Lecture 10

Labor Share

6/30

Goals

- Today, we shall focus on the labor share of income.
- The goal is to replicate some important results in Karabarbounis and Neiman's "The Global Decline of the Labor Share" (2014).

Modeling the Labor Share of Income

Cobb-Douglas

- Consider the Cobb-Douglas production function:

$$Y = AK^{\alpha}L^{1-\alpha}$$

- We assume perfect competition in all markets.
- r: rental price of capital
- w: rental price of workers = wage rate
- The price of the final good is normalized to 1.
- The firm's profit maximization problem is

$$\max_{K,L} AK^{\alpha}L^{1-\alpha} - rK - wL$$

Cobb-Douglas

- The first-order conditions are:

$$MPK = \alpha A K^{\alpha-1} L^{1-\alpha} = r$$
$$MPL = (1-\alpha) A K^{\alpha} L^{-\alpha} = w$$

- The **labor share** of income is defined as

$$\frac{wL}{Y}$$

- Substitute the FOC into above to obtain

$$\frac{wL}{Y} = \frac{(1-\alpha) A K^{\alpha} L^{1-\alpha}}{A K^{\alpha} L^{1-\alpha}} = 1-\alpha$$

- Labor supply does not matter for the result!

Cobb-Douglas

- Likewise, the capital share of income is

$$\frac{rK}{Y} = \frac{\alpha A K^{\alpha} L^{1-\alpha}}{A K^{\alpha} L^{1-\alpha}} = \alpha$$

- Perfect competition and Cobb-Douglas jointly imply a constant labor share.
  - In fact, Cobb and Douglas looked for a function that implies a constant labor share, which is consistent with data (known as the Kaldor Facts).
- The typical value is $\alpha = \frac{1}{3}$.

Cobb-Douglas

- The profit is

$$\Pi = AK^{\alpha}L^{1-\alpha} - rK - wL$$

- We substitute the FOCs in to the above

$$MPK = \alpha AK^{\alpha-1}L^{1-\alpha} = r$$
$$MPL = (1-\alpha)AK^{\alpha}L^{-\alpha} = w$$

- We can show that

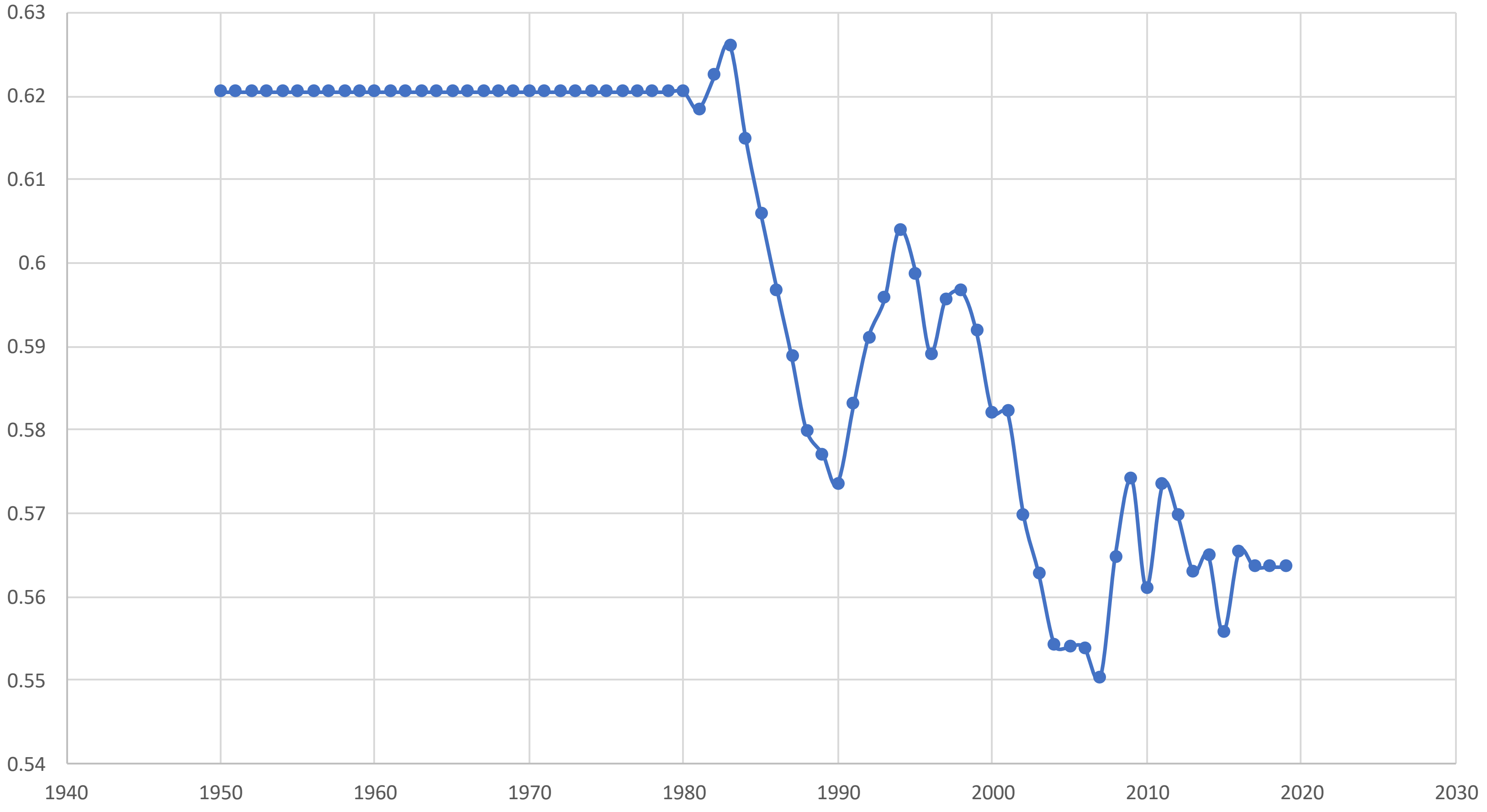
$$\Pi = 0$$

- Under perfect competition (with constant-returns-to-scale production function), firms always earn zero profit in equilibrium.

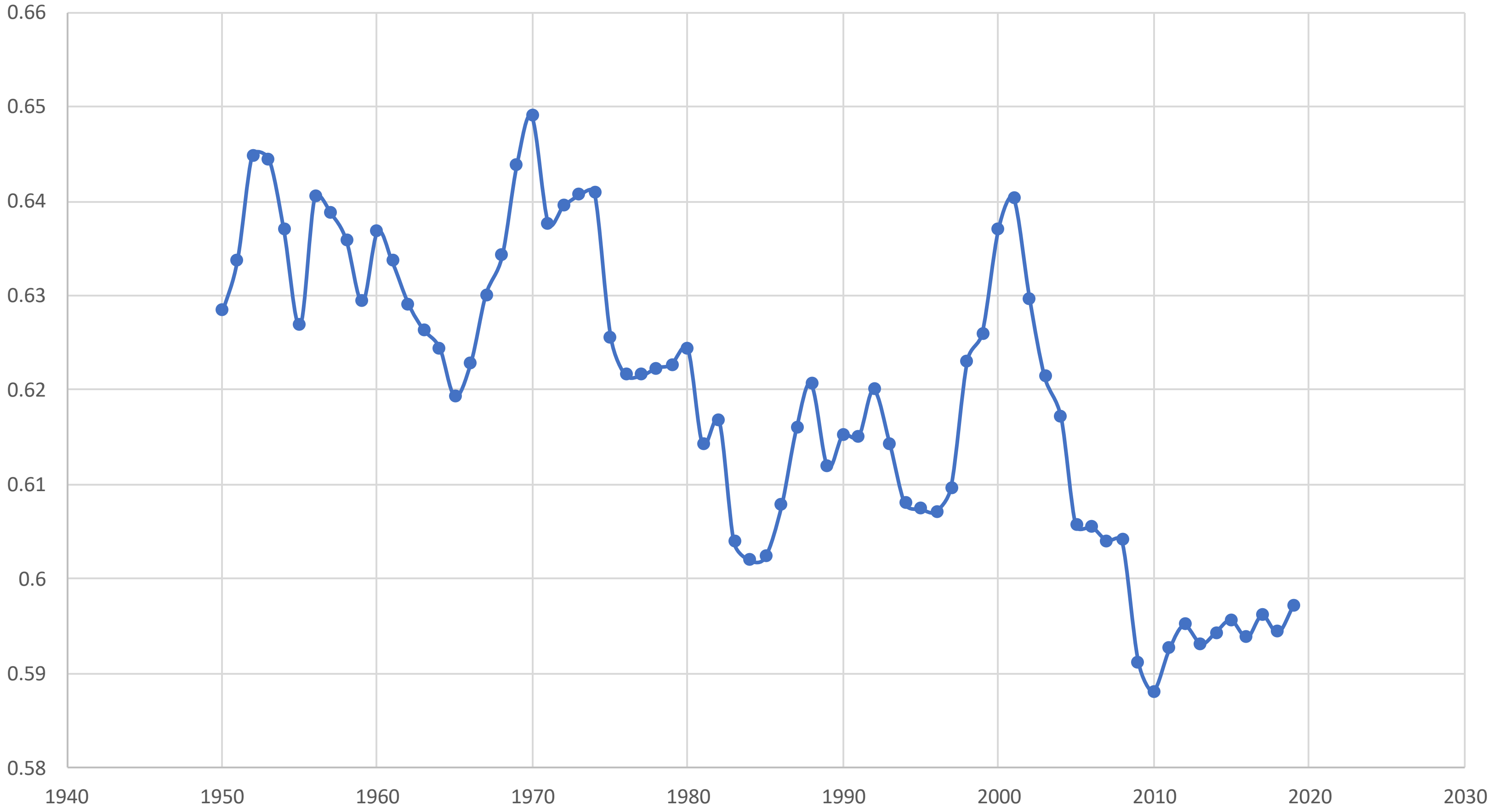
Penn World Table

- https://www.rug.nl/ggdc/productivity/pwt/?lang=en
- You can download a rich cross-country dataset on income.

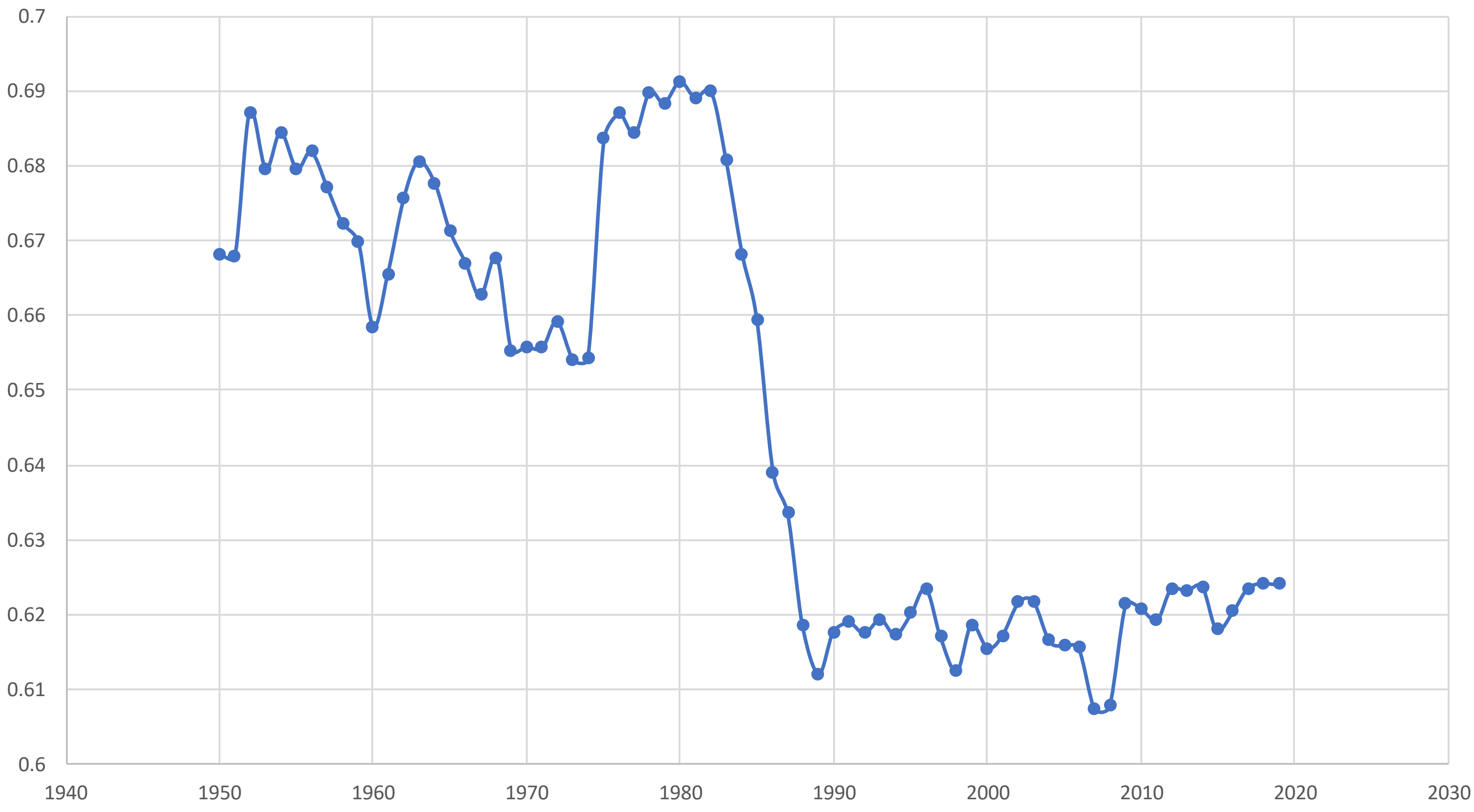
Labor Share in Japan



Labor Share in the US



Labor Share in France



CES Production Function

- To explain the decline of the labor share, we need to go beyond Cobb-Douglas.
- CES (Constant elasticity of substitution) production function:

$$Y = F(K, L) = [K^{\rho} + L^{\rho}]^{\frac{1}{\rho}},$$

where $\rho \leq 1$.

- The firm's profit maximization problem is

$$\max_{K,L} [K^{\rho} + L^{\rho}]^{\frac{1}{\rho}} - rK - wL$$

CES Production Function

- From FOCs, we obtain

$$\frac{L}{K} = \left(\frac{r}{w}\right)^{\frac{1}{1-\rho}}$$

- It is easy to verify that the elasticity of substitution between capital and labor is

$$\frac{1}{1-\rho} = \sigma$$

CES Production Function

- If $\rho > 0$ ($\sigma > 1$), then K and L are said to be **gross substitutes** in production.
- If $\rho < 0$ ($\sigma < 1$), then K and L are said to be **gross complements** in production.
- A few special cases:
  - $\rho \to 1$: K and L are **perfect substitutes**.
  - $\rho \to 0$: Cobb-Douglas production function.
  - $\rho \to -\infty$: Leontief production function.

CES Production Function

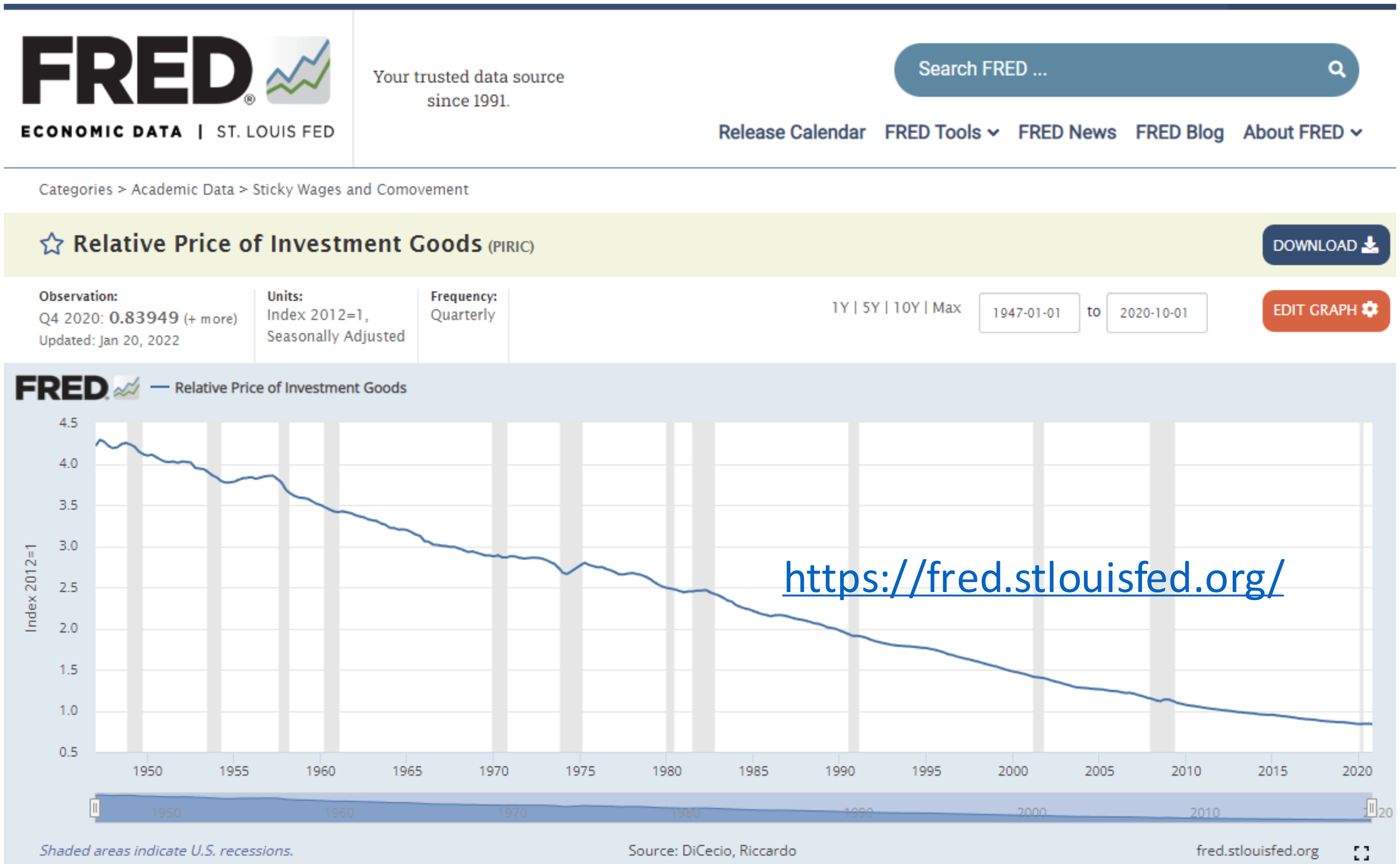
- The labor share of income is

$$\frac{wL}{Y} = \frac{L^\rho}{K^\rho + L^\rho} = \frac{\left(\frac{L}{K}\right)^\rho}{1 + \left(\frac{L}{K}\right)^\rho}$$

$$= \frac{\left(\frac{r}{w}\right)^{\frac{\rho}{1-\rho}}}{1 + \left(\frac{r}{w}\right)^{\frac{\rho}{1-\rho}}}$$

- If $\sigma > 1$, then a reduction in the relative price of capital reduces the labor share.

Machines Become Less Expensive



The Labor Share

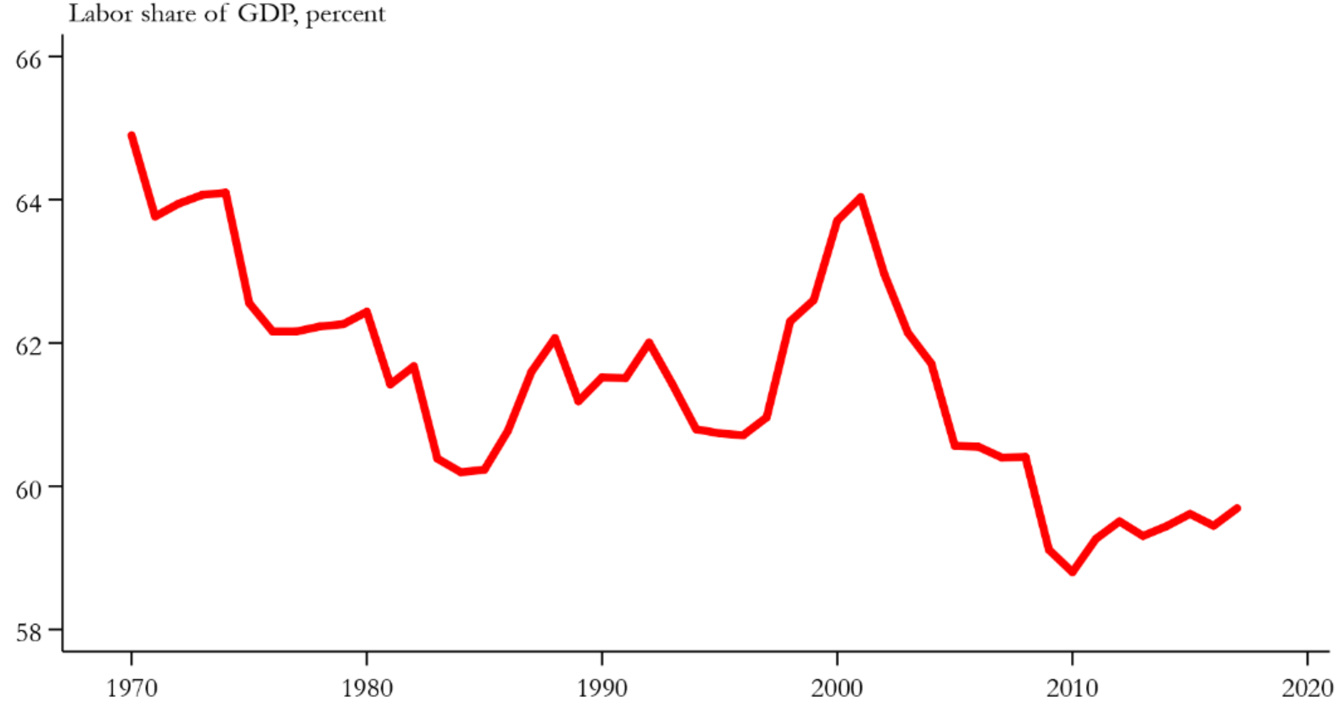


Figure XI. Labor share of GDP

Share of Labor Compensation in GDP at Current National Prices for US (Penn World Tables)

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Market Power

Perfect Competition

- Consider a perfectly competitive firm in the product market.
- Under perfect competition, the firm is a price taker.
- Let y, p, and $c(y)$ denote output, price, and the cost function. Then the firm's profit is

$$py - c(y)$$

- The profit maximization implies

$$p = c'(y) = MC$$

- Price equals the marginal cost.

Monopoly

- Now consider a monopoly firm.
- Let $y = D(p)$ denote the demand function.
- The monopolist's profit is
$$py - c(y) = pD(p) - c\big(D(p)\big)$$
- The first-order condition with respect to p is
$$D + pD'(p) - c'D'(p) = 0$$
- Assuming $D' \neq 0$, we solve it for p:
$$p = c' - \frac{D}{D'} = c' + \frac{1}{-\frac{p}{D}D'}p = c' + \frac{1}{\epsilon}p$$

Monopoly

- Rewrite it as

$$\left(1 - \frac{1}{\epsilon}\right) p = c'$$

- Because $c' > 0$, the demand elasticity must satisfy

$$\epsilon \geq 1$$

- We finally obatin

$$p = \left(1 + \frac{1}{\epsilon - 1}\right) c' = (1 + \mu) MC$$

- μ is the **markup** rate.

Markups

- The markup rate:

$$\mu = \frac{1}{demand\ elasticity - 1} \geq 0$$

- In the limit as the demand elasticity gets arbitrarily large ($\epsilon \to \infty$),

$$\mu \to 0$$
$$p \to MC$$

- This the perfectly competitive outcome.
- The markup captures the degree of market concentration (or competitiveness) of an economy.

Markups

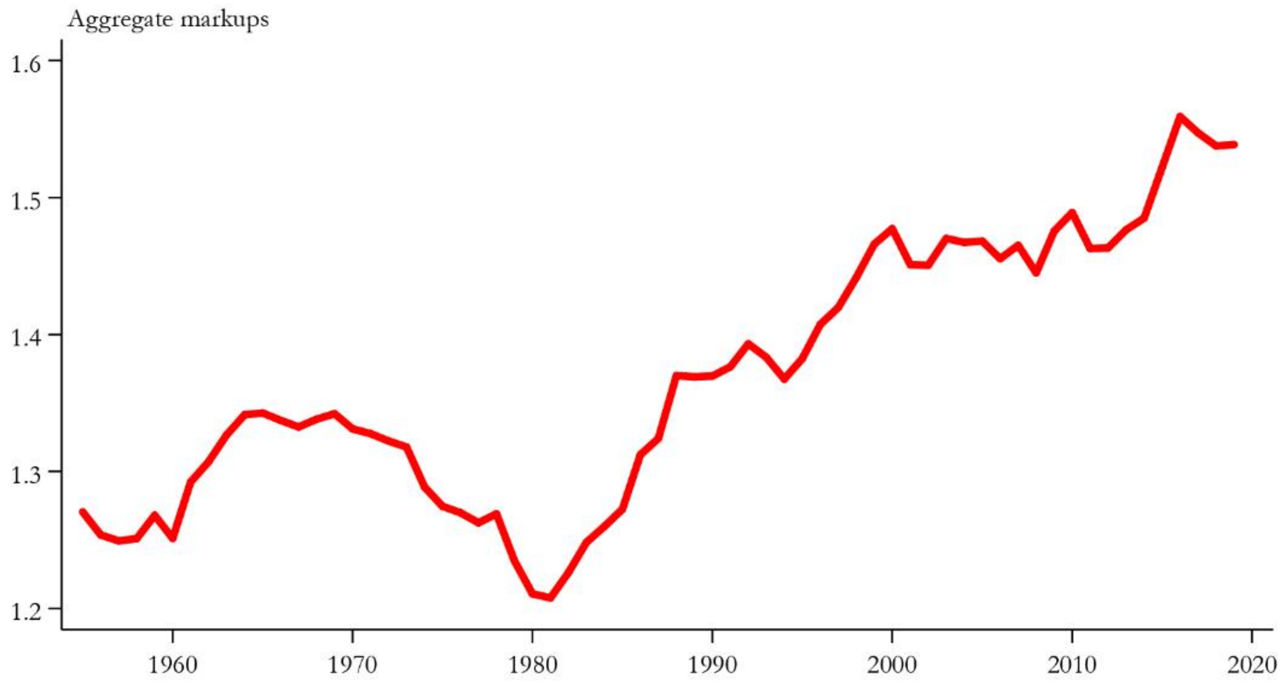

Figure 3. Aggregate markups in the United States

Revenue-weighted average markup of US publicly traded firms (source: De Loecker, Eeckhout & Unger, 2020)

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Markups

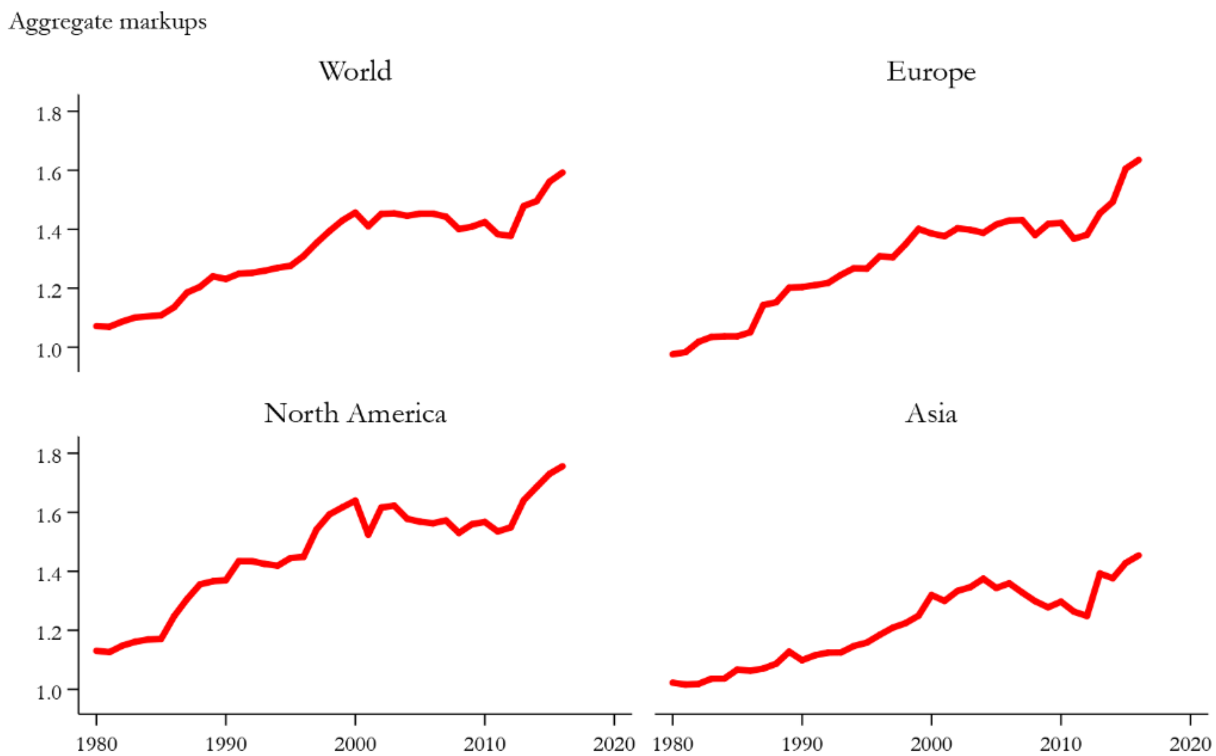

Figure 4. Aggregate global markups

Revenue-weighted average markup of publicly traded firms: World, Europe, North Amercia and Asia (source: De Loecker & Eeckhout, 2018)

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Market Power and the Labor Share

Karabarbounis and Neiman. "The global decline of the labor share." Quarterly Journal of Economics (2014) 61-103.

Monopolistic Competition

- The model is a family of the **Dixit-Stiglitz model** of monopolistic competition.
  - There is an infinity of producers. This way we can ignore strategic interaction among firms altogether.
    - Otherwise, we must use game theory.
  - Still, each firm has a monopoly power because each product is **differentiated**.
  - Thus, there is an infinity of monopolists!
- The model is so popular that there is an infinity of professional articles using the framework.

Vertical Industrial Structure

- Final goods markets are perfectly competitive:
  - Consumption good C and investment good X
  - They purchase a continuum of differentiated intermediate goods from monopolists.
- Intermediate-goods markets are monopolistic:
  - Each monopolist uses capital and labor to produce exactly one type of differentiated input, and sells it to the final goods firms.
  - They set prices above the marginal costs.

Final Consumption Goods

- The quantity of the final consumption good is C and its price is P^C.
- Let $c(z)$ denote the input of variety z.
- Production function:

$$C = \left(\int_0^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 1$ is the elasticity of substitution between any two varieties.
- The number of monopolists is normalized by 1.

Intermediate Goods

- Let $p(z)$ denote the input price of variety z.
- The input demand minimizes total expenditure:

$$\min_{c(z)} \int_0^1 p(z)c(z)dz$$

$$s.t.$$

$$C = \left(\int_0^1 c(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate Goods

- The Lagrangian is

$$\int_0^1 p(z)c(z)dz + \lambda\left[C - \left(\int_0^1 c(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}\right]$$

- FOC for variety z is

$$p(z) = \lambda\frac{\sigma}{\sigma-1}\left(\int_0^1 c(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}c(z)^{\frac{\sigma-1}{\sigma}-1}$$

- FOC for variety j is

$$p(j) = \lambda\frac{\sigma}{\sigma-1}\left(\int_0^1 c(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}c(j)^{\frac{\sigma-1}{\sigma}-1}$$

Intermediate Goods

- Taking the ratio:

$$\frac{p(z)}{p(j)} = \frac{c(z)^{\frac{\sigma-1}{\sigma}-1}}{c(j)^{\frac{\sigma-1}{\sigma}-1}} = \left(\frac{c(z)}{c(j)}\right)^{\frac{-1}{\sigma}}$$

- Thus,

$$c(j) = \left(\frac{p(z)}{p(j)}\right)^{\sigma} c(z) = p(z)^{\sigma} p(j)^{-\sigma} c(z)$$

- Thus,

$$c(j)^{\frac{\sigma-1}{\sigma}} = p(z)^{\sigma-1} p(j)^{1-\sigma} c(z)^{\frac{\sigma-1}{\sigma}}$$

Intermediate Goods

- Substitute it into the production function:

$$C = \left(\int_0^1 c(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\int_0^1 \left[p(z)^{\sigma-1} p(j)^{1-\sigma} c(z)^{\frac{\sigma-1}{\sigma}}\right] dj\right)^{\frac{\sigma}{\sigma-1}}$$

$$= p(z)^{\sigma} c(z) \left(\int_0^1 [p(j)^{1-\sigma}] dj\right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate Goods

- We obtain

$$c(z) = Cp(z)^{-\sigma}\left(\int_0^1 [p(j)^{1-\sigma}]dj\right)^{\frac{\sigma}{1-\sigma}}$$

- This is the input demand function for variety z.
- We can simplify it further…

Intermediate Goods

- Now let us use $c(j) = p(z)^{\sigma} p(j)^{-\sigma} c(z)$ to rewrite the expenditures:

$$\int_0^1 p(j)c(j)dj = c(z)p(z)^{\sigma} \int_0^1 p(j)^{1-\sigma} dj$$

- Perfect competition in the final-goods market implies zero profit: $P^C C - \int_0^1 p(j)c(j)dj = 0$
- Thus,

$$P^C C = c(z)p(z)^{\sigma} \int_0^1 p(j)^{1-\sigma} dj$$

Intermediate Goods

- Solve it for $c(z)$ as

$$c(z) = P^C C \left[\int_0^1 p(j)^{1-\sigma} dj \right]^{-1} p(z)^{-\sigma}$$

- Substitute it into the production function:

$$C = P^C C \left[\int_0^1 p(j)^{1-\sigma} dj \right]^{-1} \left(\int_0^1 p(z)^{1-\sigma} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- It simplifies to

$$1 = P^C \left[\int_0^1 p(j)^{1-\sigma} dj \right]^{\frac{1}{\sigma-1}}$$

Intermediate Goods

- Finally,

$$P^C = \left[\int_0^1 p(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$$

- RHS is the **price index** of the intermediate goods.

Intermediate Goods

- The input demand is

$$c(z) = Cp(z)^{-\sigma}\left(\int_0^1 [p(j)^{1-\sigma}]dj\right)^{\frac{\sigma}{1-\sigma}}$$
$$= Cp(z)^{-\sigma}(P^C)^{\sigma}$$

- Thus,

$$\frac{c(z)}{C} = \left[\frac{p(z)}{P^C}\right]^{-\sigma}$$

- Relative demand for input z is decreasing in the relative price of the input.

Final Investment Goods

- Let X and P^X denote the quantity of the final investment good and its price.
- Let $x(z)$ denote the input of variety z.
- The production technology is

$$X = \left(\frac{1}{\xi}\right)\left(\int_0^1 x(z)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 1$ is the elasticity of substitution between a two inputs.
- A reduction in ξ means a higher productivity.

Intermediate Goods

- Cost minimization implies

$$x(j) = p(z)^{\sigma} p(j)^{-\sigma} x(z)$$

- Zero profit in the final-goods market:

$$P^X X - \int_0^1 p(j) x(j) dj = 0$$

- Thus,

$$P^X X = x(z) p(z)^{\sigma} \int_0^1 p(j)^{1-\sigma} dj$$

Intermediate Goods

- Solve it for $x(z)$ to obtain

$$x(z) = P^X X \left[\int_0^1 p(j)^{1-\sigma} dj\right]^{-1} p(z)^{-\sigma}$$

- Substitute it into the production function:

$$\xi X = P^X X \left[\int_0^1 p(j)^{1-\sigma} dj\right]^{-1} \left(\int_0^1 p(z)^{1-\sigma} dz\right)^{\frac{\sigma}{\sigma-1}}$$

- It follows that

$$\xi = P^X \left[\int_0^1 p(j)^{1-\sigma} dj\right]^{\frac{1}{\sigma-1}}$$

Intermediate Goods

- Use P^C to obtain

$$\xi = P^X \left[\int_0^1 p(j)^{1-\sigma} dj \right]^{\frac{1}{\sigma-1}} = \frac{P^X}{P^C}$$

- A reduction in ξ = a reduction in the relative input price.
- The input demand function:

$$\frac{x(z)}{X} = \xi^{1-\sigma} \left[\frac{p(z)}{P^X} \right]^{-\sigma}$$

Intermediate Goods

- Each monopolist faces the following:

$$\frac{c(z)}{C} = \left[\frac{p(z)}{P^C}\right]^{-\sigma}$$

$$\frac{x(z)}{X} = \xi^{1-\sigma}\left[\frac{p(z)}{P^X}\right]^{-\sigma}$$

$$\xi = \frac{P^X}{P^C}$$

- Monopolist's profit is

$$\Pi(z) = p(z)y(z) - Rk(z) - Wn(z)$$
$$y(z) = c(z) + x(z)$$

Intermediate Goods

- Our normalization $P^C = 1$ implies $P^X = \xi$.
- Thus,

$$\frac{c(z)}{C} = \left[\frac{p(z)}{1}\right]^{-\sigma} = p(z)^{-\sigma}$$
$$\frac{x(z)}{X} = \xi^{1-\sigma}\left[\frac{p(z)}{\xi}\right]^{-\sigma} = \xi p(z)^{-\sigma}$$

- Profit is

$$\Pi(z) = p(z)y(z) - Rk(z) - Wn(z)$$
$$y(z) = p(z)^{-\sigma}C + \xi p(z)^{-\sigma}X$$

Intermediate Goods

- The input demand function is

$$y(z) = p(z)^{-\sigma}[C + \xi X]$$

- $C + \xi X$ is the aggregate demand, determined by the household sector. So, it is given.
- The inverse demand function is

$$p(z) = \left(\frac{y(z)}{C + \xi X}\right)^{-\frac{1}{\sigma}}$$

- The profit is

$$\Pi(z) = (C + \xi X)^{\frac{1}{\sigma}} y(z)^{\frac{\sigma - 1}{\sigma}} - Rk(z) - Wn(z)$$

Intermediate Goods

- Production function:

$$y(z) = Ak(z)^{\alpha} n(z)^{1-\alpha}$$

- Profit maximization:

$$\Pi(z) = (C + \xi X)^{\frac{1}{\sigma}} [Ak(z)^{\alpha} n(z)^{1-\alpha}]^{\frac{\sigma-1}{\sigma}} - Rk(z) - Wn(z)$$

- FOCs:

$$(C + \xi X)^{\frac{1}{\sigma}} \frac{\sigma - 1}{\sigma} [Ak^{\alpha} n^{1-\alpha}]^{\frac{-1}{\sigma}} \alpha A k^{\alpha-1} n^{1-\alpha} = R$$

$$(C + \xi X)^{\frac{1}{\sigma}} \frac{\sigma - 1}{\sigma} [Ak^{\alpha} n^{1-\alpha}]^{\frac{-1}{\sigma}} (1 - \alpha) A k^{\alpha} n^{-\alpha} = W$$

Intermediate Goods

- Rewrite them as

$$\frac{\sigma-1}{\sigma}\left[\frac{y(z)}{C+\xi X}\right]^{\frac{-1}{\sigma}}\alpha A k^{\alpha-1}n^{1-\alpha}=R$$

$$\frac{\sigma-1}{\sigma}\left[\frac{y(z)}{C+\xi X}\right]^{\frac{-1}{\sigma}}(1-\alpha)A k^{\alpha}n^{-\alpha}=W$$

- Thus,

$$\frac{\sigma-1}{\sigma}p(z)\alpha A k^{\alpha-1}n^{1-\alpha}=R$$

$$\frac{\sigma-1}{\sigma}p(z)(1-\alpha)A k^{\alpha}n^{-\alpha}=W$$

Intermediate Goods

- Solve them for input price

$$p(z) = (1 + \mu)\frac{R}{MPL}$$

$$p(z) = (1 + \mu)\frac{W}{MPK}$$

- The markup rate is

$$\mu = \frac{1}{\sigma - 1}$$

- Note: $1 + \mu$ is defined as μ in the article.

General Equilibrium

- Normalization:

$$P^C = 1$$

- All intermediate firms are symmetric:

$$p(z) = p$$
$$k(z) = k$$
$$n(z) = n$$

- As a result

$$P^C = \left[\int_0^1 p^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} = p = 1$$

General Equilibrium

- Equations determining the factor prices:

$$\frac{\sigma-1}{\sigma}p(z)\alpha A k^{\alpha-1}n^{1-\alpha}=R$$

$$\frac{\sigma-1}{\sigma}p(z)(1-\alpha)A k^{\alpha}n^{-\alpha}=W$$

- Thus,

$$\frac{Rk}{y}=\frac{1}{1+\mu}\alpha, \qquad \frac{Wn}{y}=\frac{1}{1+\mu}(1-\alpha)$$

- An increase in μ reduces both capital and labor shares.

General Equilibrium

- The sum of the capital and labor shares is

$$\frac{RK}{Y} + \frac{WN}{Y} = \frac{1}{1+\mu}\alpha + \frac{1}{1+\mu}(1-\alpha)$$
$$= \frac{1}{1+\mu} < 1$$

- What is left is **profit share**.

$$\frac{\Pi}{\mathrm{Y}} = 1 - \frac{1}{1+\mu} = \frac{\mu}{1+\mu}$$

Markups in the US

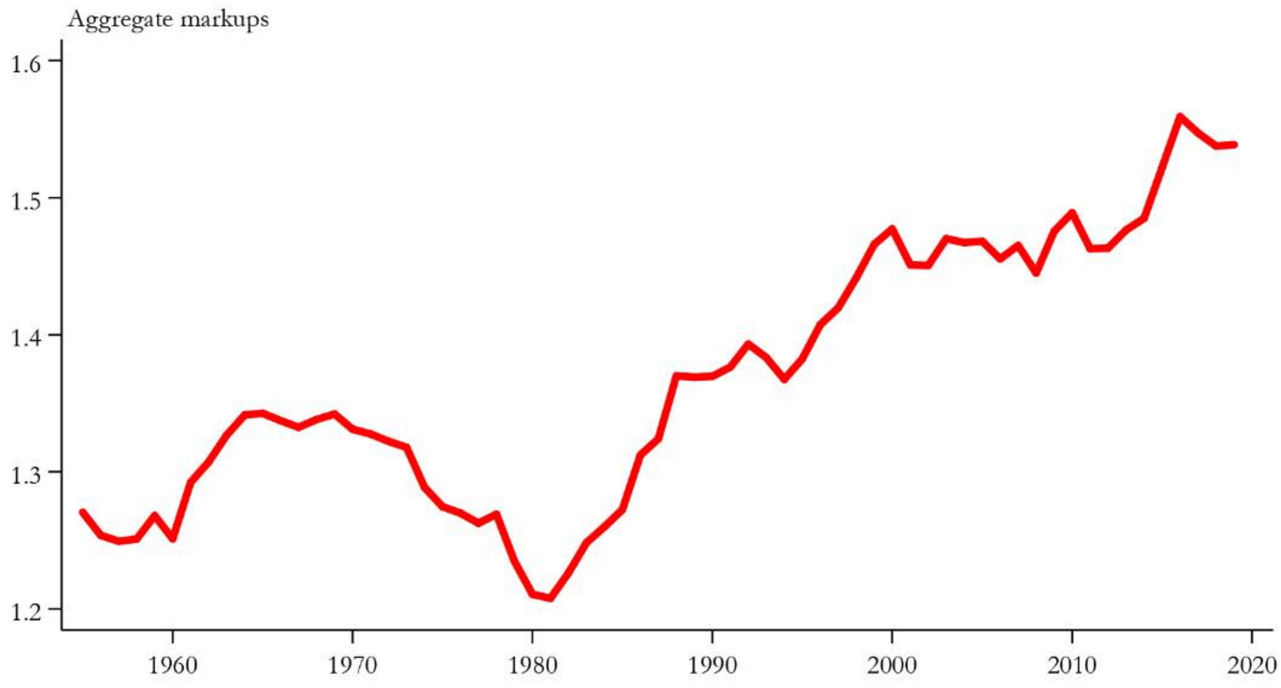

Figure 3. Aggregate markups in the United States

Revenue-weighted average markup of US publicly traded firms (source: De Loecker, Eeckhout & Unger, 2020)

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Markups and the Profit Share

- Suppose that μ increases from 0.2 to 0.5. Then,

$$\frac{\mu}{1+\mu} = \frac{0.2}{1.2} = 0.17$$

$$\frac{\mu}{1+\mu} = \frac{0.5}{1.5} = 0.33$$

- Thus, the profit share of income nearly doubles!

Profits in the US

Figure 6. Average ratio of profits to wage bill of publicly traded firms in the US

Profit to wage bill ratio is employement-weighed; 5-year moving average, annual data in dashes (source: De Loecker, Eeckhout & Unger, 2020, and own calculations)

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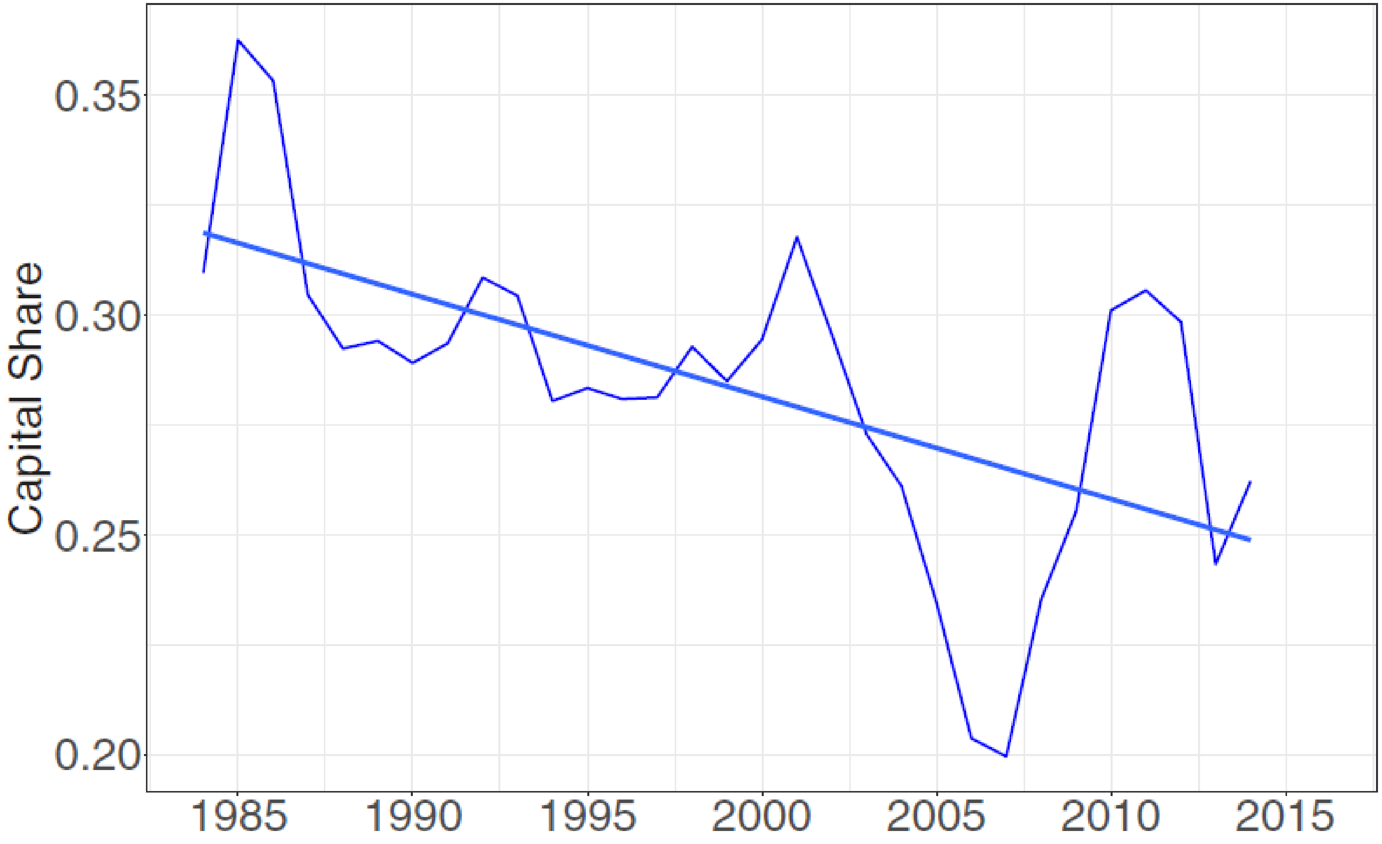
Capital Share in the US



Barkai, Simcha. "Declining labor and capital shares." *Journal of Finance* (2020) 2421-2463.

Closing the Model

- The final piece of the model is households.
- The household sector is the owner of capital and labor. Utility maximization problem is

$$\max_{\{C_t, N_t, X_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{1}{2} N_t^2 \right]$$

$$s.t.$$

$$K_{t+1} = (1-\delta)K_t + X_t$$

$$C_t + \xi_t X_t = W_t N_t + R_t K_t + \Pi_t$$

Closing the Model

- The current-value Lagrangian is

$$\sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \frac{1}{2} N_t^2 + \lambda_t [W_t N_t + R_t K_t + \Pi_t - C_t - \xi_t K_{t+1} + \xi_t (1-\delta) K_t] \right\}$$

- FOCs are:

$$\frac{1}{C_t} = \lambda_t$$
$$N_t = \lambda_t W_t$$
$$\lambda_t \xi_t = \beta \lambda_{t+1} [R_{t+1} + \xi_{t+1}(1-\delta)]$$

with the original constraints and the transversality condition ($\lim_{t\to\infty} \beta^t \lambda_t k_t = 0$).

Closing the Model

- Eliminate the multipliers to obtain:

$$N_t = \frac{1}{C_t} W_t$$

$$R_{t+1} = \frac{\lambda_t \xi_t}{\beta \lambda_{t+1}} - \xi_{t+1}(1-\delta)$$

$$= \frac{C_{t+1} \xi_t}{\beta C_t} - \xi_{t+1}(1-\delta)$$

Steady-State Equilibrium

- Drop all time subscripts:

$$NC = W$$

$$R = \xi \frac{1}{\beta} - \xi(1 - \delta)$$

$$\delta K = X$$

$$C + \xi X = WN + RK + \Pi = Y$$

- Demand = Supply:

$$k = K$$

$$n = N$$

Steady-State Equilibrium

- Factor prices

$$R = \frac{1}{1+\mu} \alpha A K^{\alpha-1} N^{1-\alpha}$$

$$W = \frac{1}{1+\mu} (1-\alpha) A K^{\alpha} N^{-\alpha}$$

- Or,

$$R = \frac{1}{1+\mu} \alpha A \left(\frac{K}{N}\right)^{\alpha-1}$$

$$W = \frac{1}{1+\mu} (1-\alpha) A \left(\frac{K}{N}\right)^{\alpha}$$

Steady-State Equilibrium

- First, notice that

$$R = \xi \frac{1}{\beta} - \xi(1 - \delta)$$

$$R = \frac{1}{1+\mu} \alpha A \left(\frac{K}{N}\right)^{\alpha - 1}$$

- The above equations uniquely pins down the capital-labor ratio.
- Let the solution be Ω, which is a number.

Steady-State Equilibrium

- Other conditions are

$$NC = W$$
$$\delta K = X$$
$$C + \xi X = AK^{\alpha}N^{1-\alpha}$$

- Or,

$$\frac{W}{N} + \xi\delta K = AK^{\alpha}N^{1-\alpha}$$

- Divide both sides by N:

$$\frac{W}{N^2} + \xi\delta\frac{K}{N} = A\left(\frac{K}{N}\right)^{\alpha}$$

Steady-State Equilibrium

- Because we know the capital-labor ratio,

$$\frac{W}{N^2} + \xi\delta\Omega = A\Omega^{\alpha}$$

$$W = \frac{1}{1+\mu}(1-\alpha)A\Omega^{\alpha}$$

- We can find N.
  - We can then find C from $NC = W$.
  - We can then find K from $\frac{K}{N} = \Omega$.

Replication

- Let us replicate Table 4 in the paper.
- Under our Cobb-Douglas specification, we can replicate columns 1, 3, and 5.

TABLE IV

EVALUATING LABOR SHARE'S DECLINE (PERCENT CHANGES ACROSS STEADY STATES)

		CD $\hat{\xi}$	CES $\hat{\xi}$	CD $\hat{\mu}$	CES $\hat{\mu}$	CD $(\hat{\xi}, \hat{\mu})$	CES $(\hat{\xi}, \hat{\mu})$
(i)	Labor share (percentage points)	0.0	−2.6	−3.1	−2.6	−3.1	−4.9
(ii)	Capital share (percentage points)	0.0	2.6	−1.9	−2.4	−1.9	−0.1
(iii)	Profit share (percentage points)	0.0	0.0	5.0	5.0	5.0	5.0
(iv)	Consumption	18.1	20.1	−5.2	−5.4	10.7	12.4
(v)	Nominal investment	18.1	30.8	−11.1	−12.7	3.7	11.9
(vi)	Labor input	0.0	−1.4	−3.2	−2.9	−3.2	−4.2
(vii)	Capital input	51.6	67.8	−11.1	−12.7	33.2	43.6
(viii)	Output	18.1	22.8	−6.3	−6.8	9.4	12.3
(ix)	Wage	18.1	19.2	−8.2	−8.2	7.1	7.7
(x)	Rental rate	−22.1	−22.1	0.0	0.0	−22.1	−22.1
(xi)	Capital-to-output	28.4	36.6	−5.2	−6.4	21.8	27.9
(xii)	Welfare equivalent consumption	18.1	22.1	−3.0	−3.4	13.2	15.8

Replication

- Parameter values are (Footnote 26 in the paper):
  - $\xi = 1$
  - $\delta = 0.1$
  - $\beta = 0.91$
  - $\alpha = 0.4$
- What is “welfare equivalent consumption”?
  - The steady state welfare is $\ln C - \frac{1}{2}N^2$, which is measured in utility units that have no interpretation.
  - We can transform welfare in consumption units by the Lagrange multiplier of the household: $\left(\ln C - \frac{1}{2}N^2\right) \times \frac{1}{\lambda}$.

Replication

- To evaluate the impact of an increase in market power on the steady-state variables, we follow the paper to consider "a markup shock that increases the profit share from an initial level of 3% to a final level of 8%".
- Thus, let the profit share be π, then, from page 50,

$$\mu = \frac{\pi}{1 - \pi}$$

Replication

- Here is a Python code computing the change in the labor share.
- My function solmodel() is a mapping from the profit share into the steady-state labor share.
- I used the function to calculate the change in the labor share.

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import root

def solmodel(ps):
    ξ = 1
    β = 0.91
    δ = 0.1
    α = 0.4
    μ = ps/(1-ps)
    A = 1
    R = ξ*(1/β -1 +δ)
    Ω = (R*(1+μ)/(α*A))**(1/(α-1))
    W = (1-α)*A*(Ω**α)/(1+μ)
    N = ((α*A*(Ω**α) - ξ*δ*Ω)/W)**(-1/2)
    C = W/N
    K = N*Ω
    Y = A*(K**α)*(N**(1-α))
    ls = W*N/Y
    cs = R*K/Y
    ps = 1-ls-cs
    welfare = np.log(C) -(N**2)/2
    return ls

change = solmodel(0.08) - solmodel(0.03)
print(f'Change in the labor share is {change*100:.2f} percentage points')
```

```
Change in the labor share is -3.00 percentage points
```

Further Readings

- Elsby, Michael WL, Bart Hobijn, and Ayşegül Şahin. "The decline of the US labor share." *Brookings Papers on Economic Activity* (2013) 1-63.
- De Loecker, Jan, Jan Eeckhout, and Gabriel Unger. "The rise of market power and the macroeconomic implications." *Quarterly Journal of Economics* (2020) 561-644.

Reading Assignment

- Matsuyama, Kiminori. "Complementarities and cumulative processes in models of monopolistic competition." *Journal of Economic Literature* (1995)
- Available from NUCT.
- Read section 3A (The Basic Model).