n p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \end{aligned}$$

Eliminating the Scale Effects

- The input demand is therefore

$$\begin{aligned} y(z) &= Y n^{\frac{1}{\sigma-1}} p(z)^{-\sigma} \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{1-\sigma}} \\ &= Y n^{-1} p(z)^{-\sigma} P^{\sigma} \end{aligned}$$

- Thus,

$$\frac{y(z)}{Y} = n^{-1} \left[\frac{p(z)}{P} \right]^{-\sigma}$$

- Firm entry reduces the input demand for all incumbent firms (= increased competition).

Eliminating the Scale Effects

- Production requires labor input $\ell(z)$:
$$y(z) = \max\{0, \varphi[\ell(z) - f]\}$$
- φ is the marginal product of labor (MPL).
- f is the **overhead cost** of production.
 - e.g.) I spend so much time on course preparation!
- The labor units needed to produce $y(z)$ is

$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Eliminating the Scale Effects

- Let w denote the wage rate. The profit is

$$\pi(z) = p(z)y(z) - w\ell(z)$$

- We assume perfectly competitive labor market.

- Input demand function:

$$\frac{y(z)}{Y} = n^{-1} \left[\frac{p(z)}{P} \right]^{-\sigma} \Leftrightarrow y(z) = n^{-1} Y P^{\sigma} p(z)^{-\sigma}$$

- Production technology:

$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Eliminating the Scale Effects

- Profit:

$$\begin{aligned}\pi(z) &= p(z)y(z) - w\ell(z) \\ &= p(z)y(z) - w \left[f + \frac{y(z)}{\varphi} \right] \\ &= n^{-1}YP^{\sigma}p(z)^{1-\sigma} - wf - \frac{w}{\varphi}n^{-1}YP^{\sigma}p(z)^{-\sigma}\end{aligned}$$

- FOC with respect to $p(z)$:

$$\begin{aligned}p(z) &= \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \\ &= \textit{markup} \times \frac{w}{MPL}\end{aligned}$$

Eliminating the Scale Effects

- Thus,

$$\begin{aligned} P &= \left[\frac{1}{n} \int_0^n \left[\frac{\sigma}{\sigma-1} \frac{w}{\varphi} \right]^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \\ &= \left[\frac{1}{n} n \left[\frac{\sigma}{\sigma-1} \frac{w}{\varphi} \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1} \frac{w}{\varphi} = p \end{aligned}$$

- P is now **independent** of n in equilibrium.

Eliminating the Scale Effects

- Input demand:

$$\begin{aligned}y(z) &= n^{-1} Y P^{\sigma} p(z)^{-\sigma} \\&= n^{-1} Y \left[\frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \right]^{\sigma} \left[\frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \right]^{-\sigma} \\&= n^{-1} Y\end{aligned}$$

- The firm's revenue is

$$r(z) = p(z)y(z) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} y(z)$$

Eliminating the Scale Effects

- The profit is

$$\begin{aligned}\pi(z) &= p(z)y(z) - w \left[f + \frac{y(z)}{\varphi} \right] \\ &= \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} y(z) - \frac{w}{\varphi} y(z) - wf \\ &= \frac{1}{\sigma - 1} \frac{w}{\varphi} y(z) - wf \\ &= \frac{r(z)}{\sigma} - wf\end{aligned}$$

Eliminating the Scale Effects

- Households supply L units of labor.
- There are n firms, and each employs ℓ workers.
- Thus,

$$L = n\ell = n \left[f + \frac{y}{\varphi} \right]$$

- Thus,

$$\frac{y}{\varphi} = \frac{L}{n} - f$$

Eliminating the Scale Effects

- The profit is

$$\begin{aligned}\pi(z) &= \frac{1}{\sigma - 1} \frac{w}{\varphi} y(z) - wf \\ &= \frac{1}{\sigma - 1} w \left[\frac{L}{n} - f \right] - wf\end{aligned}$$

- Thus, $\pi(z) \geq 0$ if and only if

$$\frac{1}{\sigma - 1} \left[\frac{L}{n} - f \right] \geq f \Leftrightarrow \frac{\mu}{1 + \mu n} \frac{L}{n} \geq f$$

- This is the condition for firm entry.

Eliminating the Scale Effects

- The free entry condition:

$$\begin{aligned}\pi(z) = 0 &\Leftrightarrow \frac{\mu}{1 + \mu} \frac{L}{n} = f \\ &\Leftrightarrow n = \frac{\mu}{1 + \mu} \frac{L}{f}\end{aligned}$$

- Thus, the equilibrium number of firms is
 - Increasing in the markup rate μ .
 - Decreasing in the overhead fixed cost f .
- $L = n\ell$ implies that an increasing in n decreases ℓ , which decreases c and increases p .

Optimal Entry

- **With scale effects**, the aggregate output is

$$Y = yn^{\frac{\sigma}{\sigma-1}}$$

- From the labor supply,

$$L = n\ell = n \left[f + \frac{y}{\varphi} \right] \Leftrightarrow y = \left(\frac{L}{n} - f \right) \varphi$$

- Thus,

$$Y = \left(\frac{L}{n} - f \right) \varphi n^{\frac{\sigma}{\sigma-1}}$$

Optimal Entry

- Consider

$$\max_n \left(\frac{L}{n} - f \right) \varphi n^{\frac{\sigma}{\sigma-1}}$$

- The optimal number of firms **under scale effects** is

$$n = \frac{L}{\sigma f}$$

- The equilibrium number of firms is

$$n = \frac{\mu}{1 + \mu} \frac{L}{f} = \frac{L}{\sigma f}$$

Optimal Entry

- **Without scale effects**, the aggregate output is

$$Y = ny$$

- From the labor supply,

$$L = n\ell = n \left[f + \frac{y}{\varphi} \right] \Leftrightarrow y = \left(\frac{L}{n} - f \right) \varphi$$

- Thus,

$$Y = \left(\frac{L}{n} - f \right) \varphi n = (L - fn)\varphi$$

- Thus, n should be as small as possible.

Optimal Entry

- With scale effects, entry **happens to be efficient** as two opposing effects causing inefficiency cancel out each other:
 - Entry increases overhead costs nf .
 - Entry increases the aggregate productivity.
- Without scale effects, the planner finds it inefficient to pay entry costs nf .
- See Section 3.E of Matsuyama (1995).

Firm Entry and Job Creation

Ebell and Haefke, “Product Market Deregulation and the US Employment Miracle,” *Review of Economic Dynamics*, 2009.

The Basic Idea

- We now introduce the DMP component into the model of firm entry.
- Unlike the DMP model, we need monopolistic **firms**.
- We shall continue to use the model without scale effects in the aggregate production function.

Competitive Labor Market

- Each monopolist's profit is

$$\pi(z) = p(z)y(z) - w\ell(z)$$

- Input demand function:

$$\frac{y(z)}{Y} = n^{-1} \left[\frac{p(z)}{P} \right]^{-\sigma}$$

- Production technology:

$$\ell(z) = f + \frac{y(z)}{\varphi}$$

- The model is **static**.

Search-Matching Frictions

- The (discrete-time) **Bellman equation**:

$$J(\ell) = \max_v \left\{ py - w(\ell)\ell - cv + \frac{1}{1+r} J(\ell_{+1}) \right\}$$

s. t.

$$\frac{y(z)}{Y} = n^{-1} \left[\frac{p(z)}{P} \right]^{-\sigma}$$

$$y = \varphi \ell$$

$$\ell_{+1} = (1 - \lambda)\ell + q(\theta)v$$

- The firm **cannot change output immediately** because of labor market frictions.

Search-Matching Frictions

- The inverse demand is

$$p = P \left(\frac{ny}{Y} \right)^{-\frac{1}{\sigma}} = P n^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}}$$

- The revenue is

$$py = P n^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} y^{1-\frac{1}{\sigma}} = P n^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} (\varphi \ell)^{1-\frac{1}{\sigma}} = \Phi \ell^{\frac{\sigma-1}{\sigma}}$$

- Thus,

$$\begin{aligned} J(\ell) &= \max_v \left\{ \Phi \ell^{\frac{\sigma-1}{\sigma}} - w(\ell)\ell - cv + \frac{1}{1+r} J(\ell_{+1}) \right\} \\ \text{s.t. } \ell_{+1} &= (1-\lambda)\ell + q(\theta)v \end{aligned}$$

Job Creation

- Consider

$$J(\ell) = \max_v \left\{ \Phi \ell^{\frac{\sigma-1}{\sigma}} - w(\ell)\ell - cv + \frac{1}{1+r} J(\ell_{+1}) \right\}$$
$$s.t. \ell_{+1} = (1-\lambda)\ell + q(\theta)v$$

- FOC with respect to v :

$$c = q(\theta) \frac{1}{1+r} J'(\ell_{+1})$$

- The Envelope condition:

$$J'(\ell) = \frac{\sigma-1}{\sigma} \Phi \ell^{\frac{\sigma-1}{\sigma}-1} - w(\ell) - w'(\ell)\ell + \frac{1-\lambda}{1+r} J'(\ell_{+1})$$

Job Creation

- Use FOC to eliminate $J'(\ell_{+1})$ from the Envelope condition to obtain

$$J'(\ell) = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} - w(\ell) - w'(\ell)\ell + \frac{(1 - \lambda)c}{q(\theta)}$$

- This the marginal firm value of employment.

Job Creation

- FOC implies that in any steady state,

$$\frac{(1+r)c}{q(\theta)} = J'(\ell)$$

- Thus, we obtain the job-creation condition

$$\frac{(1+r)c}{q(\theta)} = \frac{\sigma-1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} - w(\ell) - w'(\ell)\ell + \frac{(1-\lambda)c}{q(\theta)}$$

Job Creation

- The job-creation condition:

$$\frac{(r + \lambda)c}{q(\theta)} = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} - w(\ell) - w'(\ell)\ell$$

- Notice that we need to figure out $w'(\ell)$.

Workers

- Consider the (discrete-time) Pissarides model.
- W : Value of employment.
- U : Value of job search.
- b : unemployment benefit.
- $\delta = \frac{1}{1+r}$: the discount factor.

- Value of being employed:

$$W = w(\ell) + \lambda\delta U + (1 - \lambda)\delta W$$

- Value of job search:

$$U = b + \theta q(\theta)\delta W + [1 - \theta q(\theta)]\delta U$$

Intra-Firm Bargaining

- Nash bargaining outcome in the DMP model:

$$\beta[J - V] = (1 - \beta)[W - U].$$

- One firm versus **many workers**:

$$\beta[J(\ell) - J(\ell - \Delta)] = (1 - \beta)\Delta[W - U]$$

- One firm versus randomly selected Δ workers.
- The value of disagreement is $J(\ell - \Delta)$ because Δ workers walk away, forcing the firm to produce with $\ell - \Delta$ employees.

Intra-Firm Bargaining

- Consider:

$$\beta[J(\ell) - J(\ell - \Delta)] = (1 - \beta)\Delta[W - U].$$

- $\Delta \rightarrow 0$,

$$\beta J'(\ell) = (1 - \beta)[W - U].$$

- This means that the firm is bargaining with **one** worker. But, who is this worker?
- Each worker is treated as the **marginal** worker.
 - Whenever you negotiate with the firm, you will be treated as the ℓ th worker.

Intra-Firm Bargaining

- Consider

$$\beta J'(\ell) = (1 - \beta)[W - U]$$

- We need to know $J'(\ell)$ and $W - U$.

- The good news is, we know $J'(\ell)$:

$$J'(\ell) = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} \\ -w(\ell) - w'(\ell)\ell + \frac{(1 - \lambda)c}{q(\theta)}$$

- See page 58.

Deriving $W - U$

- Consider

$$W = w(\ell) + \lambda\delta U + (1 - \lambda)\delta W$$

$$U = b + \theta q(\theta)\delta W + [1 - \theta q(\theta)]\delta U$$

- Subtract the terms of the second equation from the other to obtain

$$\begin{aligned} W - U &= w(\ell) - b + \lambda\delta U + (1 - \lambda)\delta W \\ &\quad - \theta q(\theta)\delta W - [1 - \theta q(\theta)]\delta U \end{aligned}$$

- Thus,

$$\begin{aligned} W - U &= w(\ell) - b \\ &\quad + [1 - \lambda - \theta q(\theta)]\delta[W - U] \end{aligned}$$

Deriving $W - U$

- From $\beta J'(\ell) = (1 - \beta)[W - U]$, we obtain

$$(W - U) = \frac{\beta}{1 - \beta} J'(\ell)$$

- FOC of the firm implies that in any steady state,

$$\frac{(1 + r)c}{q(\theta)} = J'(\ell)$$

- Thus,

$$(W - U) = \frac{\beta}{1 - \beta} \frac{(1 + r)c}{q(\theta)}$$

Deriving $W - U$

- Therefore,

$$\begin{aligned} W - U &= w(\ell) - b + [1 - \lambda - \theta q(\theta)]\delta[W - U] \\ &= w(\ell) - b \\ &\quad + [1 - \lambda - \theta q(\theta)]\delta \frac{\beta}{1 - \beta} \frac{(1 + r)c}{q(\theta)} \\ &= w(\ell) - b \\ &\quad + [1 - \lambda - \theta q(\theta)] \frac{\beta}{1 - \beta} \frac{c}{q(\theta)} \end{aligned}$$

Intra-Firm Bargaining

- Consider

$$\beta J'(\ell) = (1 - \beta)[W - U]$$

- We know:

$$J'(\ell) = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} \\ -w(\ell) - w'(\ell)\ell + \frac{(1 - \lambda)c}{q(\theta)} \\ W - U = w(\ell) - b \\ + [1 - \lambda - \theta q(\theta)] \frac{\beta}{1 - \beta} \frac{c}{q(\theta)}$$

Intra-Firm Bargaining

- Thus, the wage rate must satisfy

$$\beta \left[\frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} - w(\ell) - w'(\ell)\ell + \frac{(1 - \lambda)c}{q(\theta)} \right] \\ = (1 - \beta) \left\{ w(\ell) - b + \frac{\beta}{1 - \beta} \frac{[1 - \lambda - \theta q(\theta)]c}{q(\theta)} \right\}$$

- Arrange terms to obtain:

$$w'(\ell)\ell + \frac{1}{\beta} w(\ell) = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} + \frac{1 - \beta}{\beta} b + c\theta$$

- This is an ordinary differential equation in ℓ .

Solving ODE

- Observe that

$$\begin{aligned}\frac{d \left[w(\ell) \ell^{\frac{1}{\beta}} \right]}{d\ell} &= w'(\ell) \ell^{\frac{1}{\beta}} + \frac{1}{\beta} w(\ell) \ell^{\frac{1}{\beta}-1} \\&= \left[w'(\ell) \ell + \frac{1}{\beta} w(\ell) \right] \ell^{\frac{1}{\beta}-1} \\&= \left[\frac{\sigma-1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} + \frac{1-\beta}{\beta} b + c\theta \right] \ell^{\frac{1}{\beta}-1} \\&= \frac{\sigma-1}{\sigma} \Phi \ell^{\frac{-1}{\sigma} + \frac{1}{\beta} - 1} + \left[\frac{1-\beta}{\beta} b + c\theta \right] \ell^{\frac{1}{\beta}-1}\end{aligned}$$

Solving ODE

- Consider

$$\frac{d \left[w(\ell) \ell^{\frac{1}{\beta}} \right]}{d\ell} = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma} + \frac{1}{\beta} - 1} + \left[\frac{1 - \beta}{\beta} b + c\theta \right] \ell^{\frac{1}{\beta} - 1}$$

- Integrate LHS:

$$\begin{aligned} \int_0^{\ell} \frac{d \left[w(i) i^{\frac{1}{\beta}} \right]}{di} di &= w(\ell) \ell^{\frac{1}{\beta}} - \lim_{\ell \rightarrow 0} w(\ell) \ell^{\frac{1}{\beta}} \\ &= w(\ell) \ell^{\frac{1}{\beta}} \end{aligned}$$

Solving ODE

- Integrate RHS:

$$\begin{aligned}
 & \int_0^\ell \left\{ \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma} + \frac{1}{\beta} - 1} + \left[\frac{1 - \beta}{\beta} b + c\theta \right] \ell^{\frac{1}{\beta} - 1} \right\} di \\
 &= \frac{\frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma} + \frac{1}{\beta}}}{\frac{-1}{\sigma} + \frac{1}{\beta}} + \left[\frac{1 - \beta}{\beta} b + c\theta \right] \beta \ell^{\frac{1}{\beta}} \\
 &= \frac{\frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma} + \frac{1}{\beta}}}{\frac{-1}{\sigma} + \frac{1}{\beta}} + [(1 - \beta)b + \beta c\theta] \ell^{\frac{1}{\beta}}
 \end{aligned}$$

Solving ODE

- Finally, we obtain

$$w(\ell)\ell^{\frac{1}{\beta}} = \frac{\frac{\sigma-1}{\sigma}\Phi\ell^{\frac{-1}{\sigma}+\frac{1}{\beta}}}{\frac{-1}{\sigma}+\frac{1}{\beta}} + [(1-\beta)b + \beta c\theta]\ell^{\frac{1}{\beta}}$$

- Thus,

$$w(\ell) = \beta \frac{\sigma-1}{\sigma-\beta}\Phi\ell^{\frac{-1}{\sigma}} + (1-\beta)b + \beta c\theta$$

- DMP: $w = \beta y + (1-\beta)b + \beta c\theta$
- As $\sigma \rightarrow \infty$, the two wage equations are the same.

Aggregation

- If there are n firms, then the aggregate number of vacancies is

$$nv$$

- Labor market tightness:

$$\theta = \frac{nv}{u}$$

- Labor force is 1.
- Aggregate number of employees satisfies

$$1 - u = n\ell$$

Aggregation

- Worker flows:

$$u_{t+1} = u_t + \lambda(1 - u_t) - \theta_t q(\theta_t) u_t$$

- In any steady state, (as usual) we obtain

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

- Aggregate number of employees:

$$1 - u = n\ell \Leftrightarrow u = 1 - n\ell$$

- Thus,

$$n\ell = 1 - \frac{\lambda}{\lambda + \theta q(\theta)}$$

Steady-State Equilibrium

- The job-creation condition:

$$\frac{(r + \lambda)c}{q(\theta)} = \frac{\sigma - 1}{\sigma} \Phi \ell^{\frac{-1}{\sigma}} - w(\ell) - w'(\ell)\ell$$

- The wage equation:

$$w(\ell) = \beta \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} + (1 - \beta)b + \beta c\theta$$

- Take the derivative to obtain

$$w'(\ell) = \frac{-1}{\sigma} \beta \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma} - 1}$$

Steady-State Equilibrium

- The job-creation condition:

$$w(\ell) = \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} - \frac{(r + \lambda)c}{q(\theta)}$$

- The wage equation:

$$w(\ell) = \beta \frac{\sigma - 1}{\sigma - \beta} \Phi \ell^{\frac{-1}{\sigma}} + (1 - \beta)b + \beta c\theta$$

- Worker flows:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$
$$u = 1 - n\ell$$

Closing the Model

- Remember

$$\Phi = P n^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} \varphi^{\frac{\sigma-1}{\sigma}}$$

- Output (see p.28)

$$Y = n^{\frac{-1}{\sigma-1}} \left(\int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} = ny$$

- Price index (see p.37)

$$P = n^{\frac{1}{\sigma-1}} \left[\int_0^n p^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} = p$$

Closing the Model

- Let the final consumption good be the numeraire (= units of measurement). Thus, we normalize

$$P = 1$$

- Then, $p = P = 1$. With $Y = ny$, we obtain

$$\begin{aligned}\Phi &= n^{-\frac{1}{\sigma}}(ny)^{\frac{1}{\sigma}}\varphi^{1-\frac{1}{\sigma}} \\ &= n^{-\frac{1}{\sigma}}(n\varphi\ell)^{\frac{1}{\sigma}}\varphi^{1-\frac{1}{\sigma}} \\ &= \varphi\ell^{\frac{1}{\sigma}}\end{aligned}$$

Short-Run (?) Steady State

- The job-creation condition:

$$w = \frac{\sigma - 1}{\sigma - \beta} \varphi - \frac{(r + \lambda)c}{q(\theta)}$$

- The wage equation:

$$w = \beta \frac{\sigma - 1}{\sigma - \beta} \varphi + (1 - \beta)b + \beta c\theta$$

- Worker flows:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$
$$u = 1 - n\ell$$

Relationship with the DMP Model

- Let $\sigma \rightarrow \infty$. Then, the job-creation condition is

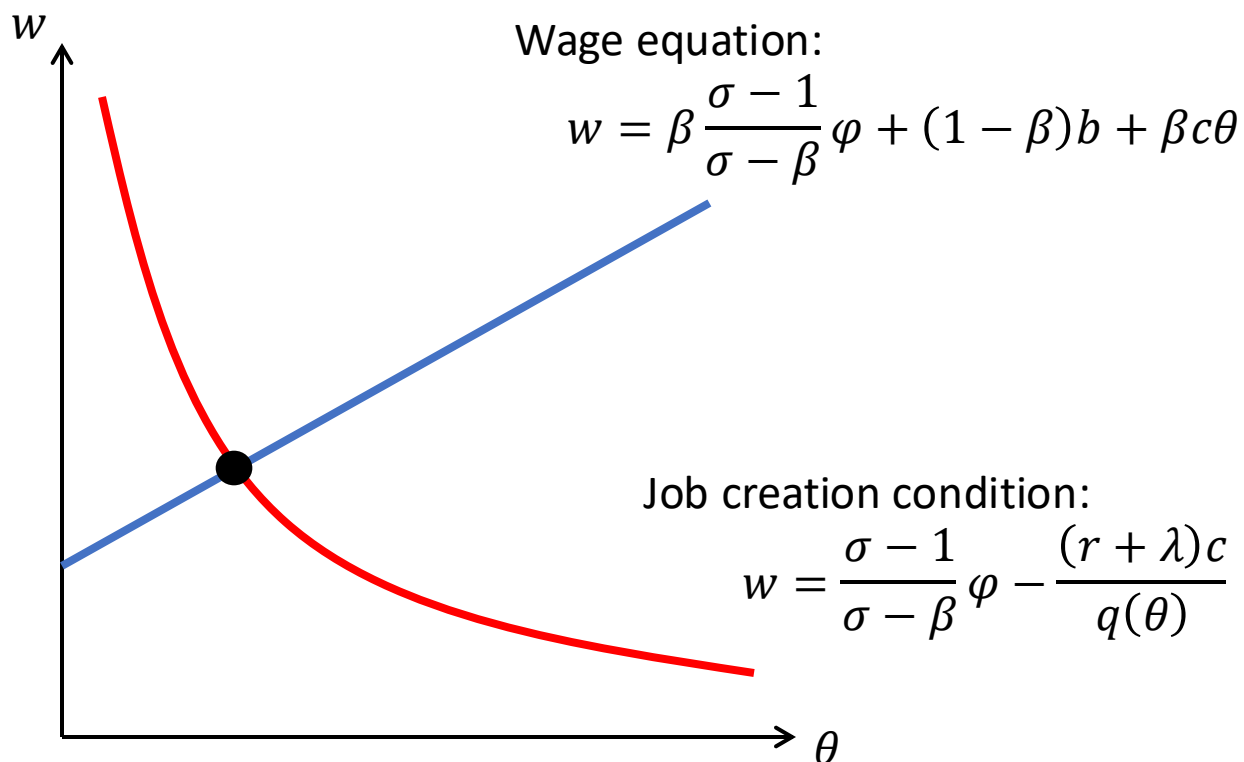
$$w = \varphi - \frac{(r + \lambda)c}{q(\theta)}$$

- The wage equation becomes

$$w = \beta\varphi + (1 - \beta)b + \beta c\theta$$

- These equations are identical to those of the textbook Pissarides model.

Short-Run (?) Steady State



Short-Run (?) Steady State

- Ebell and Haefke assumes that the number of firms n is a parameter.
- They further assume
$$\sigma = \bar{\sigma} g(n), g'(n) > 0$$
- Thus, the elasticity of substitution increases with n .
- As the number of firms increases, the markup decreases to get closer to the competitive economy.

Short-Run (?) Steady State

- Clearly, the solution pair (w, θ) depends on the number of firms n .

- Thus, JC and wage jointly imply

$$\frac{(r + \lambda)c}{q(\theta)} + \beta c\theta = (1 - \beta) \frac{\bar{\sigma}g(n) - 1}{\bar{\sigma}g(n) - \beta} \varphi - (1 - \beta)b$$

- LHS is increasing in θ .
- An increase in n increases the RHS. Thus,

$$\frac{d\theta}{dn} > 0, \frac{du}{dn} < 0$$

Firm Entry

- The value of operation with ℓ employees:

$$J(\ell) = \max_v \left\{ \Phi \ell^{\frac{\sigma-1}{\sigma}} - w(\ell)\ell - cv + \delta J(\ell_{+1}) \right\}$$
$$s.t. \quad \ell_{+1} = (1 - \lambda)\ell + q(\theta)v$$

- Thus, the value of the firm **with 0 employee** is

$$J(0) = \max_v \{ 0 - cv + \delta J(\ell_{+1}) \}$$
$$s.t. \quad \ell_{+1} = 0 + q(\theta)v$$

- FOC is

$$c = q(\theta)\delta J'(\ell_{+1})$$

Firm Entry

- Existing firms and entrants face the same condition:

$$J'(\ell_{+1}) = \frac{(1+r)c}{q(\theta)}$$

- This means that ℓ_{+1} is the same for **all** firms, incumbents and entrants.
 - In other words, the entrant creates a **huge** number of vacancies to make sure that the employees in the next period will be the same as incumbent firms.
 - There will be **no size distribution** of firms.

Firm Entry

- Given the vacancy-filling rate, each entrant creates

$$\frac{\ell_{+1}}{q(\theta)}$$

units of vacancies so that the employee will be

$$q(\theta) \times \frac{\ell_{+1}}{q(\theta)} = \ell_{+1}$$

- Thus, the value of entry is

$$J(0) = -c \frac{\ell_{+1}}{q(\theta)} + \frac{1}{1+r} J(\ell_{+1})$$

Firm Entry

- We assume each entrant must pay

$$F > 0$$

units of fixed entry cost.

- Thus, n increases if and only if

$$J(0) > F$$

- Thus, n is determined so that

$$J(0) = -c \frac{\ell_{+1}}{q(\theta)} + \frac{1}{1+r} J(\ell_{+1}) = F$$

Steady State with Firm Entry

- In any steady state,

$$-c \frac{\ell}{q(\theta)} + \frac{1}{1+r} J(\ell) = F$$

- From the Bellman equation,

$$J(\ell) = \varphi \ell - w(\ell) \ell - cv + \frac{1}{1+r} J(\ell)$$
$$\ell = (1 - \lambda) \ell + q(\theta) v$$

- Eliminate v to obtain

$$\frac{r}{1+r} J(\ell) = \varphi \ell - w(\ell) \ell - \frac{c\lambda}{q(\theta)} \ell$$

Steady State with Firm Entry

- Thus, the free entry condition for firms is

$$\varphi - w - \frac{(r + \lambda)c}{q(\theta)} = \frac{rF}{\ell}$$

- Other equations are

$$w = \frac{\sigma - 1}{\sigma - \beta} \varphi - \frac{(r + \lambda)c}{q(\theta)}$$
$$w = \beta \frac{\sigma - 1}{\sigma - \beta} \varphi + (1 - \beta)b + \beta c\theta$$
$$n\ell = 1 - \frac{\lambda}{\lambda + \theta q(\theta)}$$

Steady State with Firm Entry

- Eliminate w from the equations to obtain

$$\frac{1 - \beta}{\sigma - \beta} \varphi = \frac{rF}{\ell}$$

$$(1 - \beta) \frac{\sigma - 1}{\sigma - \beta} \varphi = \frac{(r + \lambda)c}{q(\theta)} + \beta c \theta + (1 - \beta)b$$

- Notice that under a constant σ , the first condition determines ℓ while the second condition pins down θ without any interaction between firm entry and the labor market.

Steady State with Firm Entry

- Let us now assume (as in Ebell and Haefke)

$$\sigma = \bar{\sigma}g(n), g'(n) > 0$$

- Then,

$$\frac{1 - \beta}{\bar{\sigma}g(n) - \beta} \varphi = \frac{rF}{\ell}$$

$$(1 - \beta) \frac{\bar{\sigma}g(n) - 1}{\bar{\sigma}g(n) - \beta} \varphi = \frac{(r + \lambda)c}{q(\theta)} + \beta c\theta + (1 - \beta)b$$

$$n\ell = 1 - \frac{\lambda}{\lambda + \theta q(\theta)}$$

Solving the Model

- The vacancy-filling rate: $q(\theta) = A\theta^{-\alpha}$
- Product market competitiveness: $\sigma = \bar{\sigma}g(n) = n$
- Then,

$$\frac{1 - \beta}{n - \beta} \varphi = \frac{rF}{\ell}$$

$$(1 - \beta) \frac{n - 1}{n - \beta} \varphi = \frac{(r + \lambda)c}{A\theta^{-\alpha}} + \beta c\theta + (1 - \beta)b$$

$$n\ell = 1 - \frac{\lambda}{\lambda + A\theta^{1-\alpha}}$$

Solving the Model

- Let us eliminate ℓ . Then,

$$\frac{1 - \beta}{n - \beta} \varphi = rnF \frac{\lambda + A\theta^{1-\alpha}}{A\theta^{1-\alpha}}$$

$$(1 - \beta) \frac{n - 1}{n - \beta} \varphi = \frac{(r + \lambda)c}{A\theta^{-\alpha}} + \beta c\theta + (1 - \beta)b$$

- Parameter values

$$A = 0.327, \lambda = 0.024, b = 0.626, c = 0.173, \\ \beta = 0.5, \alpha = 0.5, r = \frac{0.04}{12}, \varphi = 1, F = 0.6$$

Solving the Model

```
# Solving the Ebell-Haefke model

%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import root
plt.rcParams['figure.figsize'] = (8,8)

# Parameter values
r = 0.04/12
A = 0.327
alpha = 0.5
lambda_ = 0.024
c = 0.173
beta = 0.5
b = 0.626
phi = 1
F = 0.6

# Free Entry Condition
def FE(theta, n):
    return (1-beta)*phi/(n-beta) - r*n*F*(lambda_+A*(theta**(1-alpha)))/(A*(theta**(1-alpha)))
# Job Creation Condition
def JC(theta, n):
    return (r+lambda_)*c*(theta**alpha)/A + (1-beta)*b + beta*c*theta - (1-beta)*phi*(n-1)/(n-beta)
# Unemployment rate
def Unemp(theta):
    return lambda_/(lambda_ + A*(theta**(1-alpha)))
```

```
# Solving the model

def func(x):
    f = [FE(x[0], x[1]),
         JC(x[0], x[1])]
    return f
sol = root(func, [1, 1.5])
sol.x

eq_theta = sol.x[0]
eq_n = sol.x[1]
eq_u = Unemp(eq_theta)

print(f'Equilibrium tightness is {eq_theta:.4f}')
print(f'Equilibrium number of firms is {eq_n:.4f}')
print(f'Equilibrium unemployment rate is {eq_u:.4f}')
```

Equilibrium tightness is 1.7498
Equilibrium number of firms is 15.6422
Equilibrium unemployment rate is 0.0526

Further Readings

- Ebell and Haefke, “Product Market Deregulation and the US Employment Miracle,” *Review of Economic Dynamics*, 2009.
 - Available from NUCT.
- Blanchard and Giavazzi, “Macroeconomic Effects of Regulation and Deregulation in Goods and Labor Markets,” *Quarterly Journal of Economics*, 2003.

Reading Assignment

- Melitz, "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 2003.
 - Section 2 (Setup of the model), Section 3 (Firm Entry and Exit) and Section 4 (Equilibrium in a Closed Economy).
 - Available from NUCT.