# Lecture 8

Technological Progress 6/16

#### Goals

- Today, we introduce <u>technological progress</u> into the model of Pissarides.
- With this new component, we can ask whether a faster technological progress reduces the unemployment rate in the long run.
- We shall also study a model of Creative Destruction, in which a higher rate of technological progress causes a <u>shorter duration</u> of jobs.

# Technological Progress and Job Creation

#### Review of the DMP Model

• Consider (1.6), which is  $rV = -pc + q(\theta)(J - V)$ 

- Now consider (1.33), which is  $rV = -pc + q(\theta)(J-V) + \dot{V}$
- Let us introduce V(t) to consider the possibility that V itself grows over time.

#### Review of the DMP Model

Consider:

$$V(t) = -pc\delta t + \frac{1}{1 + r\delta t} \{ q(\theta)\delta t J(t + \delta t) + [1 - q(\theta)\delta t]V(t + \delta t) \}$$

• Multiply both sides by  $1 + r\delta t$  to obtain  $(1 + r\delta t)V(t)$ 

$$= -(1 + r\delta t)pc\delta t + \{q(\theta)\delta tJ(t + \delta t) + [1 - q(\theta)\delta t]V(t + \delta t)\}$$

Arrange terms to obtain

$$r\delta t V(t) = -(1 + r\delta t)pc\delta t + q(\theta)\delta t [J(t + \delta t) - V(t + \delta t)] + V(t + \delta t) - V(t)$$

• Divide both sides by  $\delta t$  to obtain

$$rV(t) = -(1 + r\delta t)pc + q(\theta)[J(t + \delta t) - V(t + \delta t)] + \frac{V(t + \delta t) - V(t)}{\delta t}$$

#### Review of the DMP Model

Note that

$$\lim_{\delta t \to \infty} \frac{V(t + \delta t) - V(t)}{\delta t} = \frac{dV(t)}{dt} = \dot{V}$$

- Take the limit as  $\delta t \to 0$  to obtain (1.33) as  $rV = -pc + q(\theta)(J V) + \dot{V}$
- Similarly, we derived (1.34), (1.37) and (1.38) as follows.

$$rJ = p - w + \lambda(V - J) + \dot{J}$$
  

$$rU = z + \theta q(\theta)(W - U) + \dot{U}$$
  

$$rW = w + \lambda(U - W) + \dot{W}$$

# Technological Progress

- The new ingredient is that we are going to assume that p grows over time.
- In particular,

$$p(t) = p(0)e^{gt}$$

This implies

$$\frac{\dot{p}}{p} = g$$

- Thus, the growth rate of technology is g.
  - We assume g is constant and satisfies g < r.

- Because p grows over time, we cannot find a steady state in the usual sense.
- Instead, we look for a balanced-growth
  equilibrium (BGE), in which all values grow at the
  same rate.
- Thus, we shall now impose

$$\frac{\dot{V}}{V} = \frac{\dot{J}}{J} = \frac{\dot{U}}{U} = \frac{\dot{W}}{W} = \frac{\dot{p}}{p} = g$$

Consider

$$\frac{\dot{V}}{V} = \frac{\dot{J}}{J} = \frac{\dot{U}}{U} = \frac{\dot{W}}{W} = g$$

Interestingly, this implies

$$\dot{V} = gV 
\dot{J} = gJ 
\dot{U} = gU 
\dot{W} = gW$$

• We substitute them into the Bellman equations.

In any balanced-growth equilibrium,

$$rV = -pc + q(\theta)(J - V) + gV$$

$$rJ = p - w + \lambda(V - J) + gJ$$

$$rU = z + \theta q(\theta)(W - U) + gU$$

$$rW = w + \lambda(U - W) + gW$$

• For a technical reason (to be explained), we shall assume that the unemployment benefit z is proportional to p. Thus, we assume

$$z = pb$$
,

where b > 0 is constant.

Thus, we rewrite the equations as

$$(r-g)V = -pc + q(\theta)(J-V)$$

$$(r-g)J = p - w + \lambda(V-J)$$

$$(r-g)U = pb + \theta q(\theta)(W-U)$$

$$(r-g)W = w + \lambda(U-W)$$

- Sometimes we call r-g as the **effective discount** rate.
  - I hope you now understand why we imposed the assumption g < r.
  - We want the effective discount rate to be positive.

• Notice that once we redefine r-g as  $\rho$ , say, our model looks nearly the same as the original model:

$$\rho V = -pc + q(\theta)(J - V)$$

$$\rho J = p - w + \lambda(V - J)$$

$$\rho U = pb + \theta q(\theta)(W - U)$$

$$\rho W = w + \lambda(U - W)$$

 We can now derive the job-creation condition and the wage equation from the model.

• As in the model without technological progress, we can determine a balanced-growth equilibrium by the set of  $(\theta, w, u)$  satisfying

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$
$$(r - g + \lambda) \frac{pc}{q(\theta)} = p - w$$
$$w = \beta p + (1 - \beta)pb + \beta pc\theta$$

• First, observe that the wage equation implies that the wage rate is proportional to p. To see this,

$$w = \beta p + (1 - \beta)pb + \beta pc\theta$$
$$= [\beta + (1 - \beta)b + \beta c\theta]p$$

- In other words, the wage rate grows at rate g.
  - This result is derived, not assumed.
  - For this to be true, the unemployment benefit and the vacancy cost are both proportional to p. Remember our additional assumption z=pb.

 Substitute the wage equation into the job-creation condition to obtain

$$(r - g + \lambda)\frac{pc}{q(\theta)} = p - [\beta + (1 - \beta)b + \beta c\theta]p$$

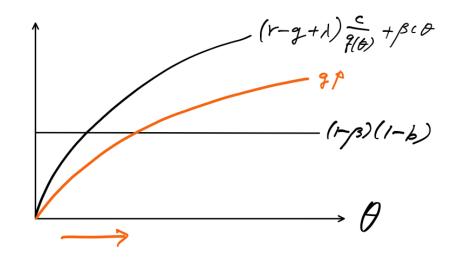
- Both sides are proportional to p.
- For this to be true, the vacancy cost must be proportional to pc, as in Pissarides.
  - In other words, without technological progress, it is OK to assume the vacancy cost to be c not pc.

• Divide both sides by p to obtain

$$(r - g + \lambda)\frac{c'}{q(\theta)} + \beta c\theta = (1 - \beta)(1 - b)$$

- Because p is dropped, all variables in this expression can be constant over time.
  - We can safely find a balanced-growth equilibrium.
- The left-hand side is increasing in  $\theta$ .
- The right-hand side is constant. The sign is positive if and only if b < 1 (i.e., z < p).

- There is a unique BGE, as shown in the diagram.
- An increase in g will increase  $\theta$  in BGE.
- Because  $u = \frac{\lambda}{\lambda + \theta q(\theta)}$  implies a negative relationship between  $\theta$  and u, an increase in g will decrease u.



## Capitalization Effect

- We have now shown that a faster technological progress will create more jobs and reduce the unemployment rate in the long run.
- This result is referred to as the capitalization effect.

# Capitalization Effect

• To fully understand the capitalization effect, consider the job-creation condition:

$$\frac{(r-g+\lambda)pc}{q(\theta)} = p - w$$

- Let  $r g + \lambda = \tilde{r}$  be the effective discount rate.
- Then we rewrite it as

$$\underbrace{\frac{1}{q(\theta)}}_{\text{Expected duration}} \times \underbrace{pc}_{\text{Vacancy cost}} = \underbrace{\frac{p-w}{\tilde{r}}}_{\text{Discounted Infinite Sum of Profits}}$$
Expected Total Vacancy Costs

# Capitalization Effect

- It is now clear that the job-creation condition balances the cost and benefit of creating a vacancy.
- The effective discount rate  $\tilde{r} = r g + \lambda$  captures two things:
  - Technological progress
  - Separations.
- Suppose that you must borrow money to finance the vacancy cost. Then,  $\frac{p-w}{\tilde{r}}$  is the maximum amount of money you can repay in the future.
  - A lower interest rate expands this amount.

# Technological Progress and Job Destruction

Mortensen, Dale T., and Christopher A. Pissarides. "Technological progress, job creation, and job destruction." Review of Economic dynamics (1998): 733-753.

#### Goals

- An implicit assumption was made in the previous model that technological progress improves the labor productivity <u>for all jobs</u>.
  - Often referred to as disembodied technological progress.
- We now consider the polar opposite in which only the newly created job can enjoy the cutting-edge technology.
  - After a job is created, there are newer, better technology outside, causing the job obsolete.

- Consider (1.33) in Pissarides, which is  $rV = -pc + q(\theta)(J V) + \dot{V}$
- This is the value of creating a vacancy.
- We shall introduce the idea of job vintage.
- Let  $J(\tau)$  denote the value of job filled in period  $\tau$ , evaluated in period t.
  - $\tau$  is the **vintage** of a particular job.
  - t is the actual time.
  - For brevity, we denote  $J(\tau)$  instead of  $J(\tau, t)$ .

We shall extend this expression as

$$rV = -pc + q(\theta)(J(t) - V - pK) + \dot{V}$$

- J(t) is the value of a job of vintage t.
- pK is the cost of starting a new job. Training cost.
- As always, we assume free entry of jobs: V=0
- Thus, this Bellman equation implies

$$\frac{c}{q(\theta)} + K = \frac{J(t)}{p}$$

- This is (3) in MP.
  - For simplicity, we will assume K=0.

- In Pissarides:  $rJ = p w + \lambda(V J) + \dot{J}$
- The value of a job of vintage  $\tau$  satisfies  $rJ(\tau) = p(\tau) w(\tau) + \lambda \big(V J(\tau)\big) + \dot{J}(\tau)$
- This is (4) in MP (when V=0).
  - We drop x by assuming x = 1.
  - Imagine an extreme example where you stop learning on the date you are hired.

• In Pissarides,

$$rW = w + \lambda(U - W) + \dot{W}$$

- The value of employment of vintage  $\tau$  is  $rW(\tau) = w(\tau) + \lambda \big(U W(\tau)\big) + \dot{W}(\tau)$
- This is (5) in MP.

In Pissarides,

$$rU = z + \theta q(\theta)(W - U) + \dot{U}$$

- The value of unemployment in MP is  $rU = pb + \theta q(\theta)(W(t) U) + \dot{U}$
- W(t) means that at the time you are hired, your job vintage is  $\tau = t$  because the date you are hired is indexed by t.
- Your job vintage is fixed at  $\tau$  while t increases over time. Thus,  $t > \tau$  and  $t \tau$  is job **tenure**.
- As  $t \tau$  expands, the job becomes **obsolete**.

• (1.17) in Pissarides:

$$W_i - U = \beta(J_i + W_i - V - U)$$

With vintage,

$$W(\tau) - U = \beta(J(\tau) + W(\tau) - V - U)$$

Or,

$$\beta[J(\tau) - V] = (1 - \beta)[W(\tau) - U]$$

- This is (7) in MP.
- Differentiate it by t to obtain

$$\beta \left[ \dot{J}(\tau) - \dot{V} \right] = (1 - \beta) \left[ \dot{W}(\tau) - \dot{U} \right]$$

Rewrite the values as

$$(r+\lambda)[J(\tau)-V] = p(\tau) - w(\tau) - rV + \dot{J}(\tau)$$

Similarly,

$$(r + \lambda)[W(\tau) - U] = w(\tau) - rU + \dot{W}(\tau)$$

Substitute these expressions into the bargaining outcome to obtain

$$\beta [p(\tau) - w(\tau) - rV + \dot{J}(\tau)]$$
  
=  $(1 - \beta) [w(\tau) - rU + \dot{W}(\tau)]$ 

Thus,

$$w(\tau) = \beta p(\tau) + (1 - \beta)rU + \beta \dot{J}(\tau) - (1 - \beta)\dot{W}(\tau)$$

The bargaining outcome implies

$$[W(\tau) - U] = \frac{\beta}{1 - \beta} [J(\tau) - V]$$

Substitute the equations on page 24 into above:

$$[W(\tau) - U] = \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)}$$

• Thus, rU (on page 27) satisfies

$$rU = pb + \frac{\beta}{1 - \beta}pc\theta + \dot{U}$$

• Substitute rU into  $w(\tau)$  to obtain

$$w(\tau) = \beta p(\tau) + (1 - \beta) \left\{ pb + \frac{\beta}{1 - \beta} pc\theta \right\} + \beta \dot{J}(\tau) - (1 - \beta) \left[ \dot{W}(\tau) - \dot{U} \right]$$

• Because  $\beta[\dot{J}(\tau)-\dot{V}]=(1-\beta)[\dot{W}(\tau)-\dot{U}]$  and  $V=\dot{V}=0$ , we obtain

$$w(\tau) = \beta p(\tau) + (1 - \beta) \left\{ pb + \frac{\beta}{1 - \beta} pc\theta \right\}$$

Consider

$$w(\tau) = \beta p(\tau) + (1 - \beta) \left\{ pb + \frac{\beta}{1 - \beta} pc\theta \right\}$$

• Collect the terms independent of  $\tau$  to define

$$\omega(\theta) = b + \frac{\beta}{1 - \beta} c\theta$$

- This is (9) in MP. Thus, the wage equation is  $w(\tau) = \beta p(\tau) + (1-\beta)p\omega(\theta)$
- This is (8).
  - We are assuming x = 1.

Consider the wage equation

$$w(\tau) = \beta p(\tau) + (1 - \beta)p\omega(\theta)$$

- Observe:
  - $p(\tau)$  is constant.
  - $p = p(0)e^{gt}$  is growing over time.
- The key here is that the wage rate for a job of vintage  $\tau$  grows over time even though the productivity of the worker-firm pair is not growing.
  - $w(\tau)$  is growing because the worker's reservation wage rU is growing because of economic growth.

#### Obsolescence and Job Destruction

- The value of a job of vintage  $\tau$  in period t satisfies  $rJ(\tau,t)=p(\tau)-w(\tau,t)+\lambda\big(V-J(\tau,t)\big)+\dot{J}(\tau,t)$
- With  $w(\tau, t)$  and V = 0, rewrite above as  $(r + \lambda)J(\tau, t) = (1 \beta)[p(\tau) p(t)\omega(\theta)] + \dot{J}(\tau, t)$
- Multiply both sides by  $e^{-(r+\lambda)t}$  to obtain  $(r+\lambda)J(\tau,t)e^{-(r+\lambda)t}-j(\tau,t)e^{-(r+\lambda)t} = (1-\beta)[p(\tau)-p(t)\omega(\theta)]e^{-(r+\lambda)t}$
- Observe that the left-hand side is

$$\frac{d\big[J(\tau,t)e^{-(r+\lambda)t}\big]}{dt} = -(r+\lambda)J(\tau,t)e^{-(r+\lambda)t} + \dot{J}(\tau,t)e^{-(r+\lambda)t}$$

#### Obsolescence and Job Destruction

 Thus,  $-\frac{d[J(\tau,t)e^{-(r+\lambda)t}]}{dt} = (1-\beta)[p(\tau)-p(t)\omega(\theta)]e^{-(r+\lambda)t}$ 

• Integrate both sides from t to  $\tau + T$  to obtain

$$e^{-(r+\lambda)t}J(\tau,t) - e^{-(r+\lambda)(\tau+T)}J(\tau,\tau+T)$$

$$= \int_{t}^{\tau+T} (1-\beta)[p(\tau) - p(s)\omega(\theta)]e^{-(r+\lambda)s}ds$$

• Multiply both sides by 
$$e^{(r+\lambda)t}$$
 to obtain 
$$J(\tau,t) = \int_t^{\tau+T} (1-\beta)[p(\tau)-p(s)\omega(\theta)]e^{-(r+\lambda)(s-t)}ds \\ + e^{-(r+\lambda)(\tau+T-t)}J(\tau,\tau+T)$$

#### Obsolescence and Job Destruction

Consider

$$J(\tau,t) = \int_{t}^{\tau+T} (1-\beta)[p(\tau) - p(s)\omega(\theta)]e^{-(r+\lambda)(s-t)}ds$$
$$+e^{-(r+\lambda)(\tau+T-t)}J(\tau,\tau+T)$$

• Let  $\tau + T$  be the date at which the job must be destroyed. Then, the terminal value of the job must satisfy (by definition)

$$J(\tau, \tau + T) = 0$$

Thus,

$$J(\tau,t) = \int_{t}^{\tau+T} (1-\beta)[p(\tau)-p(s)\omega(\theta)]e^{-(r+\lambda)(s-t)}ds$$

• Let the firm choose *T*:

$$J(\tau,t) = \max_{T} \int_{t}^{\tau+T} (1-\beta)[p(\tau) - p(s)\omega(\theta)]e^{-(r+\lambda)(s-t)}ds$$

- This is (10) in MP.
- The first-order condition with respect to T is  $(1-\beta)[p(\tau)-p(\tau+T)\omega(\theta)]e^{-(r+\lambda)(\tau+T-t)}=0$
- Equivalently,

$$p(\tau) - p(\tau + T)\omega(\theta) = 0$$

- Consider  $p(\tau) p(\tau + T)\omega(\theta) = 0$
- Because  $p(\tau + T) = p(\tau)e^{gT}$ ,  $p(\tau) = p(\tau)e^{gT}\omega(\theta)$
- Thus,

$$1 = e^{gT}\omega(\theta)$$
$$= e^{gT}\left(b + \frac{\beta}{1-\beta}c\theta\right)$$

- This is (13) in MP.
- This defines the relationship between  $\theta$  and T.

Consider

$$1 = e^{gT}\omega(\theta) = e^{gT}\left(b + \frac{\beta}{1-\beta}c\theta\right)$$

- This implies a <u>negative</u> relationship between  $\theta \& T$ .
- From page 24,

$$\frac{c}{q(\theta)}p(t) = J(t,t)$$

The right-hand side is

$$J(t,t) = \int_{t}^{t+T} (1-\beta)[p(t) - p(s)\omega(\theta)]e^{-(r+\lambda)(s-t)}ds$$

Rewrite the previous equation as

$$J(t,t)$$

$$= (1-\beta)p(t) \int_{t}^{t+T} \left[1 - e^{g(s-t)}\omega(\theta)\right] e^{-(r+\lambda)(s-t)} ds$$

$$= (1-\beta)p(t) \int_{0}^{t} \left[1 - e^{gs}\omega(\theta)\right] e^{-(r+\lambda)s} ds$$

Thus,

$$\frac{c}{q(\theta)} = (1 - \beta) \int_0^T [1 - e^{gs} \omega(\theta)] e^{-(r+\lambda)s} ds$$

Consider

$$\frac{c}{(1-\beta)q(\theta)} = \int_0^T [1 - e^{gs}\omega(\theta)]e^{-(r+\lambda)s}ds$$

 The derivative of the right-hand side with respect to T is

$$[1 - e^{gT}\omega(\theta)]e^{-(r+\lambda)T} = 0$$

- This is because  $1 = e^{gT}\omega(\theta)$ .
- This implies that this equation is independent of T.

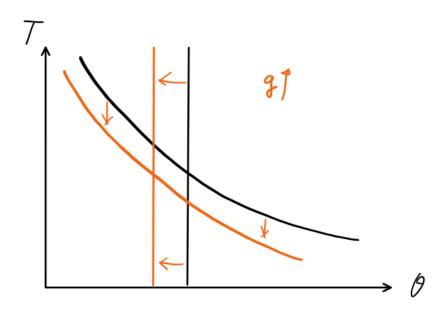
• Equilibrium is determined by a pair  $\{\theta, T\}$  satisfying

$$1 = e^{gT} \left\{ b + \frac{\beta c}{1 - \beta} \theta \right\} = e^{gT} \omega(\theta)$$

$$\frac{c}{(1 - \beta)q(\theta)} = \int_0^T [1 - e^{gs} \omega(\theta)] e^{-(r + \lambda)s} ds$$

• The first equation implies a negative relationship, while the second implies a unique  $\theta$  for each T.

- An increase in g decreases both  $\theta$  and T.
- Thus, when technology is embodied, a faster technological progress makes the life of each job shorter.
- This is called creative destruction of jobs.



- Let us consider worker flows. In the basic Pissarides model, total job creation is  $\theta q(\theta)u_t$  and total job destruction is  $\lambda(1-u_t)$ .
- With obsolescence, total job creation at t is  $C_t = \theta q(\theta) u_t$
- Because jobs are destroyed exogenously at rate  $\lambda$ ,  $\mathcal{C}$  satisfies the following differential equation:

$$\dot{C} = -\lambda C$$

• To solve it, multiply by integration factor  $e^{\lambda t}$  to get  $\dot{C}e^{\lambda t} + \lambda Ce^{\lambda t} = 0$ 

- Consider  $\dot{C}e^{\lambda t} + \lambda Ce^{\lambda t} = 0$ .
- Note that  $(Ce^{\lambda t})' = \dot{C}e^{\lambda t} + \lambda Ce^{\lambda t}$
- Thus,

$$\left(Ce^{\lambda t}\right)'=0$$

• Integrate this from t - T to t:

$$\left[Ce^{\lambda s}\right]_{t-T}^{t} = C_t e^{\lambda t} - C_{t-T} e^{\lambda(t-T)} = 0$$

• Thus,

$$C_t = C_{t-T}e^{-\lambda T}$$

Consider

$$C_t = C_{t-T}e^{-\lambda T}$$

- This is a solution to the differential equation.
- Instead of the usual initial condition at  $C_0$ , the initial condition is set at  $C_{t-T}$ .
- The expression tells us the number of jobs created at t-T that survived after T periods.
- Because the optimal tenure length is T,  $C_{t-T}e^{-\lambda T}$  is the number of jobs destroyed because of obsolescence (not by separation shock).

Total job creation at t is

$$C_t = \theta q(\theta) u_t$$

Total job destruction at t is

$$D_t = \lambda (1 - u_t) + C_{t-T} e^{-\lambda T}$$

In any steady state, two flows must be the same:

$$\theta q(\theta)u = \lambda(1-u) + \theta q(\theta)ue^{-\lambda T}$$

Solve it to obtain

$$u = \frac{\lambda}{\lambda + [1 - e^{-\lambda T}]\theta q(\theta)}$$

Consider

$$u = \frac{\lambda}{\lambda + [1 - e^{-\lambda T}]\theta q(\theta)}$$

- We have shown that an increase in g decreases both  $\theta$  and T.
- Because  $\left[1 e^{-\lambda T}\right]\theta q(\theta)$  is increasing in both  $\theta$  and T, an increase in g decreases u.
- Thus, <u>embodied technological progress increases</u> <u>unemployment</u>.

• Equilibrium is determined by a set of  $\{\theta, T, u\}$  satisfying

$$1 = e^{gT} \left\{ b + \frac{\beta c}{1 - \beta} \theta \right\} = e^{gT} \omega(\theta)$$

$$\frac{c}{q(\theta)} = (1 - \beta) \int_0^T [1 - e^{gs} \omega(\theta)] e^{-(r + \lambda)s} ds$$

$$u = \frac{\lambda}{\lambda + [1 - e^{-\lambda T}] \theta q(\theta)}$$

 The second equation is still too complicated as it includes integration.

Let us simplify the second equation.

$$\frac{c}{(1-\beta)q(\theta)} = \int_0^T [1 - e^{gs}\omega(\theta)]e^{-(r+\lambda)s}ds$$

$$= \int_0^T e^{-(r+\lambda)s}ds - \omega(\theta) \int_0^T e^{-(r-g+\lambda)s}ds$$

$$= \left[ -\frac{1}{r+\lambda}e^{-(r+\lambda)s} \right]_0^T$$

$$-\omega(\theta) \left[ -\frac{1}{r-g+\lambda}e^{-(r-g+\lambda)s} \right]_0^T$$

• Thus, we obtain

$$\frac{c}{(1-\beta)q(\theta)} = \frac{1}{r+\lambda} \left[ 1 - e^{-(r+\lambda)T} \right]$$
$$-\frac{\omega(\theta)}{r-g+\lambda} \left[ 1 - e^{-(r-g+\lambda)T} \right]$$

This equation does not contain integration.

• Equilibrium is determined by a set of  $\{\theta, T, u\}$  satisfying

$$1 = e^{gT} \left\{ b + \frac{\beta c}{1 - \beta} \theta \right\} = e^{gT} \omega(\theta)$$

$$\frac{c}{(1 - \beta)q(\theta)} = \frac{1}{r + \lambda} \left[ 1 - e^{-(r + \lambda)T} \right]$$

$$-\frac{\omega(\theta)}{r - g + \lambda} \left[ 1 - e^{-(r - g + \lambda)T} \right]$$

$$u = \frac{\lambda}{\lambda + \left[ 1 - e^{-\lambda T} \right] \theta q(\theta)}$$

# Further Readings

- Chapter 3 in Pissarides (2000).
- Mortensen, Dale T., and Christopher A. Pissarides. "Technological progress, job creation, and job destruction." *Review of Economic dynamics* (1998) 733-753.

# Reading Assignment

- Acemoglu. "Good Jobs versus Bad Jobs." *Journal of Labor Economics*, 2001.
- Downloadable from UNCT.
- Read Section I (Introduction), and Section II (The Basic Model).
- Due is on 6/23.
- 6/23 class will focus on this paper.