Labor Economics B

Lecture 1

Introduction to advanced labor economics

4/14, 2022

Official Course Names

- 「労働経済Ｂ」 for graduate program.
- 「労働経済特論」 for undergraduate program.
- 「Labor Economics A」 for G30 and NUPACE programs.

About Me

- Noritaka Kudoh, Ph.D.
- Professor of Economics at Nagoya University
- Macroeconomist
  - As a result, I will focus on “macro-labor economics”.
- Website: https://sites.google.com/site/noritakakudoh/home

Brief Description

Our Goal for this Semester

- Replication is the key to any scientific activity.
- Our goal is to replicate theoretical (= paper-pencil type) and quantitative (= computer generated) results in professional articles in the field of macro-labor economics.
- By the end of this semester, you should be able to catch up with the research frontier in the field of macro-labor economics.
- Hopefully, you can find your research topic.

Goals Ahead

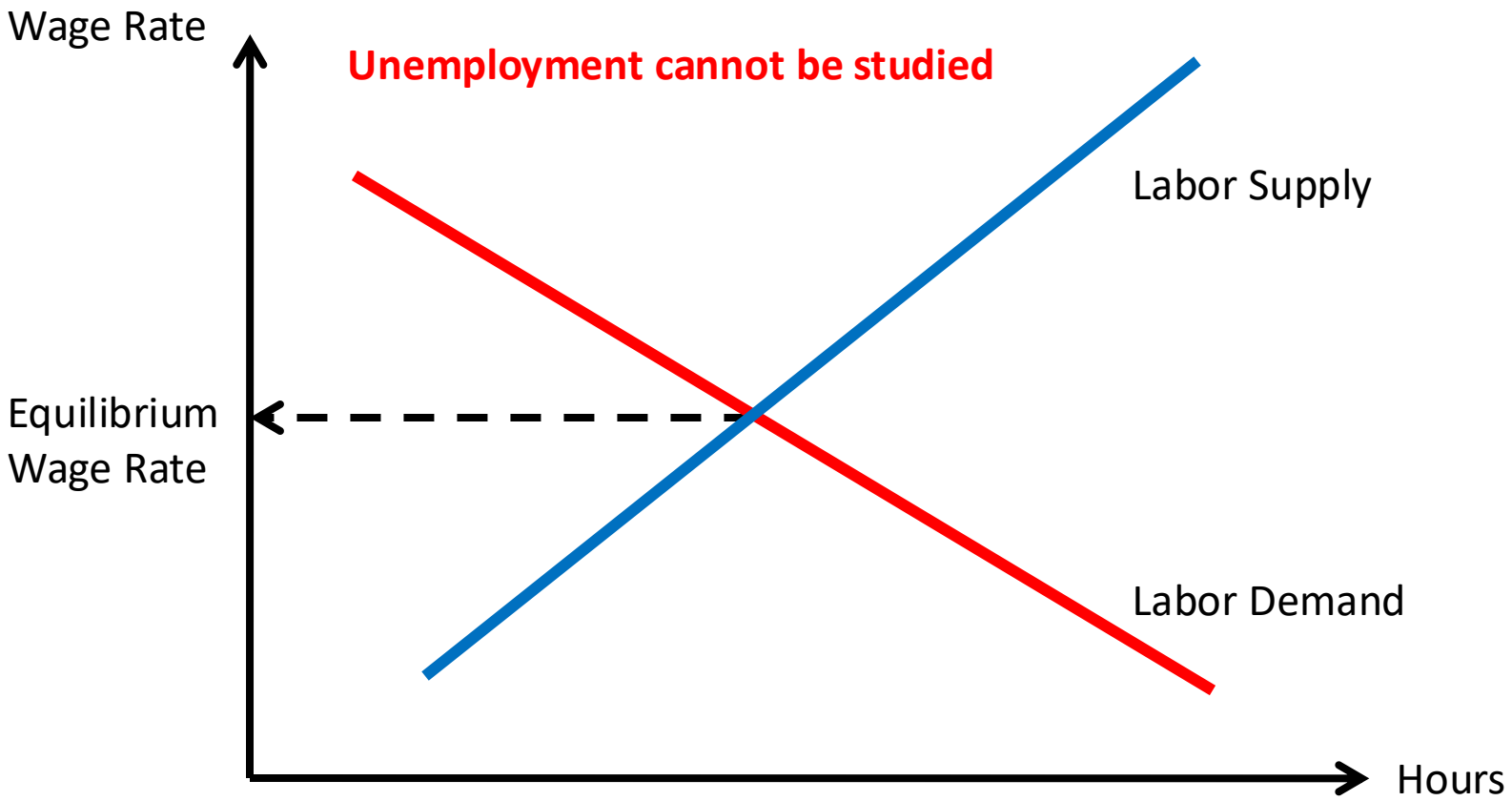
- (Usual) requirement for MA thesis
  - Fully understand and **replicate** some existing results.
- Requirement for Ph.D. dissertation
  - Introduce **innovative** assumptions or methods to some existing analyses to find something new.
  - Complete 3 papers, one of which must be published.
- This course is designed for **second-year** graduate students.
  - All serious students are welcome.

Relationship with Labor Econ A

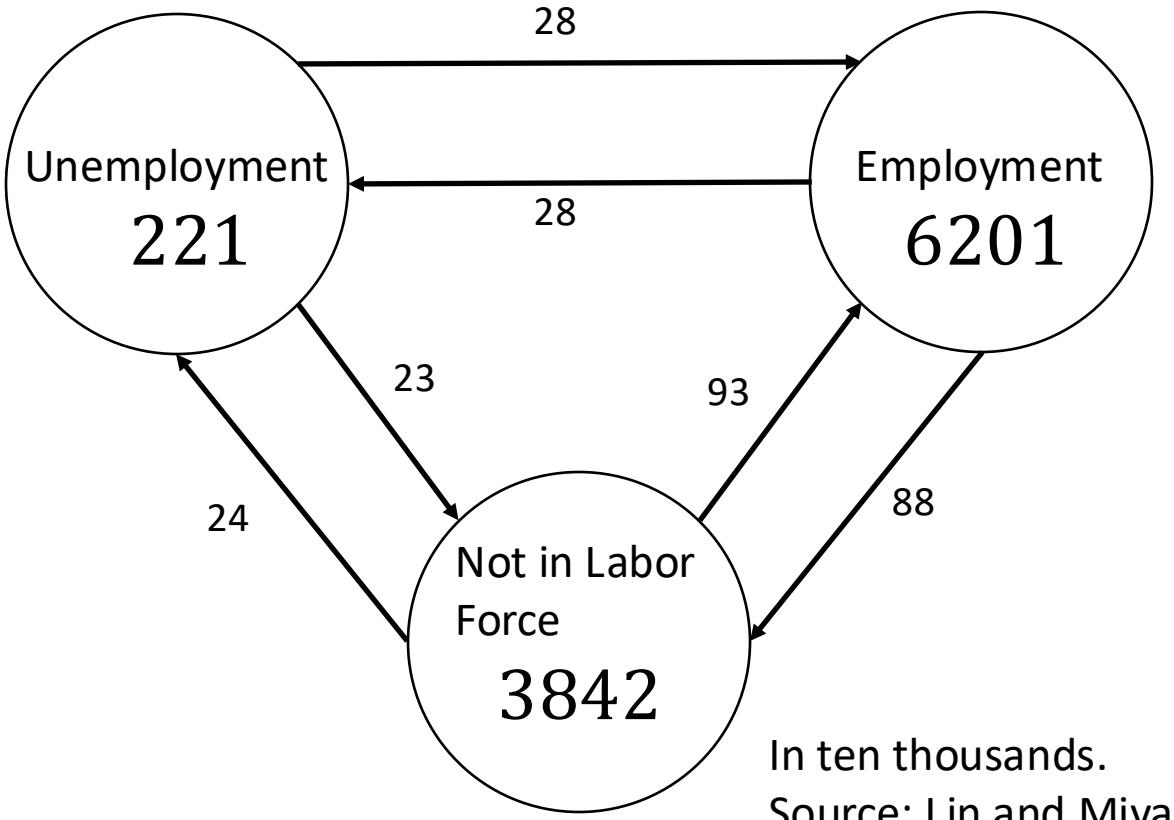
- In Labor Economics A, which I offered last year, I focused on (short-run) business cycles in the labor market:
  - Interested in time-series facts on the labor market.
  - Discrete-time models for simulation.
- In Labor Economics B, I will focus on long-run issues in the labor market:
  - Interested in technological progress, growth, income distribution, and wage inequality.
  - Continuous-time models for computing steady-states.

Detailed Description

Demand-Supply (Walrasian, or Frictionless) Paradigm



Average Monthly Worker Flows: Japan 1980-2009



In ten thousands.
Source: Lin and Miyamoto (2012)

Facts

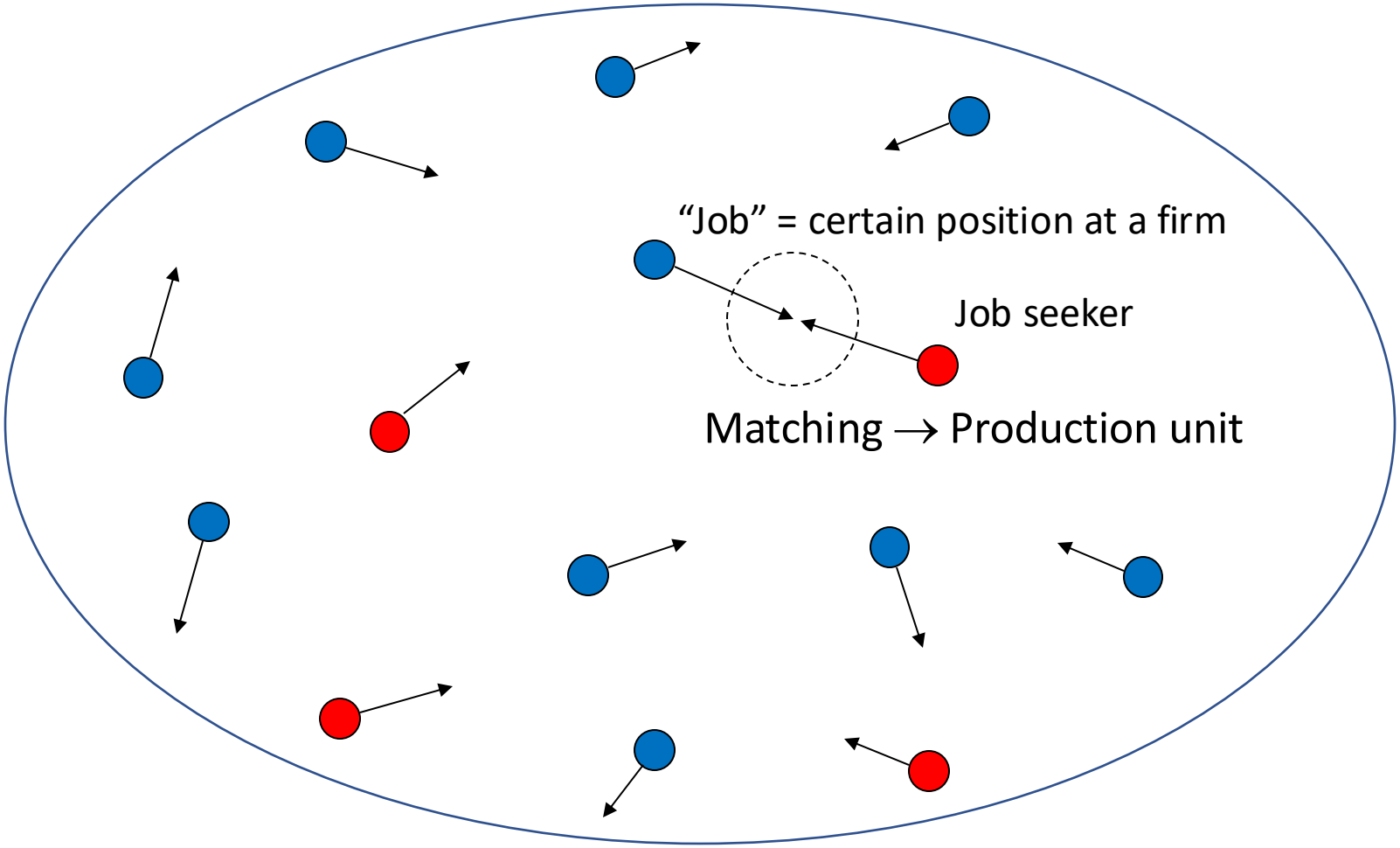
- **Unemployment exists.**
- **People change employment status over time.**

- Traditional theory cannot capture these facts.
- We need a new theory (framework, paradigm).

Search-Matching Model

- We call the standard demand-supply paradigm as the **Walrasian** model.
  - Any individual can meet everyone instantly.
- We start with the observation that it takes time to meet someone.
  - It takes even longer to meet the “right person” for you!
- We study markets with **search-matching frictions**.

Search-Matching Frictions

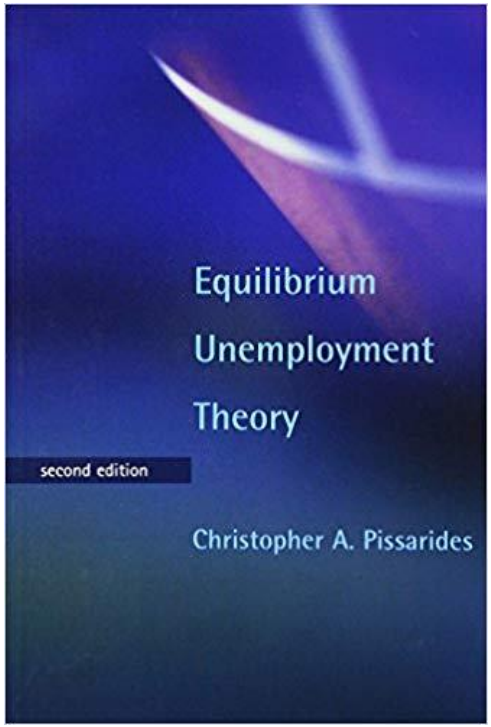


DMP Model

- Thanks to the contributions by Nobel-winning professors, Peter **D**iamond, Dale **M**ortensen, and Christopher **P**issarides, an extremely tractable framework is available.
  - The paradigm is now called the **DMP model**.
  - https://www.nobelprize.org/prizes/economic-sciences/2010/summary/
- Nicely summarized in Pissarides (2000).

Primary Reading

- Pissarides, *Equilibrium Unemployment Theory*, MIT Press, 2000.
  - Chapter 1 provides the core of macro-labor analysis.
  - Strongly recommended.
  - Chapter 1 is available from NUCT: https://ct.nagoya-u.ac.jp/x/dLy0e1



Extremely Useful Review Articles

- Rogerson, Shimer, and Wright, "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 2005.
  - https://ct.nagoya-u.ac.jp/x/d6A25f
- Mortensen, "Markets with Search Frictions and the DMP Model," *American Economic Review*, 2011.
  - https://ct.nagoya-u.ac.jp/x/cvo3tn
- See also: https://www.nobelprize.org/uploads/2018/06/advanced-economicsciences2010.pdf

Computer Languages

- 40 years ago, it was OK to write a paper without any computer simulation. Not any more. The old age has gone.
- It is increasing more important to develop a complicated (realistic) model and solve it using computer.
- There will be many assignments requiring computer simulation and data analysis.

Excel

- Very intuitive and useful.

Mathematica

- Very powerful math package.
- Especially useful for solving a system of nonlinear equations.
- Very expensive.
- You can execute short mathematica codes at https://www.wolframalpha.com/

Maxima

- A close cousin of Mathematica and this one is **free**.
- This is **not a clone** of Mathematica, but the language is similar.
- Some assignments will require Maxima.
  - No need to install it today.

Python

- The language of the next generation.
- Available for free.
- For more about this language, please visit https://quantecon.org/
- (In the long-run, python will take care of everything I need for my courses.)

Word Processors for Economists

- (For M2 students)
- Microsoft Word is fine.
- LaTeX is currently the best environment for professional articles with (many) mathematical expressions.
- There are many LaTeX packages for free:
  - TeXshop (mac)
  - TeXworks (Win and mac)
  - Overleaf (online)

Job Search and Optimal Stopping

References

- **Section 2** of Rogerson, Shimer, and Wright, "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 2005.
  - We shall refer to this as RSW.
- **Section IV** of Mortensen, "Markets with Search Frictions and the DMP Model," *American Economic Review*, 2011.

Expectations

- Suppose there are two states, g (good) and b (bad).
- State g occurs with probability p_g.
- State b occurs with probability p_b.
- Since there is no other state, $p_g + p_b = 1$.
- Your income is y_g if state g occurs.
- Your income is y_b if state b occurs.
- The expected income is

$$\mathbb{E}y = p_g y_g + p_b y_b$$

Expectations

- Suppose there are n states: $s = 1,2,3, \dots, n$.
- Let $p(s)$ be the probability that state s occurs.
- Let $y(s)$ be the income when state s.
- The expected income is
  $$\mathbb{E}y = p(1)y(1) + p(2)y(2) + \cdots + p(n)y(n)$$
- Or,
$$\mathbb{E}y = \sum_{s=1}^{n} p(s)y(s)$$

Probability Distribution

- Consider a continuous state, s.
- Suppose $s \in \left[\underline{s}, \overline{s}\right]$. We say that the support is $\left[\underline{s}, \overline{s}\right]$.
- We introduce cumulative distribution function (cdf), $F(s)$, defined on the support.
- $f(s) = F'(s)$ is the probability density function.
- The expected income is

$$\mathbb{E}y = \int_{\underline{s}}^{\overline{s}} y(s)dF(s) = \int_{\underline{s}}^{\overline{s}} y(s)f(s)ds$$

Useful Probability Distribution Functions for Search Theory

- Uniform distribution
- Pareto distribution

Uniform Distribution

- Uniform distribution on $[a, b]$.
- Cumulative distribution function:

$$F(w) = \frac{w - a}{b - a}$$

- Density:

$$f(w) = F'(w) = \frac{1}{b - a}$$

- If $y(s) = s$, the expected income is

$$\mathbb{E}w = \int_{\underline{s}}^{\overline{s}} w dF(w) = ??$$

Pareto Distribution

- Pareto distribution with shape parameter $k > 1$ and scale parameter $a > 0$ on $[a, \infty)$.
- Cumulative distribution function:

$$F(w) = 1 - \left(\frac{a}{w}\right)^k$$

- Density:

$$f(w) = F'(w) = ka^k w^{-k-1}$$

- If $y(s) = s$, the expected income is

$$\mathbb{E}w = \int_{\underline{s}}^{\overline{s}} w dF(w) = ??$$

Present Discounted Values

- Suppose you receive w yen every period for $n+1$ periods. Your lifetime income is

$$Y = w + \beta w + \beta^2 w + \cdots + \beta^n w = \sum_{t=0}^{n} \beta^t w = \frac{1-\beta^{n+1}}{1-\beta} w$$

- Let $n \to \infty$ to obtain

$$Y = \sum_{t=0}^{\infty} \beta^t \, w = \frac{w}{1-\beta}$$

- Review:
  - Consider $S = 1 + x + x^2 + \cdots + x^n$ and xS.

Recursive Formulation

- You receive w yen every period forever:

$$\begin{aligned} Y &= w + \beta w + \beta^2 w + \cdots \\ &= w + \beta\{w + \beta w + \cdots\} \\ &= w + \beta Y \end{aligned}$$

- Thus, we immediately obtain the same result

$$Y = \frac{w}{1+\beta}$$

- The core of our theory is

$$Y = w + \beta Y$$

- We call it a **Bellman equation** (in discrete time).

Job Search in Discrete Time

- Expected lifetime income:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t x_t,$$

where

$$x_t = \begin{cases} w \text{ in state } E \\ b \text{ in state } U \end{cases}$$

- Probability that you receive a job offer is α.
- Offer w is a random variable from cdf $F(w)$.

Job Search Problem

- Following the notations in RSW, we have
  $U = b + \alpha \int_0^{\bar{w}} \max\{\beta W(w), \beta U\}\, dF(w) + (1-\alpha)\beta U$
- b is the unemployment benefit.
- $W(w)$ is the value of a contract with wage w.
- If we set $\alpha = 1$, this reduces to (2) in RSW.
- Rewrite this expression as

$$(1-\beta)U = b + \alpha\beta \int_0^{\bar{w}} \max\{W(w) - U, 0\}\, dF(w)$$

Optimal Stopping

- It is optimal to "stop" (and take it) if and only if
$$W(w) > U$$
- Let R denote the **reservation wage rate**, defined by
$$W(w_R) = U$$
- Then, stop if and only if
$$w > w_R$$
- Then,
$$(1-\beta)U = b + \alpha\beta \int_{w_R}^{\bar{w}} [W(w) - U] dF(w)$$

Optimal Stopping

- The value of being employed at the wage rate w is
$$W(w) = w + \beta\{\lambda U + (1-\lambda)W(w)\}$$
- Evaluate it at w_R to obtain
$$W(w_R) = w_R + \beta\{\lambda U + (1-\lambda)W(w_R)\}$$
- λ is the separation rate. From the two equations,
$$W(w) - U = W(w) - W(w_R) = \frac{w - w_R}{1 - \beta(1-\lambda)}$$
- Thus,
$$w_R = b + \frac{\alpha\beta}{1-\beta(1-\lambda)} \int_{w_R}^{\bar{w}} [w - w_R] dF(w)$$

Optimal Stopping

- Using integration by parts, we obtain

$$\int_{w_R}^{\bar{w}} [w - w_R] dF(w) = \int_{w_R}^{\bar{w}} [1 - F(w)] dw$$

- Review of integration by parts:

$$(xy)' = x'y + xy' \Longrightarrow [xy]_a^b = \int_a^b y dx + \int_a^b x dy$$

- Thus,

$$w_R = b + \frac{\alpha\beta}{1 - \beta(1 - \lambda)} \int_{w_R}^{\bar{w}} [1 - F(w)] dw$$

Finding a solution

- The **equilibrium reservation wage rate** is the solution to

$$w_R = b + \frac{\alpha\beta}{1-\beta(1-\lambda)} \int_{w_R}^{\bar{w}} [1 - F(w)] dw$$

- If we set $\alpha = 1$ and $\lambda = 0$, then this reduces to (5) in RSW.
- Left-hand side is increasing.
- Right-hand side is decreasing.
- Diagram.

Comparative Statics

- An increase in b on w_R and u.
- An increase in α.
- An increase in β.
- An increase in λ.

Numerical Solution

- To obtain an explicit solution, we need to specify:
  - The functional form of F.
  - All parameter values.

Continuous Time

- The value of being employed at the wage rate w is
$$W(w) = w + \beta\{\lambda U + (1-\lambda)W(w)\}$$
- In continuous time,
$$W(w) = w\Delta + \frac{1}{1+r\Delta}\{\lambda\Delta U + (1-\lambda\Delta)W(w)\}$$
- Let $\Delta \to 0$ to obtain
$$rW(w) = w + \lambda[U - W(w)]$$

Continuous Time

- In discrete time, the value of job search is

$$U = b + \alpha\beta \int_0^{\bar{w}} \max\{W(w), U\}\, dF(w) + (1-\alpha)\beta U$$

- In continuous time,

$$U = b\Delta + \alpha\Delta \frac{1}{1+r\Delta} \int_0^{\bar{w}} \max\{W(w), U\}\, dF(w) + (1-\alpha\Delta) \frac{1}{1+r\Delta} U$$

- Let $\Delta \to 0$ to obtain

$$rU = b + \alpha \int_0^{\bar{w}} \max\{W(w) - U, 0\}\, dF(w)$$

Review Questions

Submission of Your Work

- All homework assignments must be obtained from and submitted to NUCT.
  - I will accept PDF files only.
- For some assignments (such as calculations), your answers must be hand-written.
  - Many scanner applications for iPhone/android are available for free.
  - You can also use a pen tablet such as iPad.
  - I strongly suggest that you test each method to see if you can create a PDF of your hand-written letter.

If you have absolutely no idea…

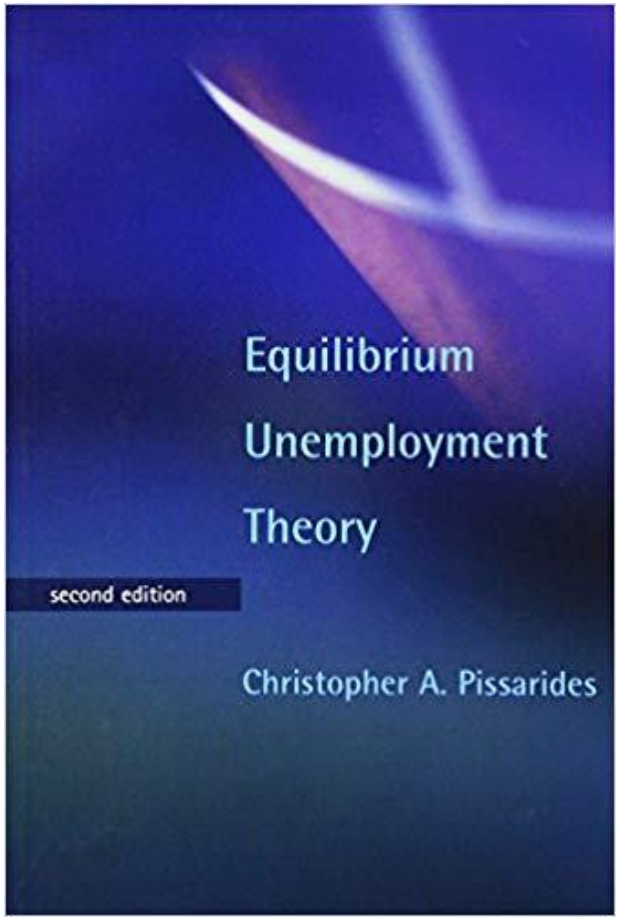
- Try one of the following:

1. My favorite is Microsoft OneNote, which is free and available under many environments.
   - I find it particularly useful on iPad with Apple Pencil.
2. My favorite scanner app is Scannable (by Evernote), but there are many apps.
3. Bring your hand-written notes to a convenience store to manually create a PDF file using a multi-purpose printer there. Costly, not recommended.

Reading Assignment

Reading Material

- Christopher A. Pissarides, *Equilibrium Unemployment Theory*, second edition, MIT Press, 2000.
  - Kindle version available.
- Chapter 1 of the book is made available at NUCT.
  - https://ct.nagoya-u.ac.jp/x/dLy0e1



Assignment

- Read Section 1.1 (Trade in the Labor Market).
- Due is 4/21.
- 4/21 Class will focus on this section.