Lecture 9

Heterogeneous Jobs

6/23

Goals

- In the Pissarides model, all jobs are **homogeneous**.
- This month, we shall introduce **heterogeneity** (in one way or another) to study **wage inequality**, a very important issue in the real world.
- As scientists, we want to understand wage inequality <u>by modelling it</u>.
- To make your life easier, (whenever possible) I modify the model notations of the original articles to be consistent with those of Pissarides.
  - This way you can quickly understand the new model.

The Big Picture

- The textbook DMP model is summarized by the set of (θ, w, u) satisfying

$$(r+\lambda)\frac{c}{q(\theta)} = p - w$$

$$w = \beta p + (1-\beta)z + \beta c\theta$$

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

- We shall study several avenues for allowing w to vary across workers (which typically requires p to vary across workers).

Heterogeneous Jobs without Labor Market Segmentation

Acemoglu, Daron. "Good jobs versus bad jobs." *Journal of labor Economics* (2001) 1-21.

Heterogeneous Jobs

- Consider (1.6) in Pissarides, which is
$$rV = -pc + q(\theta)(J - V)$$
- This is the value of creating a vacancy.
- Simplify by $c = 0$.
- With heterogeneous jobs (G and B),
$$rV_G = q(\theta)(J_G - V_G)$$
$$rV_B = q(\theta)(J_B - V_B)$$
- These are (7) in Acemoglu.

Heterogeneous Jobs

- Consider (1.8) in Pissarides, which is
$$rJ = p - w + \lambda(V - J)$$
- This is the value of a filled job.
- With job heterogeneity,
$$rJ_G = p_G - w_G + \lambda(V_G - J_G)$$
$$rJ_B = p_B - w_B + \lambda(V_B - J_B)$$
- These are (6) in Acemoglu.
- Labor productivity is 1 for both jobs. However, products are sold at different prices.

Heterogeneous Jobs

- Consider (1.11) in Pissarides, which is
$$rW = w + \lambda(U - W)$$
- This is the value of employment.
- With heterogeneous jobs,
$$rW_G = w_G + \lambda(U - W_G)$$
$$rW_B = w_B + \lambda(U - W_B)$$
- These are (5) in Acemoglu.

Heterogeneous Jobs

- Consider (1.10) in Pissarides, which is
$$rU = z + \theta q(\theta)(W - U)$$
- This is the value of job search.
- With heterogeneous jobs,
$$rU = z + \theta q(\theta)[\phi W_B + (1 - \phi)W_G - U]$$
- ϕ is the (equilibrium) proportion of B jobs.
- We are assuming that a job search is random. Thus, given a meeting, the probability that your job is B is ϕ and the probability that your job is G is $1 - \phi$.

Heterogeneous Jobs

- (1.17) in Pissarides:
$$W_i - U = \beta(J_i + W_i - V - U)$$
- For brevity, we drop subscripts.
- With heterogeneous jobs,
$$W_G - U = \beta(J_G + W_G - V_G - U)$$
$$W_B - U = \beta(J_B + W_B - V_B - U)$$
- Or,
$$(1-\beta)(W_G - U) = \beta(J_G - V_G)$$
$$(1-\beta)(W_B - U) = \beta(J_B - V_B)$$
- These are (8) in Acemoglu.

Heterogeneous Jobs

- In Pissarides, the free entry condition was
$$V = 0$$
- With heterogeneous jobs (G and B),
$$V_G = k_G$$
$$V_B = k_B$$
- In this model, there is no cost c of maintaining a vacancy. Instead, the cost of creating a vacancy must be paid at the beginning.

Heterogeneous Jobs

- In Pissarides, labor productivity p is a parameter.
- In Acemoglu, p_G and p_B are market prices.
- Consider the following aggregate production function:

$$Y = \left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{1/\rho}$$

- Y_B : amount of type-B input.
- Y_G : amount of type-G input.
- $\pi \in (0,1)$ and $\rho \leq 1$ are parameters.

Heterogeneous Jobs

- Let us assume that the product markets are competitive.
- Let the input prices be p_B and p_G.
- The representative final-output firm chooses the input demands to maximize profit:

$$\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{1/\rho} - p_B Y_B - p_G Y_G$$

- FOCs are:

$$\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{\frac{1}{\rho}-1} \pi Y_B^{\rho-1} = p_B$$

$$\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{\frac{1}{\rho}-1} (1-\pi) Y_G^{\rho-1} = p_G$$

Heterogeneous Jobs

- Take the ratio of the FOCs to obtain

$$\frac{\pi}{1-\pi}\left(\frac{Y_B}{Y_G}\right)^{\rho-1} = \frac{p_B}{p_G}$$

- Equivalently,

$$\frac{Y_B}{Y_G} = \left(\frac{1-\pi}{\pi}\frac{p_B}{p_G}\right)^{-\frac{1}{1-\rho}}$$

- The elasticity of substitution (in absolute value) is

$$\frac{1}{1-\rho}$$

Heterogeneous Jobs

- If $\rho > 0$, then Y_G and Y_B are said to be **gross substitutes** in production.
- If $\rho < 0$, then Y_G and Y_B are said to be **gross complements** in production.
- A few special cases:
  - $\rho \to 1$: Y_G and Y_B are **perfect substitutes**.
  - $\rho \to 0$: Cobb-Douglas production function.
  - $\rho \to -\infty$: Leontief production function.

Heterogeneous Jobs

- Labor force is normalized to 1.
- Let u denote the unemployment rate.
- Then, total employment at any moment is

$$1 - u$$

- Because ϕ is the proportion of B jobs and labor productivity is 1, total B products are

$$Y_B = (1 - u)\phi$$

- Similarly, total G products are

$$Y_G = (1 - u)(1 - \phi)$$

Heterogeneous Jobs

- Substitute $Y_B = (1-u)\phi$ and $Y_G = (1-u)(1-\phi)$ into the FOCs to obtain

$$p_B = \pi\phi^{\rho-1}[\pi\phi^{\rho} + (1-\pi)(1-\phi)^{\rho}]^{\frac{1-\rho}{\rho}}$$

$$p_G = (1-\pi)(1-\phi)^{\rho-1}[\pi\phi^{\rho} + (1-\pi)(1-\phi)^{\rho}]^{\frac{1-\rho}{\rho}}$$

- These are (11) in Acemoglu.
- You should be able to replicate the results.

Heterogeneous Jobs

- It will be convenient to rewrite p_G as

$$p_G = (1-\pi)(1-\phi)^{\rho-1}\left[\frac{\pi\phi^{\rho}}{(1-\pi)(1-\phi)^{\rho}} + 1\right]^{\frac{1-\rho}{\rho}} \times \left[(1-\pi)(1-\phi)^{\rho}\right]^{\frac{1-\rho}{\rho}}$$

- Thus,

$$p_G = (1-\pi)^{\frac{1}{\rho}}\left[\frac{\pi}{1-\pi}\left(\frac{\phi}{1-\phi}\right)^{\rho} + 1\right]^{\frac{1-\rho}{\rho}} = P_G(\phi)$$

- It is now easy to show that $\rho < 1$ implies

$$P_G'(\phi) > 0$$

Heterogeneous Jobs

- Similarly, rewrite p_B as

$$p_B = \pi\phi^{\rho-1}\left[1 + \frac{(1-\pi)(1-\phi)^\rho}{\pi\phi^\rho}\right]^{\frac{1-\rho}{\rho}} \times [\pi\phi^\rho]^{\frac{1-\rho}{\rho}}$$

- Thus,

$$p_B = \pi^{\frac{1}{\rho}}\left[1 + \frac{1-\pi}{\pi}\left(\frac{1-\phi}{\phi}\right)^\rho\right]^{\frac{1-\rho}{\rho}} = P_B(\phi)$$

- It is now easy to show that $\rho < 1$ implies

$$P_B'(\phi) < 0$$

Heterogeneous Jobs

- As usual, from the bargaining outcome and Bellman equations, we can obtain the wage equations:

$$w_G = \beta(p_G - rk_G) + (1-\beta)rU$$
$$w_B = \beta(p_B - rk_B) + (1-\beta)rU$$

- Replications will be your assignment.

Heterogeneous Jobs

- Let us prove that $k_G > k_B$ implies $w_G > w_B$.
- First, use

$$w_G = \beta(p_G - rk_G) + (1-\beta)rU$$
$$rV_G = q(\theta)(J_G - V_G)$$
$$rJ_G = p_G - w_G + \lambda(V_G - J_G)$$
$$V_G = k_G,$$

to obtain

$$rk_G = \frac{q(\theta)(1-\beta)(p_G - rU)}{r + \lambda + (1-\beta)q(\theta)}$$

- Do the same for B.

Heterogeneous Jobs

- Consider

$$rk_G = \frac{q(\theta)(1-\beta)(p_G - rU)}{r + \lambda + (1-\beta)q(\theta)} = \frac{p_G - rU}{Q(\theta)}$$

$$rk_B = \frac{q(\theta)(1-\beta)(p_B - rU)}{r + \lambda + (1-\beta)q(\theta)} = \frac{p_B - rU}{Q(\theta)}$$

- $Q(\theta)$ is defined for brevity of expressions.
- These are (13) and (14) in Acemoglu.
- From these expressions,

$$k_G > k_B \Leftrightarrow p_G > p_B$$

Heterogeneous Jobs

- The wage equations are

$$w_G = \beta(p_G - rU) - \beta rk_G + rU$$
$$w_B = \beta(p_B - rU) - \beta rk_B + rU$$

- Observe that

$$rk_G = \frac{p_G - rU}{Q(\theta)}, rk_B = \frac{p_B - rU}{Q(\theta)}$$

- Thus,

$$w_G = [Q(\theta) - 1]\beta rk_G + rU$$
$$w_B = [Q(\theta) - 1]\beta rk_B + rU$$

Heterogeneous Jobs

- Consider

$$w_G = [Q(\theta) - 1]\beta r k_G + rU$$
$$w_B = [Q(\theta) - 1\ ]\beta r k_B + rU$$

- From these equations,

$$w_G - w_B = [Q(\theta) - 1]\beta(rk_G - rk_B)$$

- Thus, we finally show that

$$w_G > w_B \Leftrightarrow k_G > k_B$$

- Thus, “good jobs” are jobs that are more costly to create.

Heterogeneous Jobs

- Consider
$$w_G - w_B = [Q(\theta) - 1]\beta(rk_G - rk_B)$$
- Substitute $Q(\theta)$ back into this expression to obtain
$$w_G - w_B = \frac{\beta}{1-\beta}\frac{r+\lambda}{q(\theta)}r(k_G - k_B)$$
- Wage inequality disappears if
  - $\beta = 0$ because bargaining implies $W_B = W_G = U$.
  - $q(\theta) \to \infty$ because frictions disappear.
  - $r \to 0$ because you do no care about frictions.
  - $k_G - k_B = 0$ because heterogeneity disappears.

Heterogeneous Jobs

- Consider rU:
$$rU = z + \theta q(\theta)[\phi W_B + (1-\phi) W_G - U]$$
- Substitute
$$(1-\beta)(W_G - U) = \beta(J_G - V_G)$$
$$(1-\beta)(W_B - U) = \beta(J_B - V_B)$$
$$rV_G = q(\theta)(J_G - V_G)$$
$$rV_B = q(\theta)(J_B - V_B)$$
$$V_G = k_G$$
$$V_B = k_B$$
into the above.

Heterogeneous Jobs

- Following the same procedure as in Pissarides, we obtain

$$rU = z + \theta \frac{\beta}{1-\beta}[\phi r k_B + (1-\phi) r k_G]$$

- This is supposed to be (16) in Acemoglu.
  - I do not know how to obtain (16), but I would not warry because our expression is simpler (= better).
  - In addition, I believe $\frac{\beta}{1-\beta}$ is missing in (15).
  - Those things can happen.

Heterogeneous Jobs

- rU is interpreted as the **reservation wage**. Thus,

$$rU = z + \theta \frac{\beta}{1-\beta}[\phi r k_B + (1-\phi) r k_G]$$
$$\equiv w_R(\theta, \phi)$$

- $w_R(\theta, \phi)$ satisfies $\frac{\partial w_R}{\partial \theta} > 0$ and $\frac{\partial w_R}{\partial \phi} < 0$.
- Remember that in Pissarides,

$$rU = z + \theta \frac{\beta}{1-\beta} pc$$

- These are the same if $\phi k_B + (1-\phi) k_G = \frac{pc}{r}$.

Heterogeneous Jobs

- Substitute the reservation wage $rU = w_R(\theta, \phi)$ and prices, $p_B = P_B(\phi)$ and $p_G = P_G(\phi)$, into
$$rk_G = \frac{q(\theta)(1-\beta)(p_G - rU)}{r + \lambda + (1-\beta)q(\theta)}$$
$$rk_B = \frac{q(\theta)(1-\beta)(p_B - rU)}{r + \lambda + (1-\beta)q(\theta)}$$
- Then, a steady-state equilibrium is given by
$$rk_G = \frac{q(\theta)(1-\beta)\big(P_G(\phi) - w_R(\theta, \phi)\big)}{r + \lambda + (1-\beta)q(\theta)}$$
$$rk_B = \frac{q(\theta)(1-\beta)\big(P_B(\phi) - w_R(\theta, \phi)\big)}{r + \lambda + (1-\beta)q(\theta)}$$

Heterogeneous Jobs

- Consider

$$rk_G = \frac{q(\theta)(1-\beta)\big(P_G(\phi) - w_R(\theta,\phi)\big)}{r + \lambda + (1-\beta)q(\theta)}$$

- Rewrite it as

$$\left[\frac{r+\lambda}{(1-\beta)q(\theta)} + 1\right] rk_G = P_G(\phi) - w_R(\theta,\phi)$$

- With $q'(\theta) < 0$, $P_G'(\phi) > 0$, $\frac{\partial w_R}{\partial \theta} > 0$, and $\frac{\partial w_R}{\partial \phi} < 0$, the equation above implies $\frac{d\theta}{d\phi} > 0$. This is the upward-sloping "good-job locus" in Acemoglu.

Heterogeneous Jobs

- Consider

$$rk_B = \frac{q(\theta)(1-\beta)\big(P_B(\phi) - w_R(\theta,\phi)\big)}{r + \lambda + (1-\beta)q(\theta)}$$

- Rewrite it as

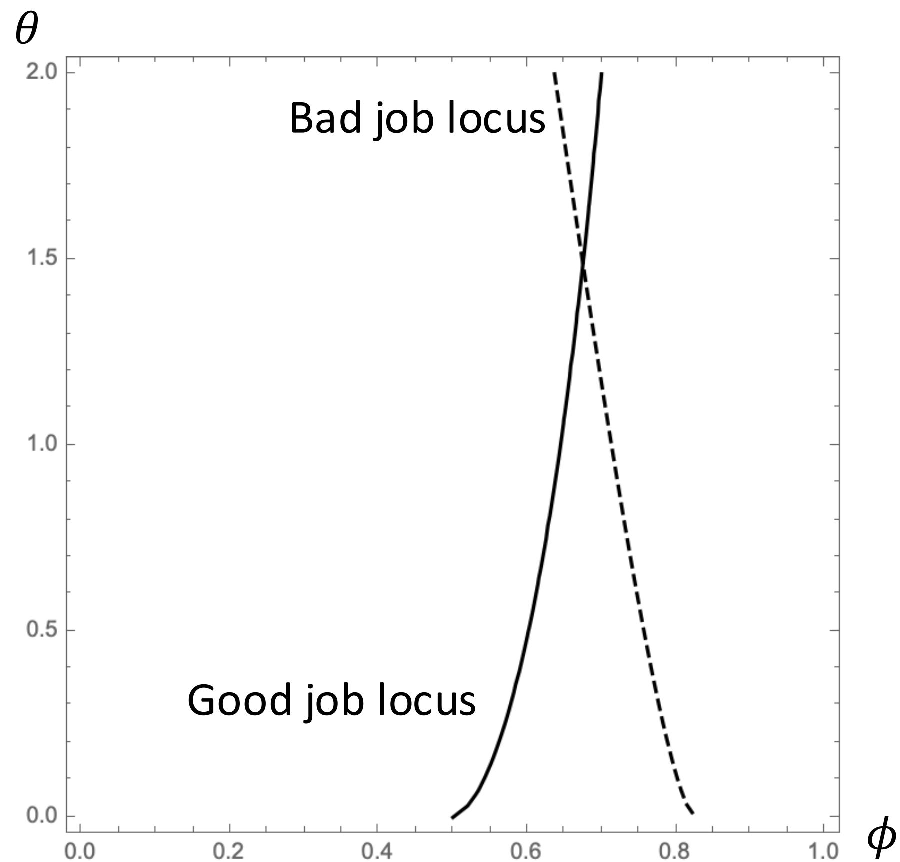
$$\left[\frac{r+\lambda}{(1-\beta)q(\theta)} + 1\right] rk_B = P_B(\phi) - w_R(\theta,\phi)$$

- Note that $P_B'(\phi) < 0$ (page 18). The "bad job locus" is downward sloping if and only if

$$P_B'(\phi) - \frac{\partial w_R}{\partial \phi}(\theta,\phi) < 0$$

Heterogeneous Jobs

- Let $q(\theta) = A\theta^{-\alpha}$
- Parameters:
  - $A = 1$
  - $\alpha = 0.6$
  - $r = 0.1$
  - $\lambda = 0.4$
  - $\beta = 0.6$
  - $\rho = -0.8$
  - $\pi = 0.7$
  - $k_G = 2$
  - $k_B = 1$
  - $z = 0.1$



Heterogeneous Jobs with Labor Market Segmentation

Miyamoto, Hiroaki. "Growth and non-regular employment." *The BE Journal of Macroeconomics* (2016) 523-554.

Heterogeneous Jobs

- There are two contracts (i.e., jobs):
  - Type G: **Regular employment** = Long-term contracts = "Good" jobs.
  - Type B: **Non-regular employment** = Short-term contracts = "Bad" jobs.
- The separation rates for B and G satisfy $\lambda_B > \lambda_G$.
- Let w_G denote the wage rate for G jobs. $f_G \in [0,1]$ is a parameter. **Firing costs** for G jobs are $f_G w_G$. This amount must be paid to the worker when separated. Later, we will assume $f_G = 0$.

Search-Matching Frictions

- Labor force L is normalized to $L = 1$.
- What is new today is that there are two **segmented labor markets**, one for G and the other for B. Thus,
$$M_G = m(u_G, v_G)$$
$$M_B = m(u_B, v_B)$$
- The key assumption is that workers choose which labor market to enter.
- u_G: number of job seekers in G sector.
- u_B: number of job seekers in B sector.

Search-Matching Frictions

- Tightness for each market must be specified:

$$\frac{v_G}{u_G} = \theta_G, \qquad \frac{v_B}{u_B} = \theta_B$$

- The vacancy-filling rate for G is

$$\frac{m(u_G, v_G)}{v_G} = m(\theta_G^{-1}, 1) \equiv q(\theta_G)$$

- The vacancy-filling rate for B is

$$\frac{m(u_B, v_B)}{v_B} = m(\theta_B^{-1}, 1) \equiv q(\theta_B)$$

Search-Matching Frictions

- The proportion of B labor force (= employees + job seekers) is ϕ.
  - Note: In Miyamoto, ϕ is the proportion of G labor force, not B. I choose this notation to make the model consistent with Acemoglu.
- Total G employees: $Y_G = (1 - \phi) - u_G$
- Total B employees: $Y_B = \phi - u_B$
- In this model (also in Acemoglu), total G (or B) employees and total type G (or B) inputs are the same thing. Just a matter of interpretation.

Production Technology

- As in Acemoglu, the production function is:

$$Y = p\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{1/\rho}$$

- Y_B : amount of type-B input.
- Y_G : amount of type-G input.
- $\pi \in (0,1)$ and $\rho \leq 1$ are parameters.
- p is disembodied technology: $p(t) = p(0)e^{gt}$, or

$$\frac{\dot{p}}{p} = g$$

Input Prices

- Let us assume that the product markets are competitive. There is no search frictions.
- Let the input prices be p_B and p_G.
- The representative final-output firm chooses the input demands to maximize profit:

$$p\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{1/\rho} - p_B Y_B - p_G Y_G$$

- FOCs are:

$$p\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{\frac{1}{\rho}-1} \pi Y_B^{\rho-1} = p_B$$

$$p\left[\pi Y_B^{\rho} + (1-\pi) Y_G^{\rho}\right]^{\frac{1}{\rho}-1} (1-\pi) Y_G^{\rho-1} = p_G$$

Input Prices

- Simplify the FOCs to obtain

$$p\left[\pi + (1-\pi)\left(\frac{Y_G}{Y_B}\right)^{\rho}\right]^{\frac{1}{\rho}-1} \pi = p_B$$

$$p\left[\pi\left(\frac{Y_B}{Y_G}\right)^{\rho} + (1-\pi)\right]^{\frac{1}{\rho}-1} (1-\pi) = p_G$$

Value of Employment

- With segmented labor markets for G and B,
$$rW_G = w_G + \lambda_G(f_G w_G + U_G - W_G) + gW_G$$
$$rW_B = w_B + \lambda_B(U_B - W_B) + gW_B$$
- Define $r - g \equiv \tilde{r}$ and rewrite the equations as
$$(\tilde{r} + \lambda_G)[W_G - U_G] = (1 + \lambda_G f_G)w_G - \tilde{r}U_G$$
$$(\tilde{r} + \lambda_B)[W_B - U_B] = w_B - \tilde{r}U_B$$

Value of Unemployment

- With segmented labor markets,

$$rU_G = pb + \theta_G q(\theta_G)(W_G - U_G) + gU_G$$
$$rU_B = pb + \theta_B q(\theta_B)(W_B - U_B) + gU_B$$

- Or,

$$\tilde{r}U_G = pb + \theta_G q(\theta_G)(W_G - U_G)$$
$$\tilde{r}U_B = pb + \theta_B q(\theta_B)(W_B - U_B)$$

Values of Jobs

- Consider

$$rJ_G = p_G - w_G + \lambda_G(-f_G w_G + V_G - J_G) + gJ_G$$
$$rJ_B = p_B - w_B + \lambda_B(V_B - J_B) + gJ_B$$

- Define $r - g \equiv \tilde{r}$ and rewrite the equations as

$$(\tilde{r} + \lambda_G)[J_G - V_G] = p_G - (1 + \lambda_G f_G)w_G - \tilde{r}V_G$$
$$(\tilde{r} + \lambda_B)[J_B - V_B] = p_B - w_B - \tilde{r}V_B$$

- These are useful for wage bargaining.

Values of Vacancies

- Consider

$$rV_G = -pc_G + q_G(J_G - V_G) + gV_G$$
$$rV_B = -pc_B + q_B(J_B - V_B) + gV_B$$

- We assume $c_B < c_G$ and

$$V_G = 0$$
$$V_B = 0$$

- The equations for vacancies imply:

$$J_G = \frac{pc_G}{q_G}, J_B = \frac{pc_B}{q_B}$$

Job-Creation Conditions

- From the previous 2 pages, we obtain the job-creation conditions as

$$(\tilde{r} + \lambda_G) \frac{pc_G}{q_G} = p_G - (1 + \lambda_G f_G) w_G$$

$$(\tilde{r} + \lambda_B) \frac{pc_B}{q_B} = p_B - w_B$$

Sectoral Choice

- We assume that workers may freely choose a sector to enter by comparing the values of job search.
- If $U_G > U_B$, then job-seekers in B sector want to switch sectors to G. This will decrease θ_G and hence U_G. Thus, in any equilibrium, we have
$$U_G = U_B$$
- See also III-B (Directed Search) in Acemoglu.

Sectoral Choice

- G sector is more costly to enter.
- To be a type-G job-seeker, you need to pay a **training cost** pe, which is proportional to productivity p.
- Thus,

$$U_G - pe = U_B$$

- $e \geq 0$ is a parameter.

Wage Bargaining

- As in Acemoglu, wages are determined by Nash bargaining:

$$(1-\beta)(W_G - U_G) = \beta(J_G - V_G)$$
$$(1-\beta)(W_B - U_B) = \beta(J_B - V_B)$$

- As usual, we can obtain the wage equations:

$$(1+\lambda_G f_G)w_G = \beta p_G + (1-\beta)\tilde{r}U_G$$
$$w_B = \beta p_B + (1-\beta)\tilde{r}U_B$$

- Wage inequality may occur because

$$p_G \neq p_B$$

Balanced-Growth Equilibrium

- Total G jobs created at any moment is
$$\theta_G q(\theta_G) u_G$$
- Total G jobs destroyed at any moment is
$$\lambda_G(1-\phi-u_G)$$
- Thus, in any balanced-growth equilibrium,
$$\theta_G q(\theta_G) u_G = \lambda_G(1-\phi-u_G)$$
- Thus,
$$u_G = \frac{\lambda_G(1-\phi)}{\lambda_G + \theta_G q(\theta_G)}$$

Balanced-Growth Equilibrium

- Total B jobs created at any moment is
$$\theta_B q(\theta_B) u_B$$
- Total B jobs destroyed at any moment is
$$\lambda_B(\phi - u_B)$$
- Thus, in any balanced-growth equilibrium,
$$\theta_B q(\theta_B) u_B = \lambda_B(\phi - u_B)$$
- Thus,
$$u_B = \frac{\lambda_B \phi}{\lambda_B + \theta_B q(\theta_B)}$$

Balanced-Growth Equilibrium

- We rewrite the input ratio Y_B/Y_G as

$$\frac{Y_B}{Y_G} = \frac{\phi - u_B}{1-\phi-u_G}$$
$$= \frac{\phi - \frac{\lambda_B\phi}{\lambda_B+\theta_B q(\theta_B)}}{1-\phi-\frac{\lambda_G(1-\phi)}{\lambda_G+\theta_G q(\theta_G)}} = \frac{\phi}{1-\phi}\frac{1+\frac{\lambda_G}{\theta_G q(\theta_G)}}{1+\frac{\lambda_B}{\theta_B q(\theta_B)}}$$
$$\equiv y(\theta_G,\theta_B,\phi)$$

- Evidently,

$$\frac{\partial y}{\theta_G} < 0, \frac{\partial y}{\theta_B} > 0, \frac{\partial y}{\phi} > 0$$

Balanced-Growth Equilibrium

- Input prices are:

$$p_B = p[\pi + (1-\pi)[y(\theta_G, \theta_B, \phi)]^{-\rho}]^{\frac{1}{\rho}-1}\pi$$
$$\equiv pP_B(\theta_G, \theta_B, \phi)$$

$$p_G = p[\pi[y(\theta_G, \theta_B, \phi)]^{\rho} + (1-\pi)]^{\frac{1}{\rho}-1}(1-\pi)$$
$$\equiv pP_B(\theta_G, \theta_B, \phi)$$

- Evidently,

$$\frac{\partial p_B}{\theta_G} > 0, \frac{\partial p_B}{\theta_B} < 0, \frac{\partial p_B}{\phi} < 0, \frac{\partial p_G}{\theta_G} < 0, \frac{\partial p_G}{\theta_B} > 0, \frac{\partial p_G}{\phi} > 0$$

Balanced-Growth Equilibrium

- As always, we can rewrite the reservation wages as

$$\tilde{r}U_G = pb + pc_G \frac{\beta}{1-\beta}\theta_G$$

$$\tilde{r}U_B = pb + pc_B \frac{\beta}{1-\beta}\theta_B$$

- Substitute them and the input prices into the wage equations to obtain

$$(1+\lambda_G f_G)w_G = \beta p P_G(.) + (1-\beta)pb + pc_G\beta\theta_G$$

$$w_B = \beta p P_B(.) + (1-\beta)pb + pc_B\beta\theta_B$$

Balanced-Growth Equilibrium

- Substitute the wage equations into the job-creation conditions to obtain

$$\frac{(\tilde{r}+\lambda_G)c_G}{q(\theta_G)} = (1-\beta)[P_G(.)-b] - \beta c_G \theta_G$$

$$\frac{(\tilde{r}+\lambda_B)c_B}{q(\theta_B)} = (1-\beta)[P_B(.)-b] - \beta c_B \theta_B$$

Balanced-Growth Equilibrium

- Substitute the expressions for $\tilde{r}U_G$ and $\tilde{r}U_B$ into $U_G - pe = U_B$ to obtain
$$c_G \frac{\beta}{1-\beta}\theta_G - \tilde{r}e = c_B \frac{\beta}{1-\beta}\theta_B$$
- When $e = 0$, we obtain
$$c_G\theta_G = c_B\theta_B$$
- Thus,
$$c_G > c_B \Leftrightarrow \theta_G < \theta_B$$
- B jobs are easier to find.

Balanced-Growth Equilibrium

- A balanced-growth equilibrium is determined by

$$\frac{(\tilde{r}+\lambda_G)c_G}{q(\theta_G)} = (1-\beta)[P_G(\theta_G,\theta_B,\phi)-b]-\beta c_G\theta_G$$

$$\frac{(\tilde{r}+\lambda_B)c_B}{q(\theta_B)} = (1-\beta)[P_B(\theta_G,\theta_B,\phi)-b]-\beta c_B\theta_B$$

$$c_G\theta_G = c_B\theta_B$$

- Let

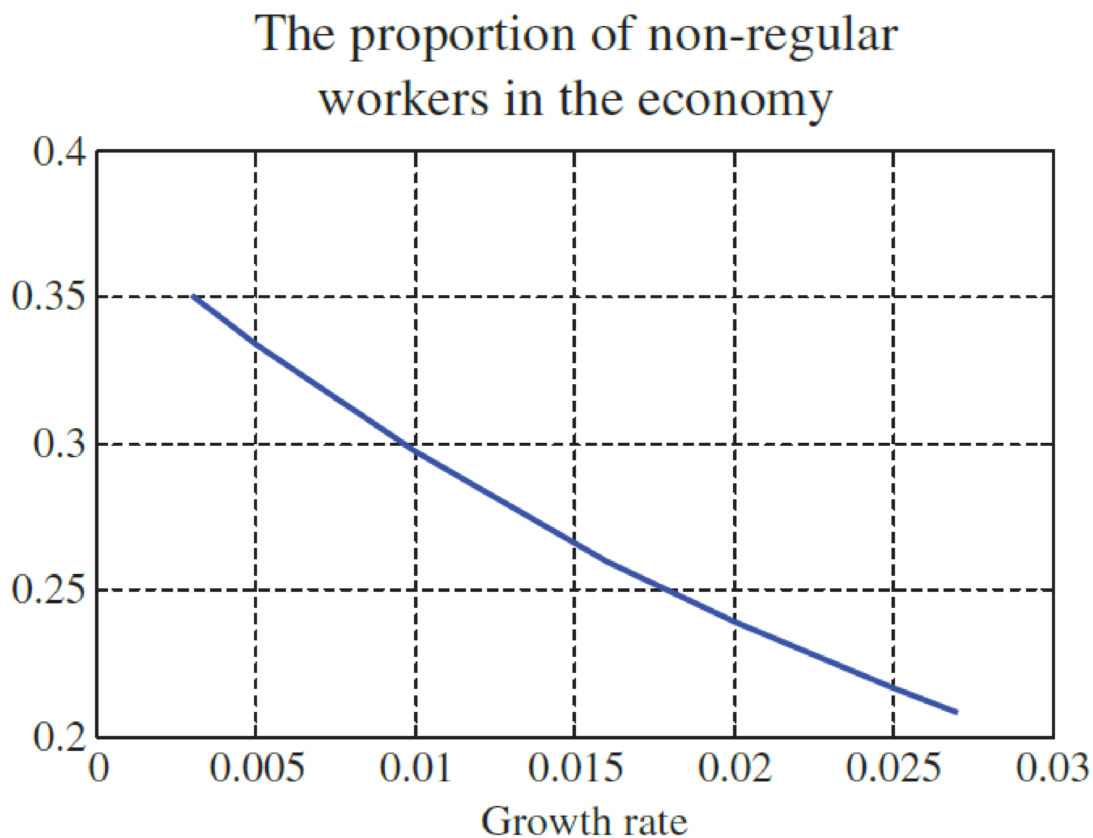
$$\frac{c_B}{c_G} = c < 1$$

- Then, $\theta_G = c\theta_B$.

Balanced-Growth Equilibrium

- A balanced-growth equilibrium is determined by
$$\frac{(\tilde{r}+\lambda_G)c_G}{q(c\theta_B)} = (1-\beta)[P_G(c\theta_B,\theta_B,\phi)-b]-\beta c_B\theta_B$$
$$\frac{(\tilde{r}+\lambda_B)c_B}{q(\theta_B)} = (1-\beta)[P_B(c\theta_B,\theta_B,\phi)-b]-\beta c_B\theta_B$$
- For brevity set $f_G = 0$. Given the solution, we can calculate the wage premium by
$$\frac{w_G}{w_B} = \frac{\beta P_G(.)+(1-\beta)b+c_G\beta\theta_G}{\beta P_B(.)+(1-\beta)b+c_B\beta\theta_B}$$
- In Miyamoto (2016), the wage premium is $\frac{0.668}{0.401}$.

Productivity Slowdown and the Rise of Temporary Contracts



Reading Assignment

- Karabarbounis, Loukas, and Brent Neiman. "The global decline of the labor share." *Quarterly Journal of Economics* (2014) 61-103.
- Downloadable from UNCT.
- Read Section I (Introduction), Section II (Trends in Labor Shares and Investment Prices), and Section III (A Model of the Labor Share).
- Due is on 6/30.
- 6/30 class will focus on this paper.