Lecture 5

Pissarides, Equilibrium Unemployment Theory

1-4: Wage Determination

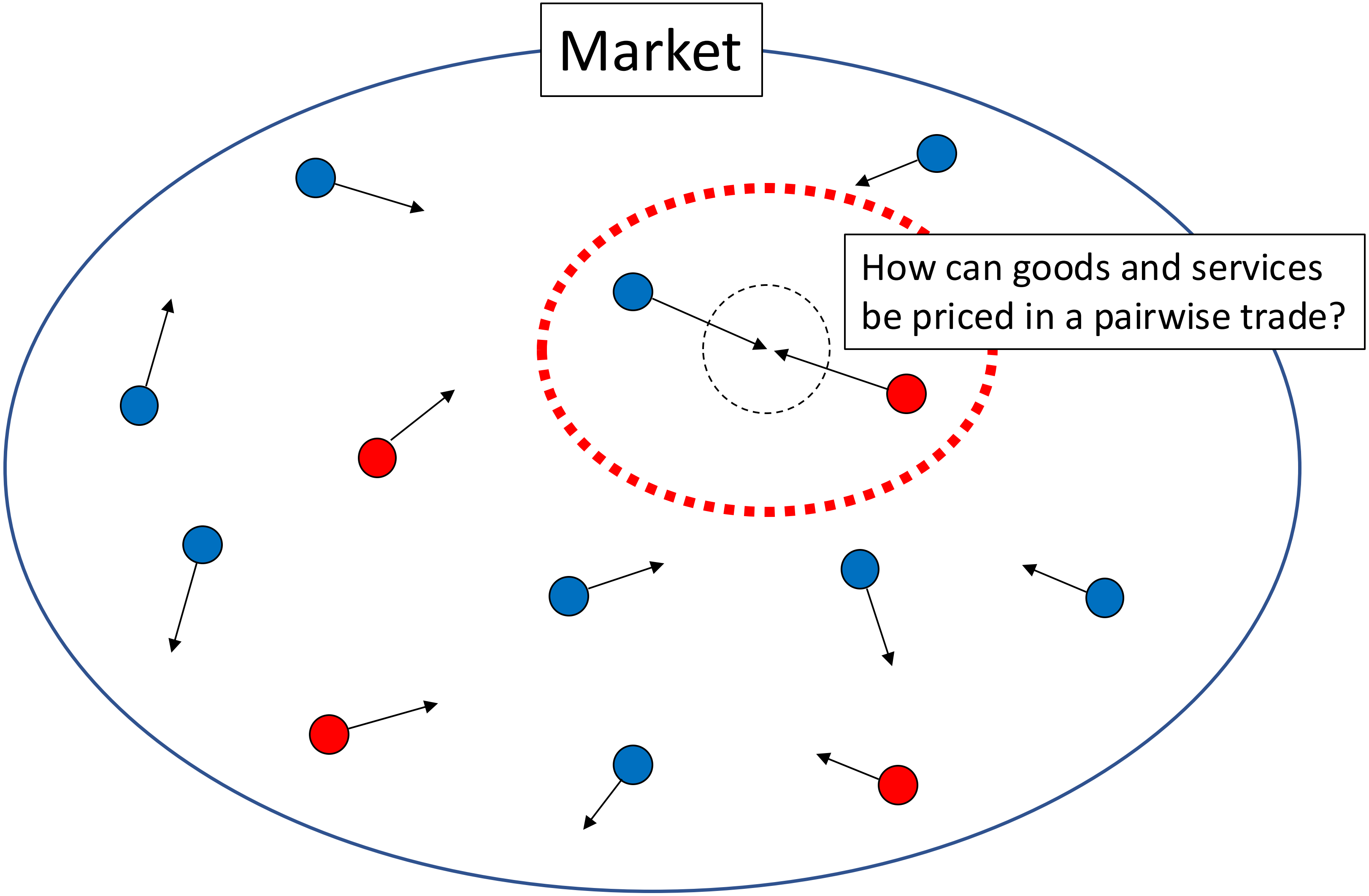
5/19

Goals

- Today, we want to fully understand section 1-4 in Pissarides book.
  - As always, I will assume that you read this section.
- We have two specific goals:
  - We want to understand **Nash bargaining**, which is a powerful device for modeling how prices are determined in markets with search frictions.
  - We will discuss **strategic foundations** of the Nash solution.

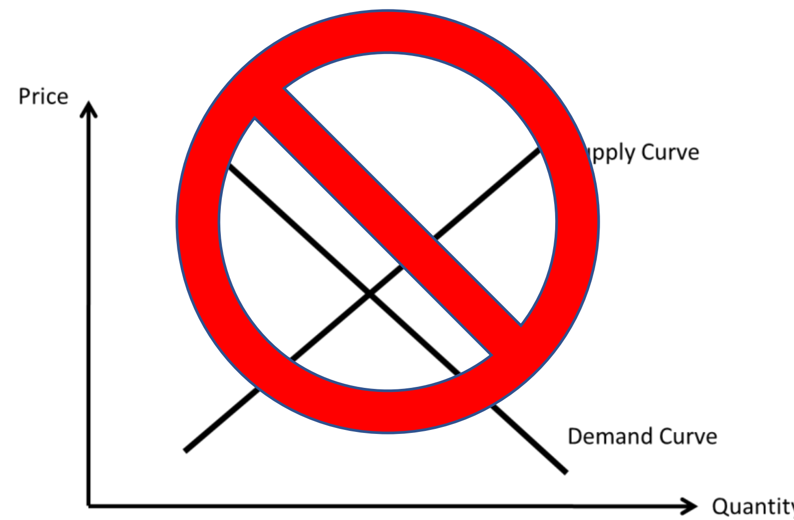
Nash Bargaining

Pricing in a Frictional Market



Pricing in a Frictional Market

- We cannot use the competitive price mechanism because it requires a perfectly competitive market, in which there is an infinity of buyers and sellers in the same place at the same time.
- We need **bargaining theory**, which is part of **game theory**.
  - Game theory is a science of **strategic interactions** among individuals.



Bargaining Problem

- Consider a situation in which two individuals (player A and B) bargain over their shares of a pie of size (normalized to) <u>one</u>.
- A **bargaining problem** is a pair (S, d) such that
  - S is the set of all utility pairs (s_A, s_B) that correspond to **agreement**.
  - d is the utility pair (d_A, d_B) that corresponds to **disagreement**.
- A **bargaining solution** is <u>a function that maps a bargaining problem to a unique allocation</u>.

The Nash Bargaining Solution

- The **Nash Bargaining Solution** the unique pair of utilities that solves:

$$\underset{s_A \geq d_A, s_B \geq d_B}{\operatorname{argmax}} \; (s_A - d_A)(s_B - d_B)$$

  - $\operatorname{argmax} X$ means the argument that maximizes X.
  - $(s_A - d_A)(s_B - d_B)$ is called the **Nash product**.
  - (s_A, s_B) denote utilities from agreement.
  - (d_A, d_B) denote utilities from disagreement.
- It is derived from four axioms (i.e., assumptions).

The Nash Bargaining Solution

- $d = (d_A, d_B)$ is referred to as the **threat point**.
  - Threat point must be smaller than the size of the surplus to be divided. Otherwise, there is no need to start a negotiation in the first place.
- $s_A - d_A$ is referred to as Player A's net surplus, while s_A is the gross surplus.
  - For example, consider two firms, A and B, dividing 100 yen from a joint project. If firm A has an opportunity to earn 60 yen ($d_A = 60$) without the project, then A is not happy about $s_A = 50$.
- Thus, d matters a lot.

Nash's Theorem

- Nash assumed that a bargaining situation satisfies the following **four** axioms (i.e., assumptions):
  - Axiom I: Order-preserving linear transformations of u to v such that $v = au + b$ (a and b are parameters) do not change the solution
  - Axiom II (Symmetry): If $d_A = d_B$, then $s_A = s_B$.
  - Axiom III (Independence of Irrelevant Alternatives): If (S, d) and (T, d) are bargaining problems with $S \subset T$ and the solution to (T, d) is an element of S, then the two bargaining problems lead to the same bargaining solution.
  - Axiom IV (Pareto efficiency): If $s \in S$, $t \in S$, and $t_A > s_A$ and $t_B > s_B$. Then $s = (s_A, s_B)$ is not a bargaining solution.
- Theorem: The Nash Bargaining Solution is the unique solution satisfying the four axioms.
  - <u>Do not worry about understanding the meaning of each axiom</u>.
  - It is sufficient to know that the Nash bargaining solution has some foundation.

Application: Dividing a Pie

- Player A and player B bargain over 1 unit of a pie:
$$s_A + s_B = 1$$
- Then the (symmetric) Nash bargaining problem is
$$\max_{s_A} (s_A - d_A)(1 - s_A - d_B)$$
- The first-order condition is
$$1 - s_A - d_B - (s_A - d_A) = 0$$
- Solve it for s_A to obtain
$$s_A = \frac{1 + d_A - d_B}{2}$$
- Find the condition under which $s_A > s_B$ holds.

Application: Dividing a Pie

- It is easy to show that

$$s_A > s_B \Leftrightarrow \frac{1 + d_A - d_B}{2} > 1 - \frac{1 + d_A - d_B}{2}$$

- Arrange terms to obtain

$$s_A > s_B \Leftrightarrow d_A > d_B$$

- Thus, **the threat point plays a central role** in determining the bargaining outcome.
- Evidently, when $d_A = d_B$, we obtain $s_A = s_B = \frac{1}{2}$.

Asymmetric Nash Bargaining

- For any $\beta \in (0,1)$, consider:
$$\max_{s_A \geq d_A, s_B \geq d_B} (s_A - d_A)^{\beta} (s_B - d_B)^{1-\beta}$$
- We refer to the problem as the **asymmetric** (or, **generalized**) **Nash bargaining**.
- This solution satisfies axioms I, III, and IV.
- Because the real-world negotiations are not necessarily symmetric, the asymmetric Nash bargaining is employed in many applications.

Application: Dividing a Pie

- Player A and player B bargain over 1 unit of a pie:
$$s_A + s_B = 1$$
- The asymmetric Nash bargaining problem is
$$\max_{s_A} (s_A - d_A)^{\beta} (1 - s_A - d_B)^{1-\beta}$$
- The first-order condition is
$$\beta (s_A - d_A)^{\beta-1} (1 - s_A - d_B)^{1-\beta}$$
$$-(s_A - d_A)^{\beta} (1-\beta)(1 - s_A - d_B)^{-\beta} = 0$$
- Arrange terms to obtain
$$\beta (1 - s_A - d_B) = (1-\beta)(s_A - d_A)$$

Application: Dividing a Pie

- Solve it for s_A as
$$s_A = \beta(1 - d_B) + (1 - \beta)d_A$$
- When $d_A = d_B = 0$, we obtain
$$s_A = \beta$$
$$s_B = 1 - \beta$$
- Thus, an increase in β alters player A's share of a pie in favor of him/her even in the absence of d.
  - In this sense, β is referred to as player A's **exogenous bargaining power**. This one is exogenous because it is artificially imposed to alter the original Nash solution.
  - Note that A's threat d_A can also be interpreted as A's bargaining power. This one is **endogenous**.

Application: Dividing a Pie

- Consider once again

$$s_A = \beta(1 - d_B) + (1 - \beta)d_A$$

- We can rewrite it as

$$s_A - d_A = \beta(1 - d_A - d_B)$$

- Interpretation:
    - Because $s_A + s_B = 1$, we observe that $1 - d_A - d_B = s_A - d_A + s_B - d_B$ is the sum of the net surpluses for A and B. This sum is the **total (net) surplus** to be shared.
    - This expressions states that the share of player A's surplus is β.

Strategic Foundations

Motivation

- Nash bargaining is an ingenious theory, but it is a **cooperative game theory** and lacks strategic foundations.
- We wish to describe the details of how people interact with each other, using non-cooperative game theory, which is (once again) pioneered by the same genius, John Nash.
- We shall then verify that the Nash solution can be constructed by an appropriately designed strategic environment.

Take-it-or-leave-it offer

- Consider the bargaining problem.
- If player A has the right to make an offer and player B has no right to make a counteroffer, then this offer is a **take-it-or-leave-it offer**. (e.g., vending machines)
- Any offer must be acceptable, so it must be that

$$s_B \geq d_B$$

- Because A has no incentive to give anything to B, the offer must make B indifferent between "accept" and "reject". Thus,

$$s_B = d_B$$

- Thus, the equilibrium outcome is $s = (1 - d_B, d_B)$.

Nash's Demand Game

- Suppose that player A can make a take-it-or-leave-it offer with probability ½ and B can make an offer with probability ½. There is no counter offer.
- A's offer must make B indifferent between "accept" and "reject". Thus, $s_B = d_B \Leftrightarrow s_A = 1 - d_B$.
- B's offer must make A indifferent. Thus, $s_A = d_A$.
- The expected surplus for A is

$$\frac{1}{2}(1 - d_B) + \frac{1}{2}d_A = \frac{1 + d_A - d_B}{2}$$

  - This corresponds to the Nash solution on page 10.

Nash's Demand Game

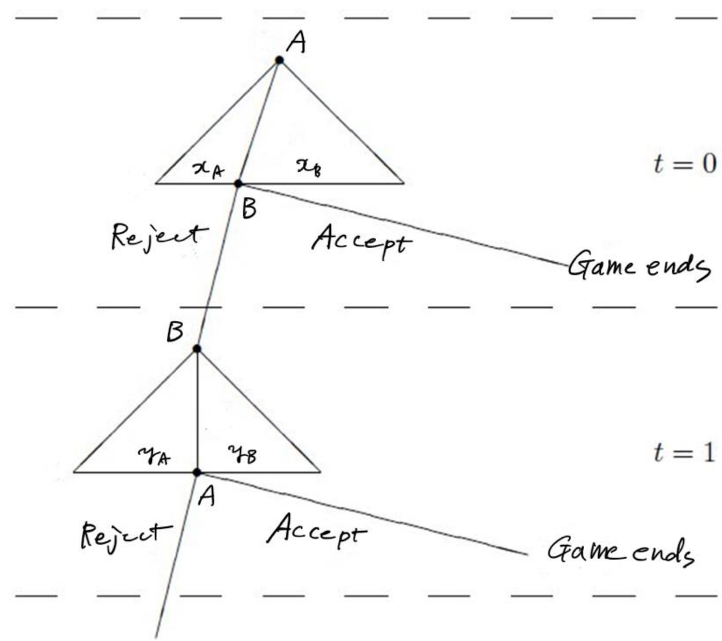
- Suppose that player A can make a take-it-or-leave-it offer with probability β and B can make an offer with probability $1 - \beta$. There is no counter offer.
  - A's offer must make B indifferent between "accept" and "reject". Thus, $s_B = d_B \Leftrightarrow s_A = 1 - d_B$.
  - B's offer must make A indifferent. Thus, $s_A = d_A$.
- The expected surplus for A is

$$\beta(1 - d_B) + (1 - \beta)d_A$$

  - This is the Nash bargaining outcome on page 15.
- This game is called the **Nash demand game**.

Rubinstein’s (1982) Alternating Offer Bargaining Game

- In period 0, player A makes the first offer, and player B chooses to accept or reject.
  - If accept, the game ends.
  - If reject, the game continues.
- In period 1, player B makes a **counteroffer**, and player A chooses to accept or reject.

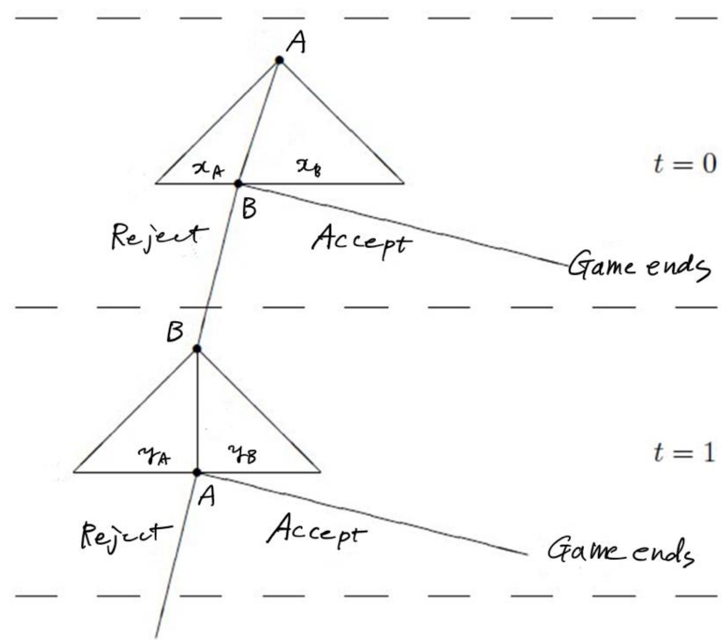


Subgame Perfect Equilibrium

- Rubinstein (1982) proved that there is a unique subgame perfect equilibrium (SPE) for this game.
- SPE satisfies:
  - No delay: Whenever a player makes an offer, his/her offer is immediately accepted by the other player.
  - Stationarity: Whenever a player makes an offer, he/she makes the same offer.

Subgame Perfect Equilibrium

- $x = (x_A, x_B)$ is A's offer.
- $y = (y_A, y_B)$ is B's offer.
- $\delta_A < 1$: Discount factor for A.
- $\delta_B < 1$: Discount factor for B.
- A's offer makes B indifferent between "accept" and "reject".
- B's offer makes A indifferent between "accept" and "reject".



Subgame Perfect Equilibrium

- A's offer (x_A, x_B) must make B indifferent between "accept" and "reject".
- From B's point of view,
  - The value of "accept" = x_B
  - The value of "reject" = the value of making a counteroffer (y_A, y_B), which will be accepted = $\delta_B y_B$
- Thus,

$$x_B = \delta_B y_B$$

Subgame Perfect Equilibrium

- B's offer (y_A, y_B) must make A indifferent between "accept" and "reject".
- From A's point of view,
  - The value of "accept" = y_A
  - The value of "reject" = the value of making a counteroffer (x_A, x_B), which will be accepted = $\delta_A x_A$
- Thus,

$$y_A = \delta_A x_A$$

Subgame Perfect Equilibrium

- SPE is a solution to

$$x_B = \delta_B y_B$$
$$y_A = \delta_A x_A$$
$$x_A + x_B = 1$$
$$y_A + y_B = 1$$

- Find the solution.

Subgame Perfect Equilibrium

- Let us reduce the number of equations:

$$x_B = \delta_B y_B = \delta_B(1 - y_A) = \delta_B(1 - \delta_A x_A)$$
$$y_A = \delta_A x_A$$
$$x_A + x_B = 1$$
$$y_A + y_B = 1$$

- Thus, we obtain

$$1 - x_A = \delta_B(1 - \delta_A x_A)$$

- Solve it for x_A as

$$x_A = \frac{1 - \delta_B}{1 - \delta_A \delta_B}$$

Subgame Perfect Equilibrium

- Thus, x_B is

$$x_B = 1 - \frac{1 - \delta_B}{1 - \delta_A \delta_B}$$

- When $\delta_A = \delta_B = \delta$, we obtain

$$x_A = \frac{1 - \delta}{1 - \delta^2} = \frac{1}{1 + \delta} > \frac{1}{2} > x_B$$

- This occurs because player A happens to be the first mover in this game.
- This result is referred to as the **first-mover advantage**.

Removing the First-Mover Advantage

- In many applications, it is inappropriate (or impossible) to specify who makes the first move.
- One way to deal with this issue is to consider a continuous-time environment.
  - Length of each bargaining round is Δ. The first-mover advantage should disappear as $\Delta \to 0$.
- Then the discount factor is $\delta^{\Delta} < 1$. Then,

$$x_A = \frac{1}{1 + \delta^{\Delta}}$$

- Evidently, $\lim_{\Delta \to 0} x_A = 1/2$.

Random Proposer Model

- To neutralize the first-mover advantage even in a discrete-time environment, suppose that at each node of the game, player A makes an offer with probability π.
- SPE is a solution to

$$
\begin{aligned}
x_B &= \delta\{(1-\pi)y_B + \pi x_B\} \\
y_A &= \delta\{\pi x_A + (1-\pi)y_A\} \\
x_A &+ x_B = 1 \\
y_A &+ y_B = 1
\end{aligned}
$$

- This extends expressions on page 26.

Random Proposer Model

- Consider

$$x_B = \delta\{(1-\pi)y_B + \pi x_B\}$$
$$y_A = \delta\{\pi x_A + (1-\pi)y_A\}$$
$$x_A + x_B = 1$$
$$y_A + y_B = 1$$

- After several lines of calculation, we obtain

$$x_A = 1 - \delta(1-\pi)$$
$$x_B = \delta(1-\pi)$$
$$y_A = \delta\pi$$
$$y_B = 1 - \delta\pi$$

Random Proposer Model

- Because A makes an offer with probability π to get $1 - \delta(1-\pi)$ and accepts B's offer with probability $1-\pi$ to get $\delta\pi$, the expected payoff for A is
$$s_A = \pi \times [1 - \delta(1-\pi)] + (1-\pi) \times \delta\pi = \pi$$
- Similarly,
$$s_B = \pi \times \delta(1-\pi) + (1-\pi) \times (1 - \delta\pi) = 1 - \pi$$
- This corresponds to the asymmetric Nash bargaining solution for $d_A = d_B = 0$ on page 14 if we replace π with β.
- Thus, the exogenous bargaining power in the Nash bargaining can be interpreted as the likelihood of making offers.

Wage Determination

Brief Summary of the Model

- We derived (1.6),
$$rV = -pc + q(\theta)(J - V)$$
- We also derived
$$rJ = p - w + \lambda(V - J)$$
- With free entry ($V = 0$), this implies (1.8).
- We derived (1.10),
$$rU = z + \theta q(\theta)(W - U)$$
- We also derived (1.11),
$$rW = w + \lambda(U - W)$$

Understanding (1.16)

- We are now ready to study the **Nash wage bargaining** problem.
- Consider a pair of an employee and a firm instead of players A and B.
- The set of agreement payoffs is (W, J).
- The set of disagreement payoffs is (U, V):
  - Disagreement of a wage negotiation means unemployment for the worker and vacant for the firm.

Understanding (1.16)

- The **Nash wage bargaining** problem is given
$$\max_{w_i}(W_i - U)^{\beta}(J_i - V)^{1-\beta}$$
- This is (1.16) in Pissarides.
  - Subscript i reflects the fact that we are looking at one particular pair from an infinity of pairs in the economy.
  - There is no subscript i for U and V because the values of unemployment and vacancies are common for all workers and jobs.
  - More importantly, each bargaining pair cannot influence the threat point (U, V). This is outside of bargaining.

Derivation of (1.17)

- Let us now solve the problem:

$$\max_{w_i}(W_i - U)^{\beta}(J_i - V)^{1-\beta}$$

- Because each bargaining pair cannot influence the threat point (U, V), the pair takes U and V as given.
- Remember that the Bellman equations satisfy

$$rW_i = w_i + \lambda(U - W_i)$$
$$rJ_i = p - w_i + \lambda(V - J_i)$$

- Let us construct $W_i - U$ and $J_i - V$ in terms of w_i and parameters alone.

Derivation of (1.17)

- First, consider $rW_i = w_i + \lambda(U - W_i)$.
- Arrange terms to obtain $(r + \lambda)W_i = w_i + \lambda U$.
- Subtract $(r + \lambda)U$ from both sides to write
$$(r + \lambda)(W_i - U) = w_i - rU$$
- Similarly, from $rJ_i = p - w_i + \lambda(V - J_i)$, we obtain
$$(r + \lambda)(J_i - V) = p - w_i - rV$$
- Thus, we can rewrite the Nash product as
$$\left(\frac{w_i - rU}{r + \lambda}\right)^{\beta}\left(\frac{p - w_i - rV}{r + \lambda}\right)^{1-\beta}$$

Derivation of (1.17)

- Consider the problem:

$$\max_{w_i}\left(\frac{w_i - rU}{r+\lambda}\right)^{\beta}\left(\frac{p - w_i - rV}{r+\lambda}\right)^{1-\beta}$$

- This problem is quite intuitive:
  - For the worker, w_i is the payoff, rU is the reservation wage, so $w_i - rU = w_i - w_R$ is the net surplus from bargaining. This surplus lasts forever with separation rate λ. Thus, the surplus must be discounted by $r + \lambda$.

Derivation of (1.17)

- Drop the constant terms from the problem and consider:

$$\max_{w_i}(w_i - rU)^{\beta}(p - w_i - rV)^{1-\beta}$$

- The first-order condition is

$$\beta(w_i - rU)^{\beta-1}(p - w_i - rV)^{1-\beta} - (w_i - rU)^{\beta}(1-\beta)(p - w_i - rV)^{-\beta} = 0$$

- Simplify this condition as

$$\beta(p - w_i - rV) = (1-\beta)(w_i - rU)$$

Derivation of (1.17)

- Consider
$$\beta(p - w_i - rV) = (1-\beta)(w_i - rU)$$
- By arranging terms, we obtain (1.18).
- To obtain (1.17), remember
$$(r+\lambda)(W_i - U) = w_i - rU$$
$$(r+\lambda)(J_i - V) = p - w_i - rV$$
- Substitute them back into the above to obtain
$$\beta(J_i - V) = (1-\beta)(W_i - U)$$
- Arrange terms to obtain (1.17) as
$$W_i - U = \beta(J_i + W_i - V - U)$$

Derivation of (1.17)

- Consider (1.17):

$$W_i - U = \beta(J_i + W_i - V - U)$$

- It states that the share of worker's surplus is β.
- Let us go back to the previous expression:

$$\beta(J_i - V) = (1-\beta)(W_i - U)$$

- With free entry of jobs ($V = 0$), this reduces to

$$W_i - U = \frac{\beta}{1-\beta} J_i = \frac{\beta}{1-\beta} \frac{pc}{q(\theta)}$$

- $J_i = pc/q(\theta)$ is from (1.7).

Derivation of (1.20)

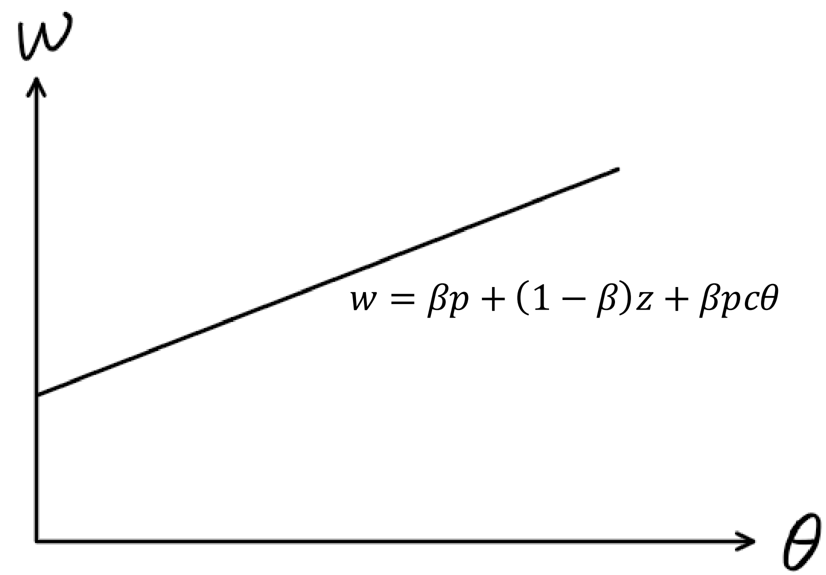
- Consider once again
$$\beta(p - w_i - rV) = (1 - \beta)(w_i - rU)$$
- Solve it for w_i and impose the free entry condition ($V = 0$) to obtain (1.18):
$$w_i = \beta p + (1 - \beta)rU$$
- (1.10) implies $rU = z + \theta q(\theta)(W_i - U)$.
- Thus, substitute it into the other to get
$$w_i = \beta p + (1 - \beta)[z + \theta q(\theta)(W_i - U)]$$

Derivation of (1.20)

- We are almost there. Consider
$$w_i = \beta p + (1-\beta)[z + \theta q(\theta)(W_i - U)]$$
- From page 26, we know
$$W_i - U = \frac{\beta}{1-\beta}\frac{pc}{q(\theta)}$$
- Thus, we finally obtain the **wage equation** as
$$w_i = \beta p + (1-\beta)z + \beta pc\theta = w$$
- Because the terms on the right-hand side are independent of i, we no longer need it.
- This is (1.20).

Wage Equation

- The wage equation:
  $w = \beta p + (1-\beta)z + \beta pc\theta$
- This is linear in θ.
- The interpretation of the wage equation is found on page 17 in Pissarides.



Further Readings

- Binmore, Rubinstein, and Wolinsky. “The Nash Bargaining Solution in Economic Modelling.” *Rand Journal of Economics*, 1986.
- Osborne & Rubinstein, *Bargaining and Markets*, 1990.
- Muthoo, *Bargaining Theory with Applications*, 1999.

Reading Assignment

Reading Assignment

- Christopher A. Pissarides, *Equilibrium Unemployment Theory*, second edition, MIT Press, 2000.
- Read Section 1.5 (Steady-State Equilibrium).
- 5/26 Class will focus on this section.