Lecture 2

Pissarides, Equilibrium Unemployment Theory

1-1: Trade in the Labor Market

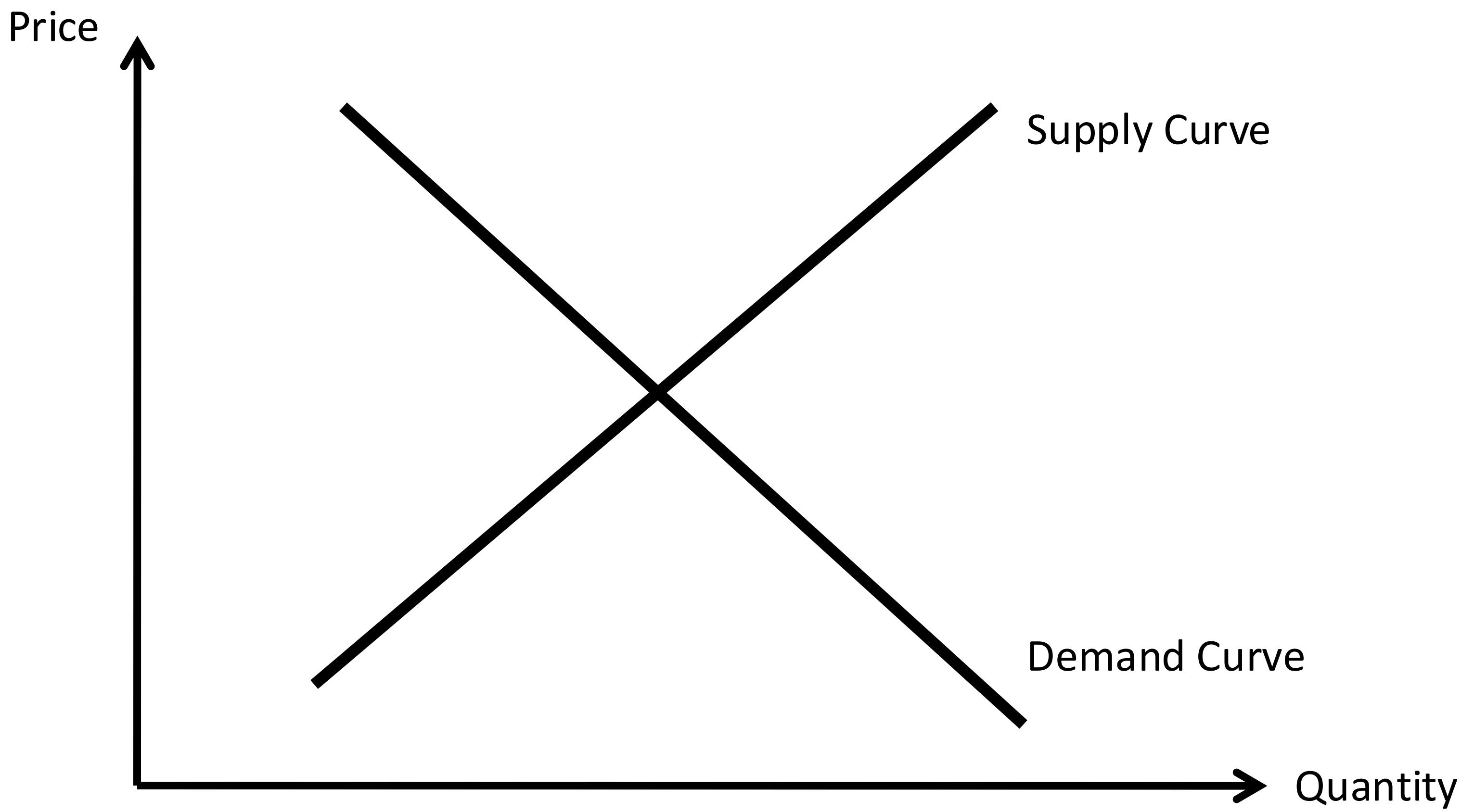
4/21

Goals

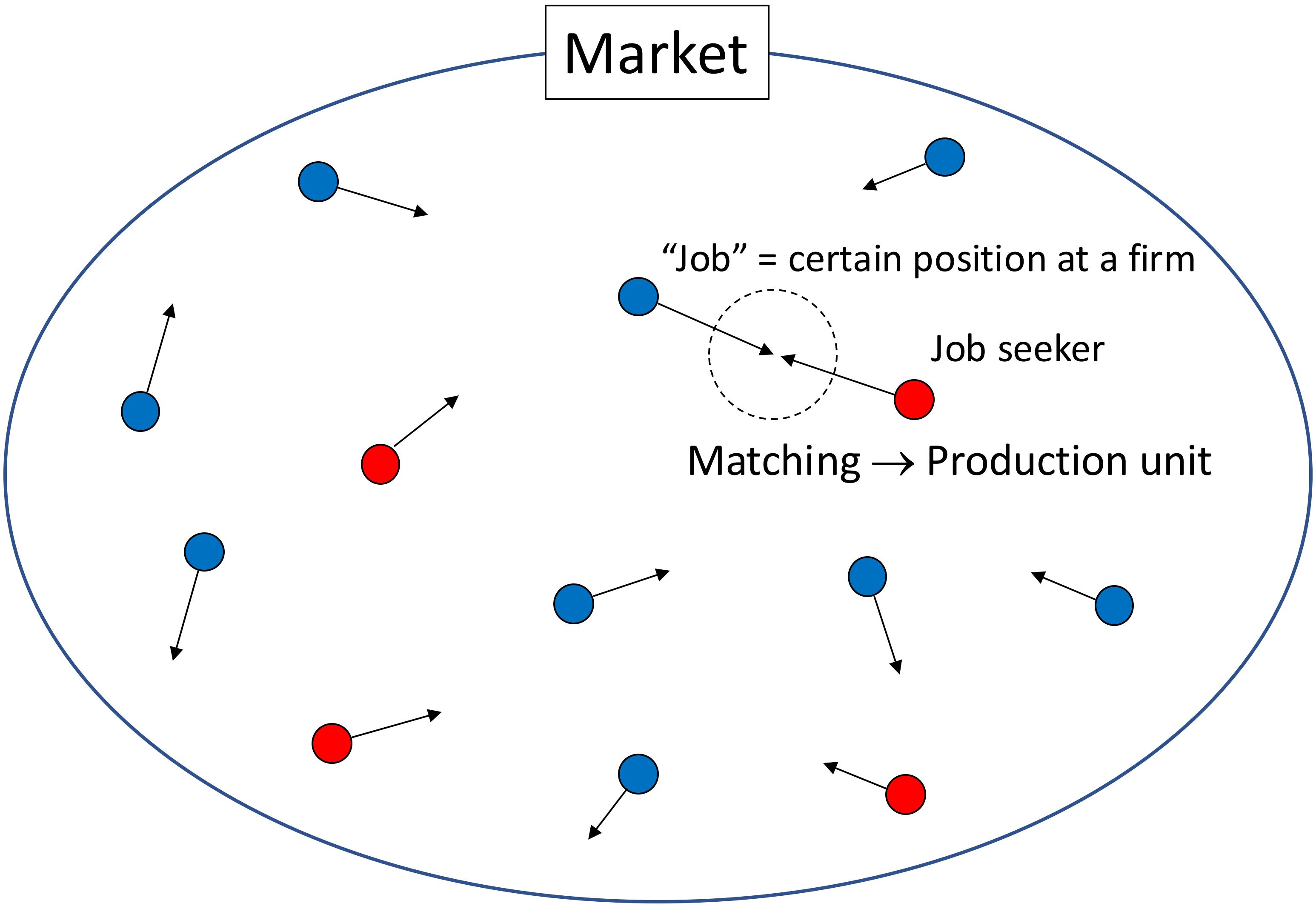
- Today, we want to fully understand section 1-1 in Pissarides book.
- We have two specific goals:
  - First, we want to understand the **matching function**, which will be used almost every class of this course.
  - Second, we want to familiarize **worker flows** <u>in continuous time</u>.

Matching Function

Forget This One



Imagine This



Frictions in the Market

- In a frictional market, you are like a small particle in the air.
- There are countless many others in the air, and overtime, you and someone encounter face to face.
  - Each meeting is <u>one to one</u>.
  - Whether you find someone today or not is a <u>random event</u>.
- When you find someone, you can trade with him/her or just walk away.

Trade in the Market

- “Trade” in frictional markets could mean many things:
  - Exchange of goods and money.
  - Job interview and hiring.
  - Marriage.
- The (time-consuming) process of finding the right opportunity (or partner) in a frictional market is referred to as **search**.

Single-Sided Search

- Consider shopping.
- You visit several stores to find your perfect T-shirt.
- While time-consuming, the decision is all yours to make. If you are happy about the T-shirt you tried, then you buy it. This is an example of **single-sided search**.
- The traditional labor search theory assumes that search is single-sided.
  - Implicit is that you will not be rejected (!).
  - Theory is insightful, but with obvious limitations.

Two-Sided Search

- In many partnership formations in the real world, search is two-sided.
  - You can freely reject someone.
  - You can also be get rejected.
- Mathematical modeling of this process could be quite complicated.
- What we need is a simple framework that captures the core of two-sided search.
  - Nobel-winning professors, Diamond, Mortensen, and Pissarides, made it possible.

Matching Function

- The breakthrough is made by introducing the concept of **matching function**.
  - The matching function is a function similar to the production function in macroeconomics.
- Suppose there are B single males and G single females in the marriage market.
- Then, the number of couples made is given by

$$m(B, G)$$

- B and G (those who are searching for partners) are inputs.
- This way, we no longer need to describe rejections.

Matching Function

- In the labor market, those who are searching for partners are workers and firms.
- To be more specific, workers who are searching are **unemployed**.
  - Implicit is that <u>those with jobs are not searching</u>.
  - Of course, you can write down a bigger model with **on-the-job-search**. There are many models in the literature.
- Those who are searching for workers are **job vacancies**.
  - It is convenient to <u>ignore the notion of firms</u>.

Matching Function

- Let L be the number of labor force. Then,
$$L = \text{Employed} + \text{Unemployed}$$
- The **unemployment rate** is defined by
$$u = \frac{\text{Unemployed}}{\text{Employed} + \text{Unemployed}} = \frac{\text{Unemployed}}{L}$$
- Similarly, the **vacancy rate** is defined by
$$v = \frac{\text{Number of job vacancies}}{L}$$

Matching Function

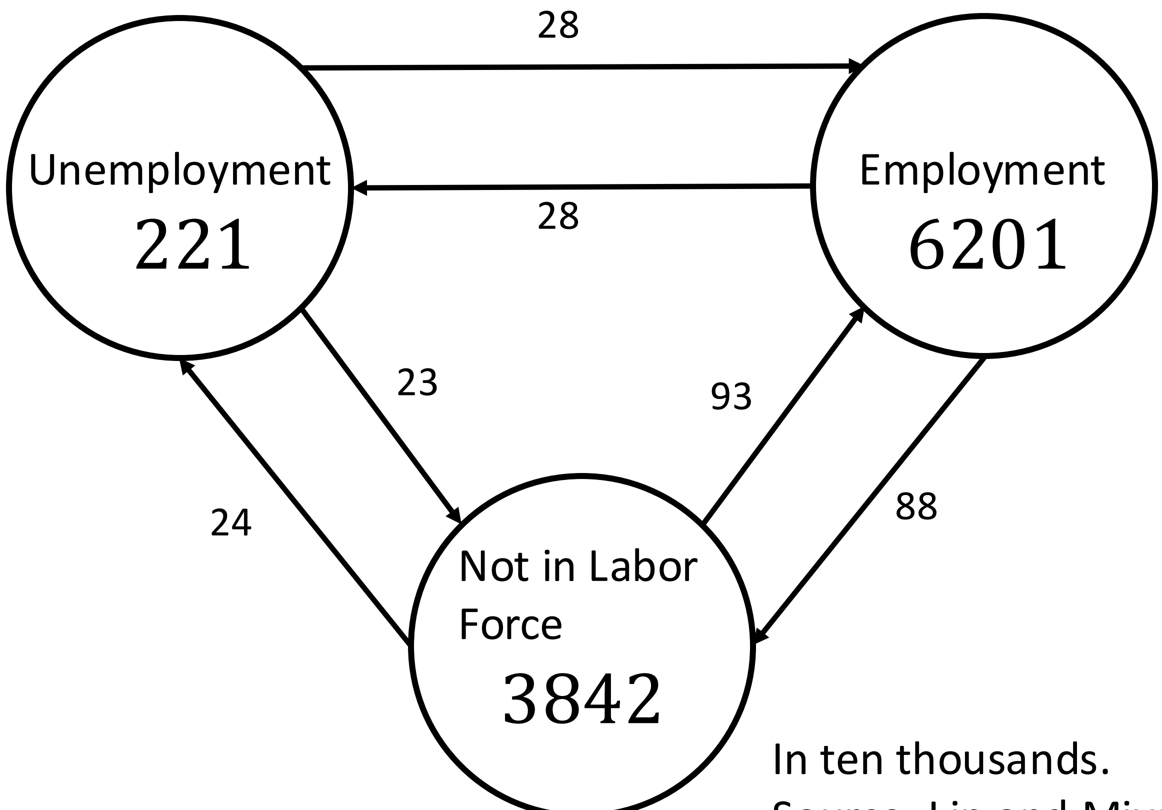
- Number of matches made is
  $M = m(\text{Unemployed}, \text{Vacancies}) = m(uL, vL)$
  - See Equation (1.1) in Pissarides.
- Function m satisfies:
  - Increasing in both arguments.
  - Concave.
  - Homogeneous of degree one (constant-returns to scale).
- These are the same properties as those of the aggregate production function in macroeconomics.

Matching Function

- Often assumed is Cobb-Douglas:
$$m(uL, vL) = A(uL)^{\alpha}(vL)^{1-\alpha}$$
  - $A > 0$ is a parameter that captures the "productivity" of the matching process.
  - α is a parameter satisfying $0 < \alpha < 1$.
- Is this function a reasonable approximation of reality?
  - Particularly important is the assumption of constant returns to scale.
  - There are many empirical studies on the matching function, including Petrongolo and Pissarides (2001).

Worker Flows

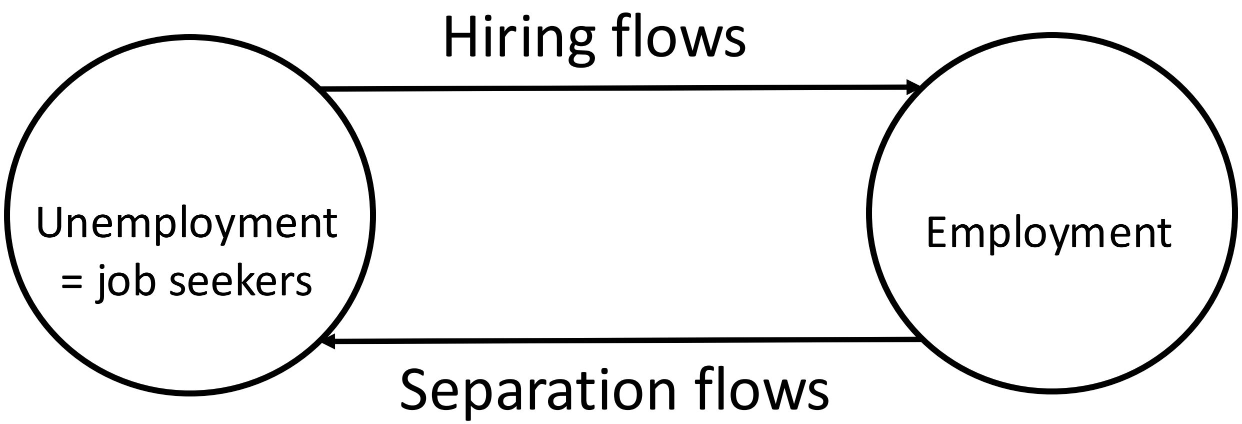
Average Monthly Worker Flows in Japan 1980-2009



In ten thousands.
Source: Lin and Miyamoto (2012)

Simplification

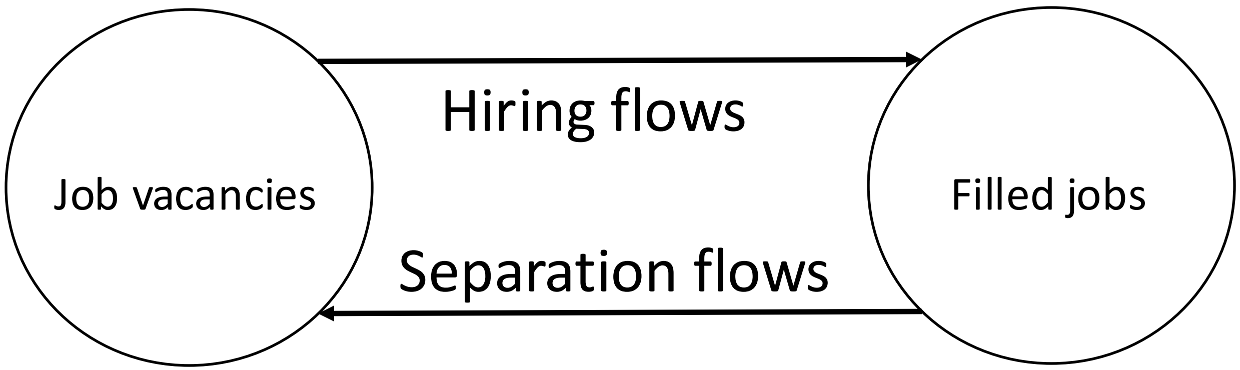
- We shall ignore “not in labor force”.



- The description of how workers change states over time is referred to as **worker flows**.

Job Flows

- There are 2 states for "Jobs".



- Vacant jobs cannot produce anything.
- The description of how jobs change states over time is referred to as **job flows**.

Transition Equations

- Suppose that the unit of time is a month.
- Then, the facts on page 16 imply that the monthly **job-finding probability** is calculated as

$$\frac{28}{221} = 0.13$$

- This is because (on average) 280 thousand workers out of 2210 thousands find jobs within a month.
- Is the <u>weakly</u> or <u>yearly</u> job-finding probability the same as the monthly job-finding probability?

Transition Equations

- The answer should be no.
- As you search longer, it is more likely that you find the right partner.
    - Consider the “lifetime probability of getting married” versus “daily probability of getting married”.
- Thus, the job-finding probability depends (positively) on the length of a period.

Transition Equations

- Let δt denote the length of one period.
- Thus, the job-finding probability during an interval δt is given by

$$\frac{m(uL, vL)}{uL} \times \delta t$$

- Similarly, the probability that a vacancy is filled during an interval δt is

$$\frac{m(uL, vL)}{vL} \times \delta t$$

Transition Equations

- Consider the time-independent components of the transition probabilities.
- Because function m satisfies constant returns to scale,

$$\frac{m(uL, vL)}{vL} = m\left(\frac{u}{v}, 1\right)$$

- We define θ as

$$\theta = \frac{v}{u}$$

- θ is often referred to as the **labor market tightness**.

Transition Equations

- Thus, we obtain the **vacancy-filling rate** as

$$\frac{m(uL, vL)}{vL} = m\left(\frac{u}{v}, 1\right) = m\left(\frac{1}{\theta}, 1\right) = q(\theta)$$

- This is (1.2) in Pissarides.
- Clearly, $q(\theta)$ is <u>decreasing</u> in θ.
  - In words, it is more difficult to fill a vacancy when the vacancy-unemployment ratio is higher.
- Thanks to the property of constant returns to scale, we obtain a function that depends only on θ.

Transition Equations

- Similarly, we obtain the **job-finding rate** as

$$\frac{m(uL, vL)}{uL} = \frac{v}{u}\frac{m(uL, vL)}{vL} = \theta q(\theta)$$

- The job-finding rate is also expressed as

$$\theta q(\theta) = \frac{m(uL, vL)}{uL} = m\left(1, \frac{v}{u}\right) = m(1, \theta)$$

- This implies that $\theta q(\theta)$ is <u>increasing</u> in θ.
  - In words, for a job-seeker, it is easier to find a job when the labor market tightness is higher.

Transition Equations

- When the matching function is Cobb-Douglas (see page 14), the vacancy-filling rate is
$$\frac{m(uL, vL)}{vL} = \frac{A(uL)^{\alpha}(vL)^{1-\alpha}}{vL} = A\left(\frac{v}{u}\right)^{-\alpha} = A\theta^{-\alpha}$$
- Evidently,
$$q'(\theta) = -\alpha A\theta^{-\alpha-1} < 0$$
- Similarly, the job-finding rate is
$$\theta q(\theta) = A\theta^{1-\alpha}$$
- Thus,
$$\big(\theta q(\theta)\big)' = (1-\alpha)A\theta^{-\alpha} > 0$$

Transition Equations

- Elasticity of $q(\theta)$ with respect to θ is defined by

$$\frac{dq(\theta)}{d\theta}\frac{\theta}{q(\theta)} = \frac{q'(\theta)\theta}{q(\theta)} = -\eta(\theta)$$

- With Cobb-Douglas specification,

$$\frac{q'(\theta)\theta}{q(\theta)} = -\frac{\alpha A\theta^{-\alpha-1} \times \theta}{A\theta^{-\alpha}} = -\alpha$$

- Similarly, the elasticity of $\theta q(\theta)$ with respect to θ is

$$\frac{d[\theta q(\theta)]}{d\theta}\frac{\theta}{\theta q(\theta)} = \frac{q(\theta) + q'(\theta)\theta}{q(\theta)} = 1 + \frac{q'(\theta)\theta}{q(\theta)}$$
$$= 1 - \eta(\theta)$$

Transition Equations

- Consider worker flows.
- The job-finding probability during an interval δt is given by
$$\theta q(\theta)\delta t$$
- In this event, the job-seeker becomes an employee.
- A pair of such employee and job is called a **match**.
- In this economy, each match is the unit of production.

Transition Equations

- Unfortunately, no relationship can last forever.
- The probability that a match is destroyed during an interval δt is

$$\lambda \times \delta t$$

- λ is referred to as the **separation rate**.
  - We assume that λ is a parameter.
  - Of course, you can write down a bigger model with endogenous job destruction.
- When this event occurs, the employed worker becomes unemployed (and search for new jobs).

Transition Equations

- Let $u(t)$ denote the unemployment rate at time t.
- After a small interval δt, the unemployment rate becomes $u(t + \delta t)$, which satisfies
$$u(t + \delta t)L$$
$$= u(t)L + \lambda \delta t \big(1 - u(t)\big)L - \theta q(\theta) \delta t u(t)L$$
- We can drop L to obtain
$$u(t + \delta t) - u(t) = \lambda \delta t \big(1 - u(t)\big) - \theta q(\theta) \delta t u(t)$$
- Divide both sides by δt to obtain
$$\frac{u(t + \delta t) - u(t)}{\delta t} = \lambda \big(1 - u(t)\big) - \theta q(\theta) u(t)$$

Transition Equations

- Take the limit as $\delta t \to 0$:

$$\lim_{\delta t \to 0} \frac{u(t+\delta t) - u(t)}{\delta t} = \frac{du(t)}{dt} = \dot{u}$$

- Thus, the transition equation for the unemployment rate <u>in continuous time</u> is

$$\dot{u} = \lambda\big(1 - u(t)\big) - \theta q(\theta) u(t)$$

- Or, more compactly,

$$\dot{u} = \lambda(1 - u) - \theta q(\theta) u$$

- This is (1.3) in Pissarides.

Duration

- Suppose that everyone spends exactly 2 months to find a job, then the monthly job-find rate must be

$$\frac{1}{2} = 0.5$$

- Or,

$$\frac{1}{\text{Duration of unemployment}} = \theta q(\theta)$$

- Thus, the **duration** of unemployment is given by

$$\frac{1}{\theta q(\theta)}$$

Steady State

- Transition equation $\dot{u} = \lambda(1-u) - \theta q(\theta)u$ is a **differential equation**.
- Instead of studying how to solve differential equations, we shall focus on a very special solution, the **steady state**.
  - It is a state in which u is constant over time.
  - It corresponds to the notion of long run equilibrium.
- A steady state of this transition equation is obtained by $\dot{u} = 0$.

Steady State

- Thus, the steady-state unemployment rate satisfies
$$\lambda(1-u)=\theta q(\theta)u$$
- Thus, steady state requires that the number of workers flow out of "unemployment state" equals the number of workers flow into that state.
- The solution is
$$u=\frac{\lambda}{\lambda+\theta q(\theta)}$$
- Because λ is constant and $\theta q(\theta)$ is increasing in θ, we know that (in the long run) u is <u>decreasing</u> in θ.

The Beveridge Curve

- Rewrite the equation as $\lambda u + \theta q(\theta) u = \lambda$.
- From this expression, we want to derive the relationship between u and v.
- With $\theta = v/u$, we obtain
$$\lambda u + q\left(\frac{v}{u}\right) v = \lambda$$
- We totally differentiate this equation to obtain the relationship between u and v.
- Before going to the next page, you should spend 10 minutes to do this calculation.

The Beveridge Curve

$$\lambda u + q\left(\frac{v}{u}\right)v = \lambda$$

$$\Longrightarrow$$

$$\lambda du + q(\theta)dv + q'(\theta)\left[\frac{1}{u}dv - \frac{v}{u^2}du\right]v = 0$$

$$\Longleftrightarrow$$

$$\left(\lambda - q'(\theta)\theta^2\right)du + \left(q(\theta) + q'(\theta)\theta\right)dv = 0$$

$$\frac{dv}{du} = -\frac{\lambda - q'(\theta)\theta^2}{q(\theta) + q'(\theta)\theta}$$

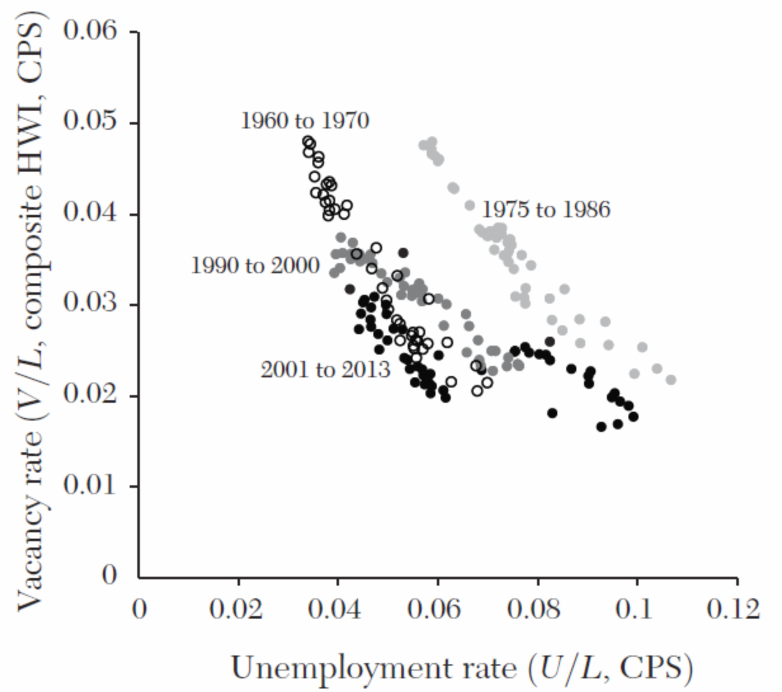
$$= -\frac{\lambda - \frac{q'(\theta)}{q(\theta)}\theta \cdot q(\theta)\cdot\theta}{q(\theta)\left[1 - \eta(\theta)\right]}$$

$$= -\frac{\lambda + \eta(\theta) \times q(\theta)\cdot\theta}{q(\theta)\left[1 - \eta(\theta)\right]} < 0$$

The Beveridge Curve

- The relationship between unemployment and vacancies is often referred to as the **Beveridge curve**.

US Beveridge Curve

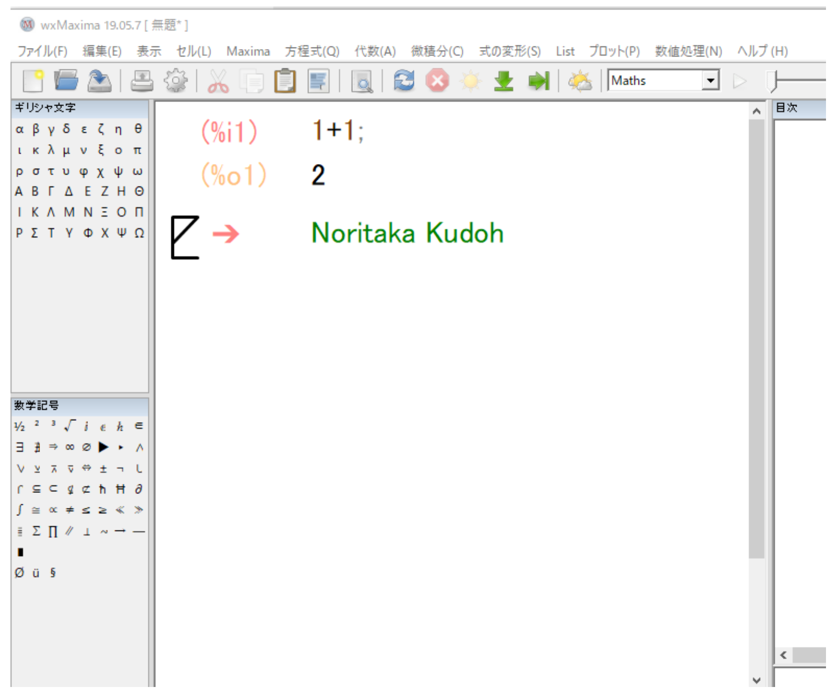


Further Readings

- Petrongolo and Pissarides, “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature*, 2001.
- Elsby, Michaels, and Ratner, “The Beveridge Curve: A Survey,” *Journal of Economic Literature*, 2015.

Get Ready for Maxima

- Visit https://wxmaxima-developers.github.io/wxmaxima/
- Install wxMaxima in your computer.
- Execute “1+1;”to see it works.
    - “Shift + Enter” to execute any command.



Get Ready for Python (Optional)

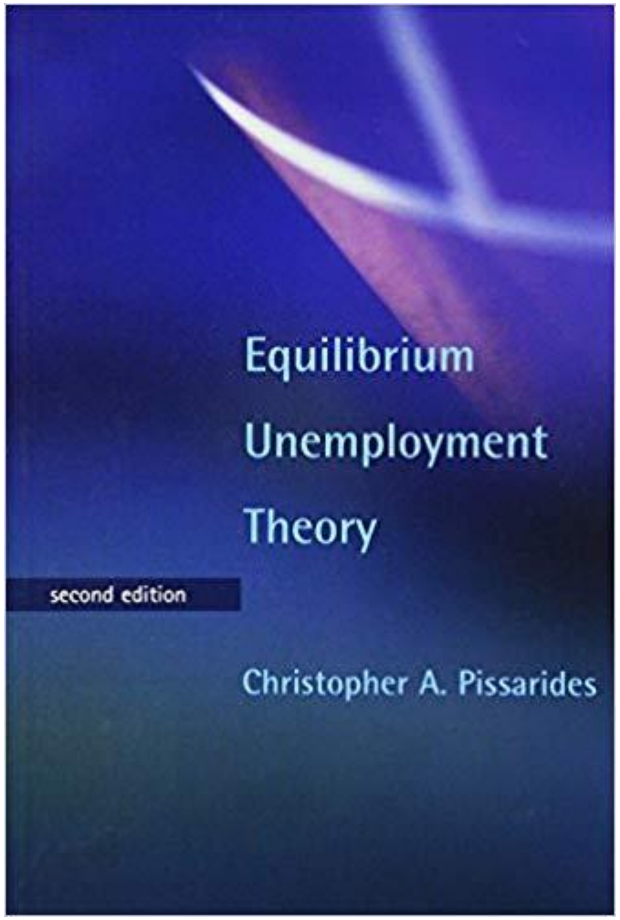
- Install Anaconda (= Python) on your PC.
- https://python-programming.quantecon.org/getting_started.html
- Python is optional, but strongly recommended if you failed to install Maxima.

ANACONDA

Reading Assignment

Reading Material

- Christopher A. Pissarides, *Equilibrium Unemployment Theory*, second edition, MIT Press, 2000.
  - Kindle version available.
- Chapter 1 of the book is made available at NUCT.
  - https://ct.nagoya-u.ac.jp/x/dLy0e1



Assignment

- Read Section 1.2 (Job Creation).
- Due is 4/28.
- 4/28 Class will focus on this section.