NAGOYA UNIVERSITY

Electromagnetics

Answers to Homework

Exercise 1.3.2 (Homework)

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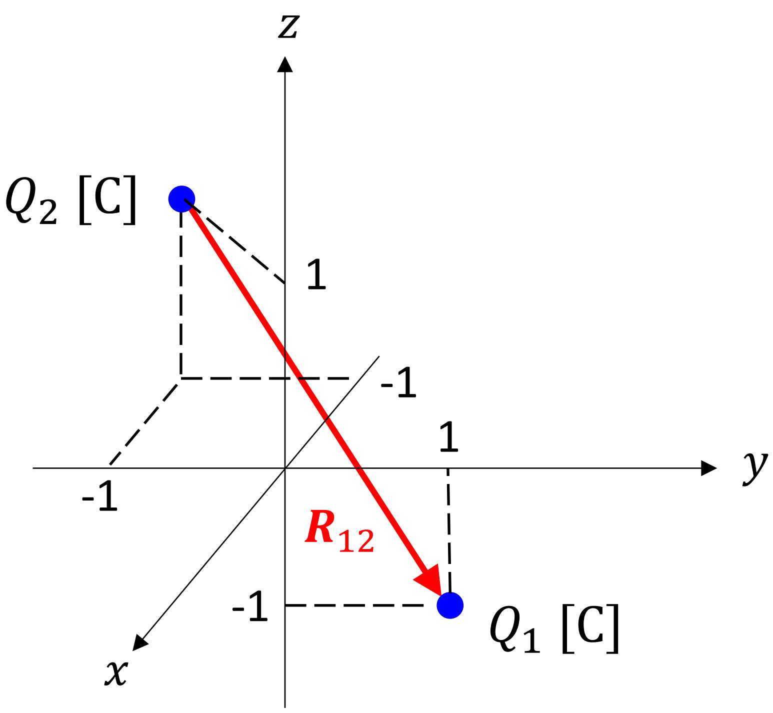
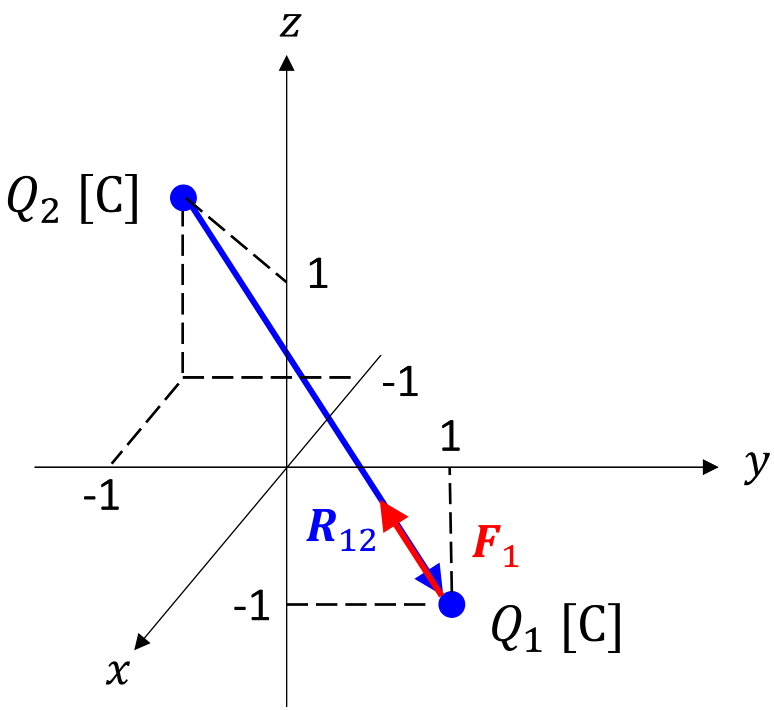
Find the force on a point charge $Q_1 = -30$ [μC], due to a point charge $Q_2 = 10$ [μC], where Q_1 is at (0, 1, -1) [m] and Q_2 is at (-1, -1, 1) [m]. Both Q_1 and Q_2 are in the air.



Exercise 1.3.2 (Answer)



$$\boldsymbol{R}_{12} = 1\boldsymbol{a}_x + 2\boldsymbol{a}_y - 2\boldsymbol{a}_z$$

$$|\boldsymbol{R}_{12}| = \sqrt{1^2 + 2^2 + 2^2} = 3 \text{ [m]}$$

$$\boldsymbol{a}_{12} = \frac{1\boldsymbol{a}_x + 2\boldsymbol{a}_y - 2\boldsymbol{a}_z}{3}$$

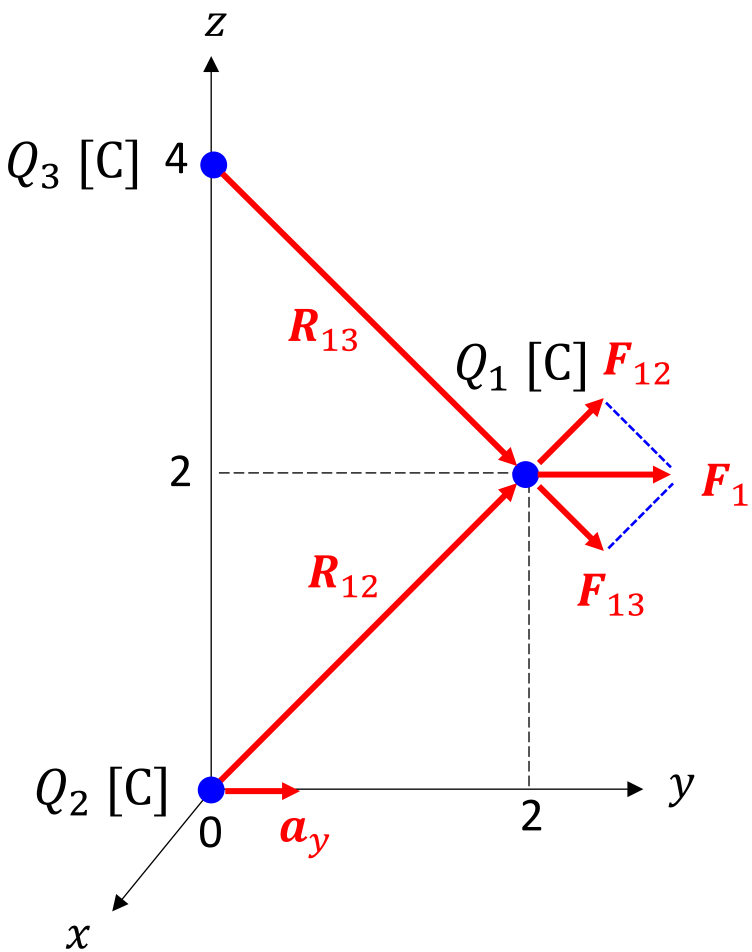
$$\begin{aligned}\boldsymbol{F}_1 &= \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{R_{12}^2}\boldsymbol{a}_{12} \\ &\approx 9 \times 10^9 \times \frac{-3 \times 10^{-10}}{3^2} \times \frac{1\boldsymbol{a}_x + 2\boldsymbol{a}_y - 2\boldsymbol{a}_z}{3} \\ &= -0.1\boldsymbol{a}_x - 0.2\boldsymbol{a}_y + 0.2\boldsymbol{a}_z \text{ [N]}\end{aligned}$$

Exercise 1.4 (Homework)

Exercise 1.4 (Homework)

Find the force on a point charge $Q_1 = 10\ [\mu\mathrm{C}]$, due to point charges $Q_2 = Q_3 = 100\ [\mu\mathrm{C}]$. The locations of Q_1, Q_2, and Q_3 are at (0, 2, 2) $[\mathrm{m}]$, (0, 0, 0) $[\mathrm{m}]$, and (0, 0, 4) $[\mathrm{m}]$, respectively. The charges are in the air.

Exercise 1.4 (Answer)



$$\boldsymbol{R}_{12} = 0\boldsymbol{a}_x + 2\boldsymbol{a}_y + 2\boldsymbol{a}_z$$
$$\boldsymbol{R}_{13} = 0\boldsymbol{a}_x + 2\boldsymbol{a}_y - 2\boldsymbol{a}_z$$

$$|\boldsymbol{R}_{12}| = |\boldsymbol{R}_{13}| = \sqrt{0^2 + 2^2 + 2^2} = 2\sqrt{2}\ [\mathrm{m}]$$

$$\boldsymbol{a}_{12} = \frac{2\boldsymbol{a}_y + 2\boldsymbol{a}_z}{2\sqrt{2}}$$

$$\boldsymbol{a}_{13} = \frac{2\boldsymbol{a}_y - 2\boldsymbol{a}_z}{2\sqrt{2}}$$

$$\begin{aligned}\boldsymbol{F}_1 &= \frac{1}{4\pi\varepsilon}\frac{Q_1Q_2}{R_{12}^2}\boldsymbol{a}_{12} + \frac{1}{4\pi\varepsilon}\frac{Q_1Q_3}{R_{13}^2}\boldsymbol{a}_{13} \\ &\approx 9\times10^9\times\frac{10^{-9}}{\left(2\sqrt{2}\right)^2}\times\frac{(2+2)\boldsymbol{a}_y + (2-2)\boldsymbol{a}_z}{2\sqrt{2}} \\ &= \frac{9\sqrt{2}}{8}\boldsymbol{a}_y \approx 1.6\boldsymbol{a}_y\ [\mathrm{N}]\end{aligned}$$

Exercise 1.6.3 (Homework)

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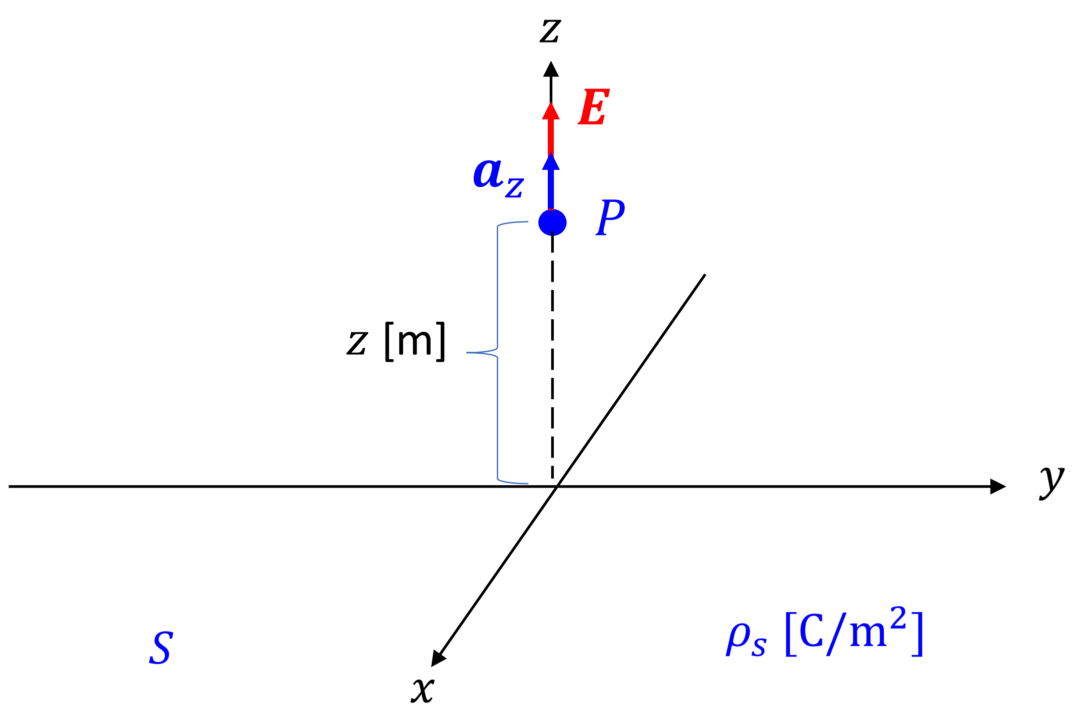
Exercise 1.6.3 (Homework)

Show that the total electric field intensity at a point P produced by charge distributed with uniform density ρ_s $[\mathrm{C/m^2}]$ over an infinite $x - y$ plane S is given by

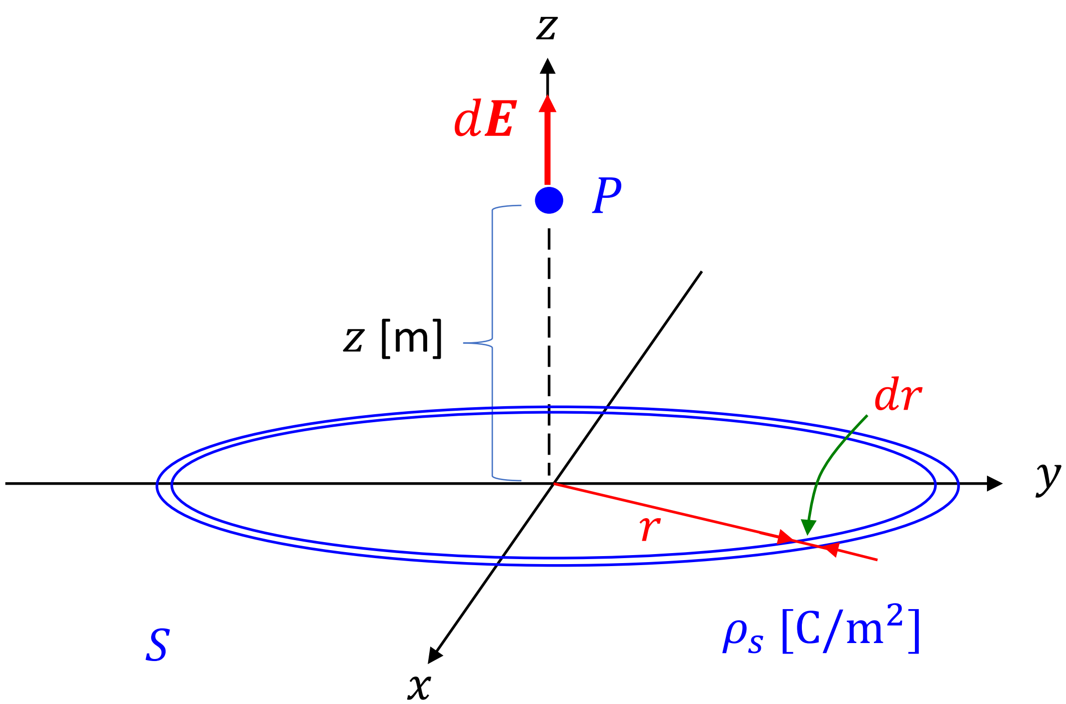
$$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon}\boldsymbol{a}_z$$

where $\boldsymbol{a}_z$ is the unit vector in parallel to the z axis. The distance between P and the plane is z [m]. Note that $\boldsymbol{E}$ is not a function of z.

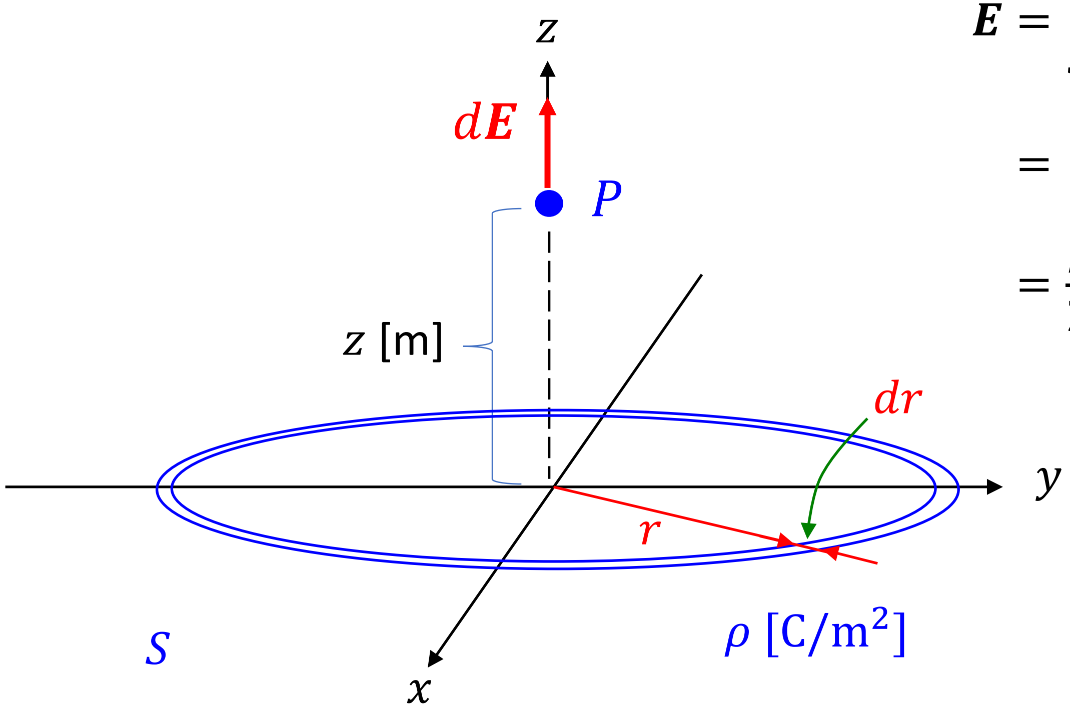


Exercise 1.6.3 (Homework)

Hint : Use the result of Exercise 1.6.2. You can choose differential charge distributed along a circle of radius r placed in the $x-y$ plane. Integrate $d\boldsymbol{E}$ from 0 to ∞ with respect to r.



Exercise 1.6.3 (Answer)



$$d\boldsymbol{E} = \frac{\rho_s z}{2\varepsilon}\frac{r dr}{(r^2+z^2)^{3/2}}\boldsymbol{a}_z$$

$$\begin{aligned}\boldsymbol{E} &= \int_0^\infty \frac{\rho_s z}{2\varepsilon}\frac{r}{(r^2+z^2)^{3/2}} dr\boldsymbol{a}_z \\ &= \left[\frac{\rho_s z}{2\varepsilon}\frac{-1}{\sqrt{r^2+z^2}}\right]_0^\infty \boldsymbol{a}_z \\ &= \frac{\rho_s}{2\varepsilon}\boldsymbol{a}_z\end{aligned}$$

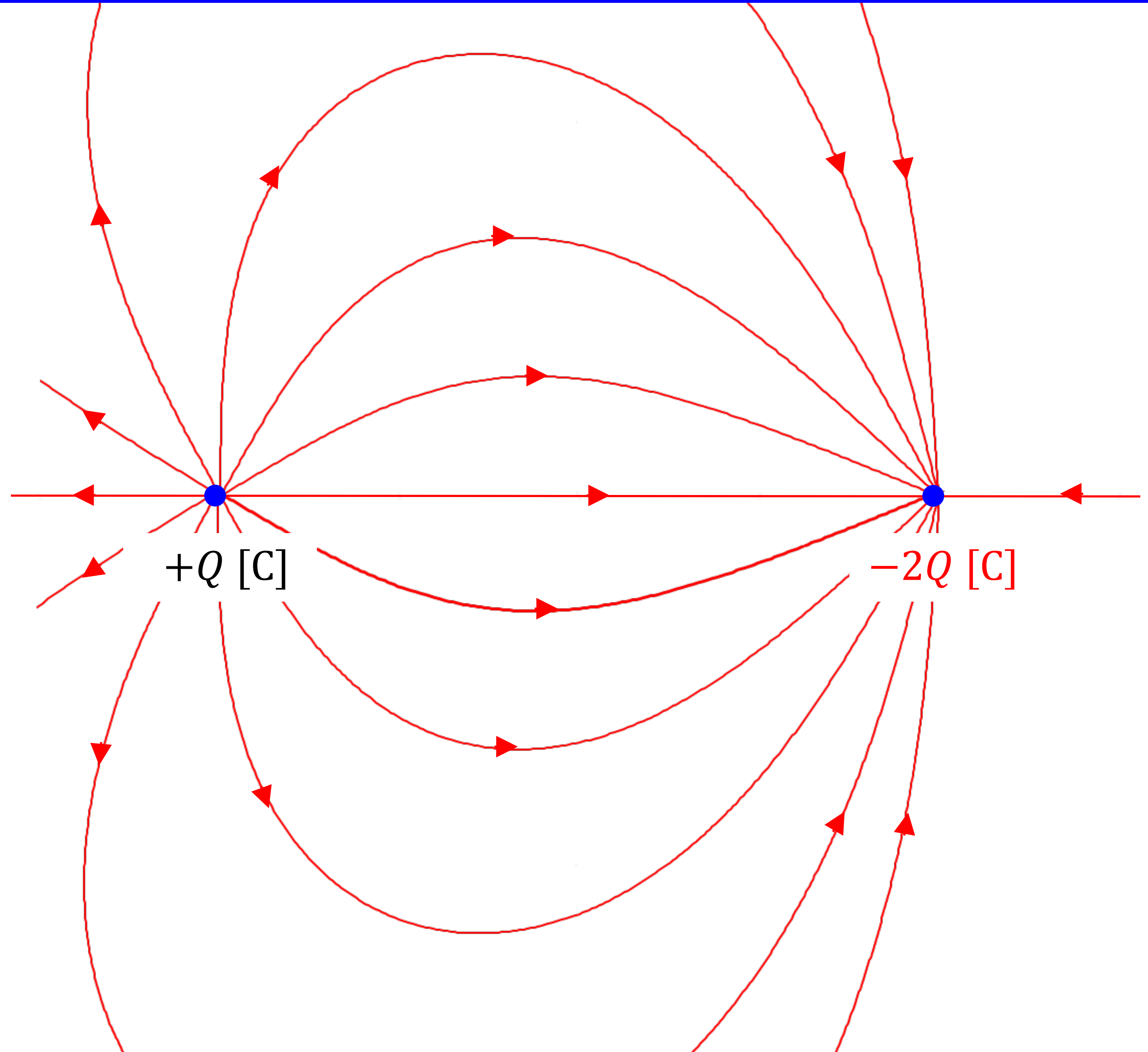
Exercise 2.1.1 (Homework)

Exercise 2.1.1 (Homework)

There are two point charges as below. Draw electric flux lines.

$+Q$ [C] $\qquad\qquad\qquad\qquad$ $-2Q$ [C]

Exercise 2.1.1(Answer)



Exercise 2.1.2 (Homework)

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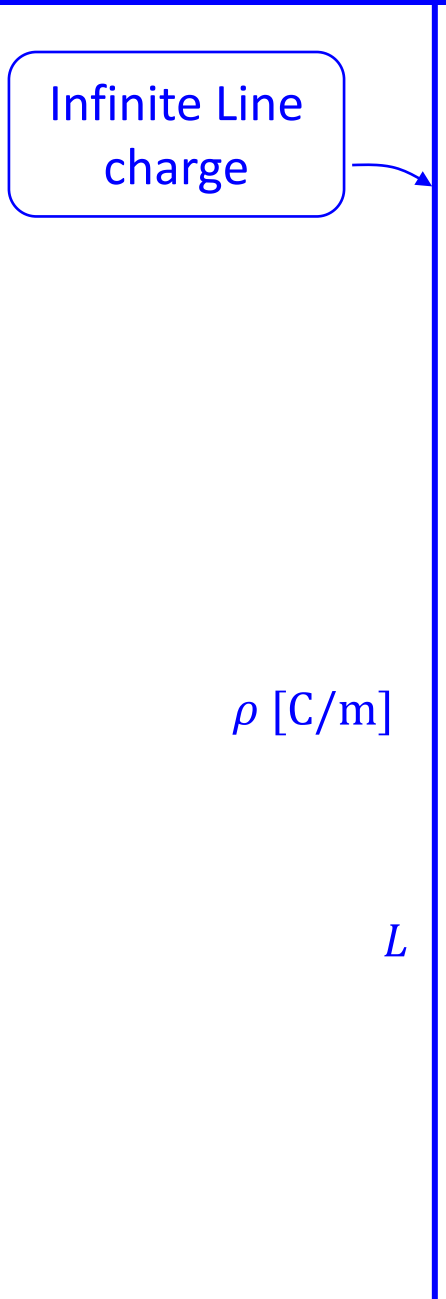
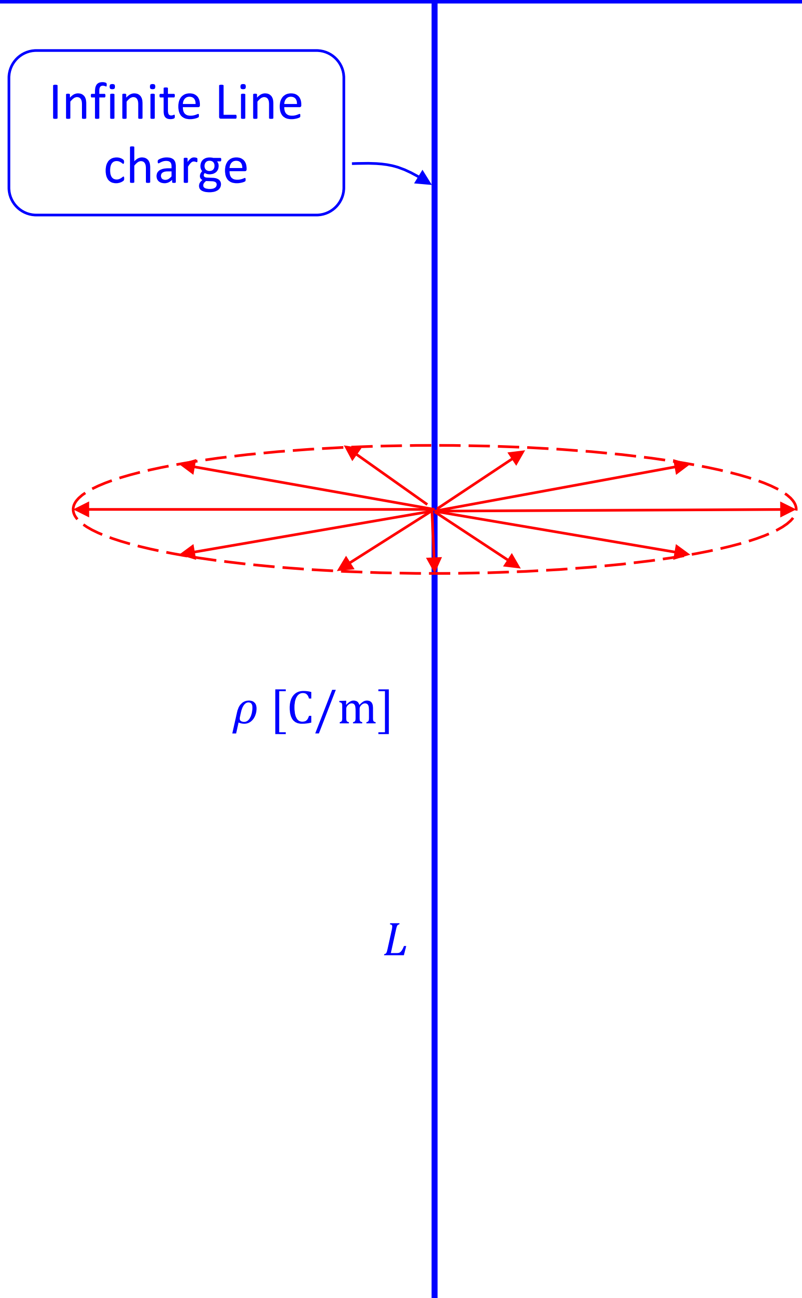


There is an infinite line charge along an infinite line L with zero thickness. The charge density is ρ [C/m].
Draw electric flux lines.

Exercise 2.1.2 (Answer)



According to the result of Exercise 1.6.1, an electric field intensity generated by an infinite line charge is radial and perpendicular to the line. Thus, the electric flux line is also radial and perpendicular to the line L. These lines leave the line charge and terminate at infinity.

Exercise 2.2 (Homework)

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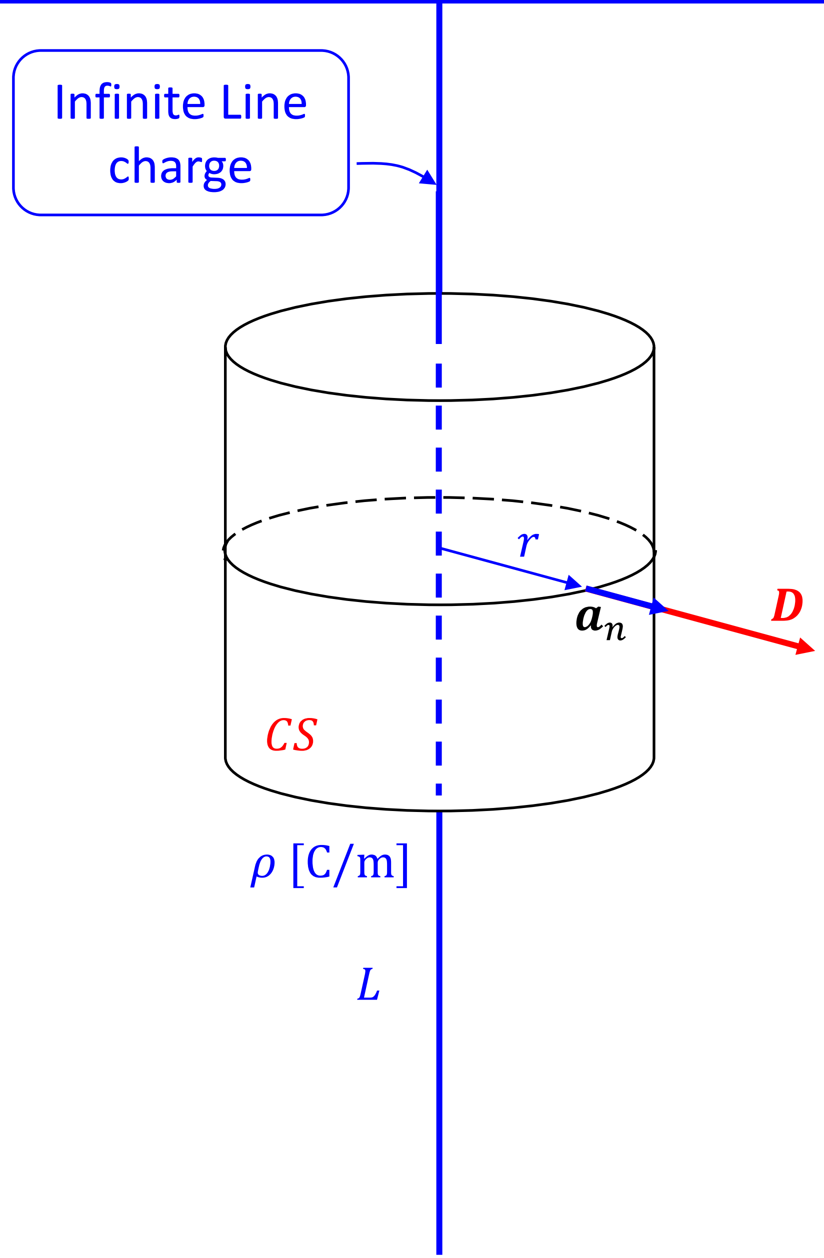
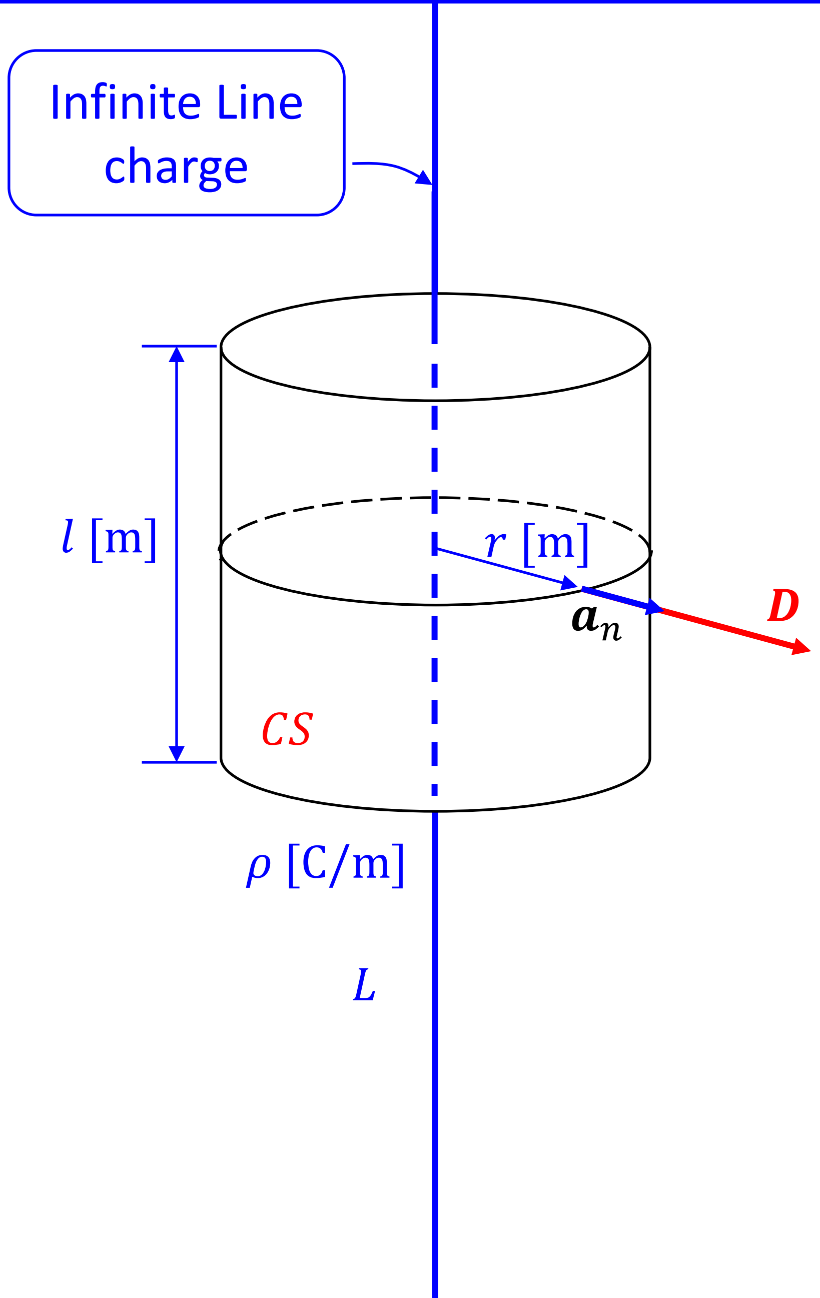


There is an infinite line charge along an infinite line L with zero thickness. The charge density is ρ [C/m]. Show that the electric flux density at the surface of a cylindrical closed surface CS of radius r [m] is given by

$$\boldsymbol{D} = \frac{\rho}{2\pi r}\boldsymbol{a}_n \quad [\mathrm{c/m^2}]$$

where $\boldsymbol{a}_n$ is the unit vector normal to the cylindrical surface and has the same direction as $\boldsymbol{D}$.

Exercise 2.2 (Answer)



According to the result of Exercise 1.6.1, an electric flux line by an infinite line charge is radial and perpendicular to the line. The height of cylinder is set to be l [m]. Then, the total electric flux lines Ψ crossing the cylindrical surface is given by

$$\Psi = \rho l \text{ [C]}.$$

The surface of the side of cylinder S, which does not include the top and bottom surfaces, is

$$S = 2\pi r l \quad [m^2].$$

Thus, the electric flux density $\boldsymbol{D}$ is

$$\boldsymbol{D} = \frac{\Psi}{S}\boldsymbol{a}_n = \frac{\rho}{2\pi r}\boldsymbol{a}_n \ \ [\mathrm{c/m^2}].$$

Exercise 2.3.2 (Homework)

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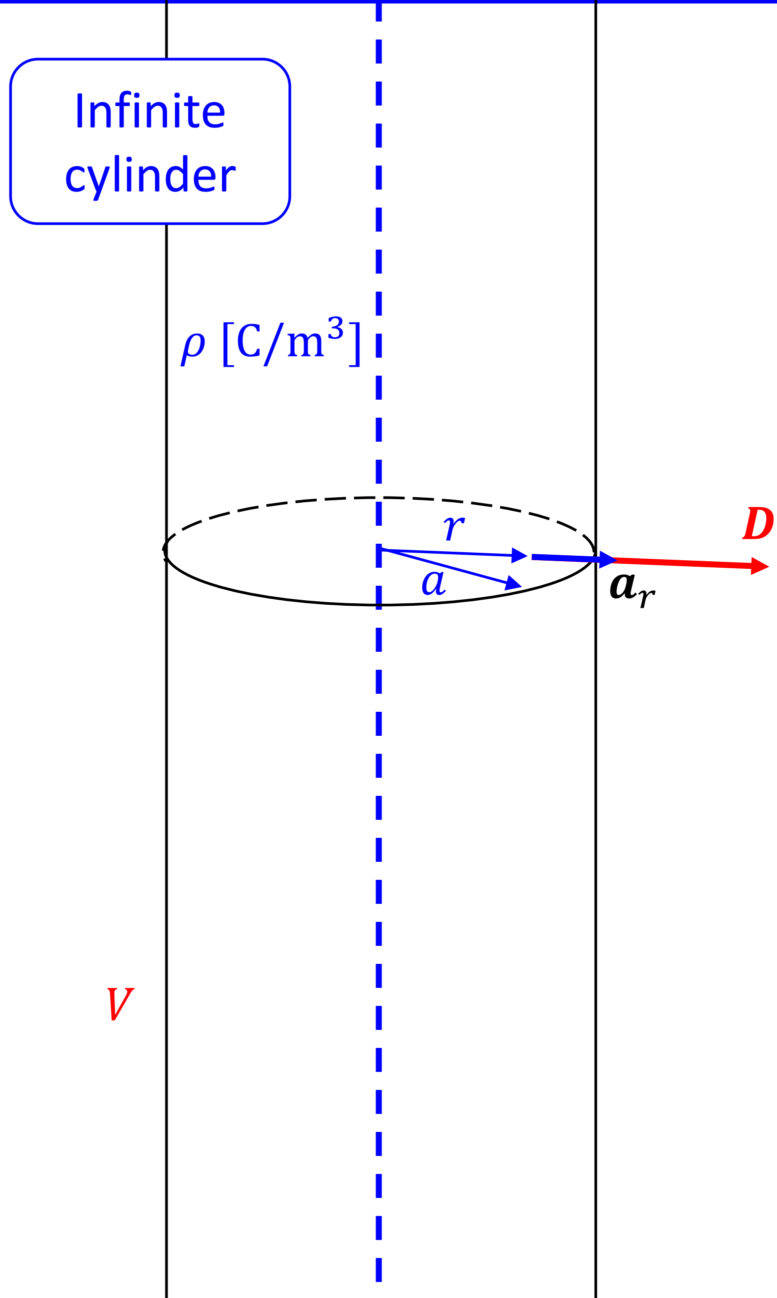
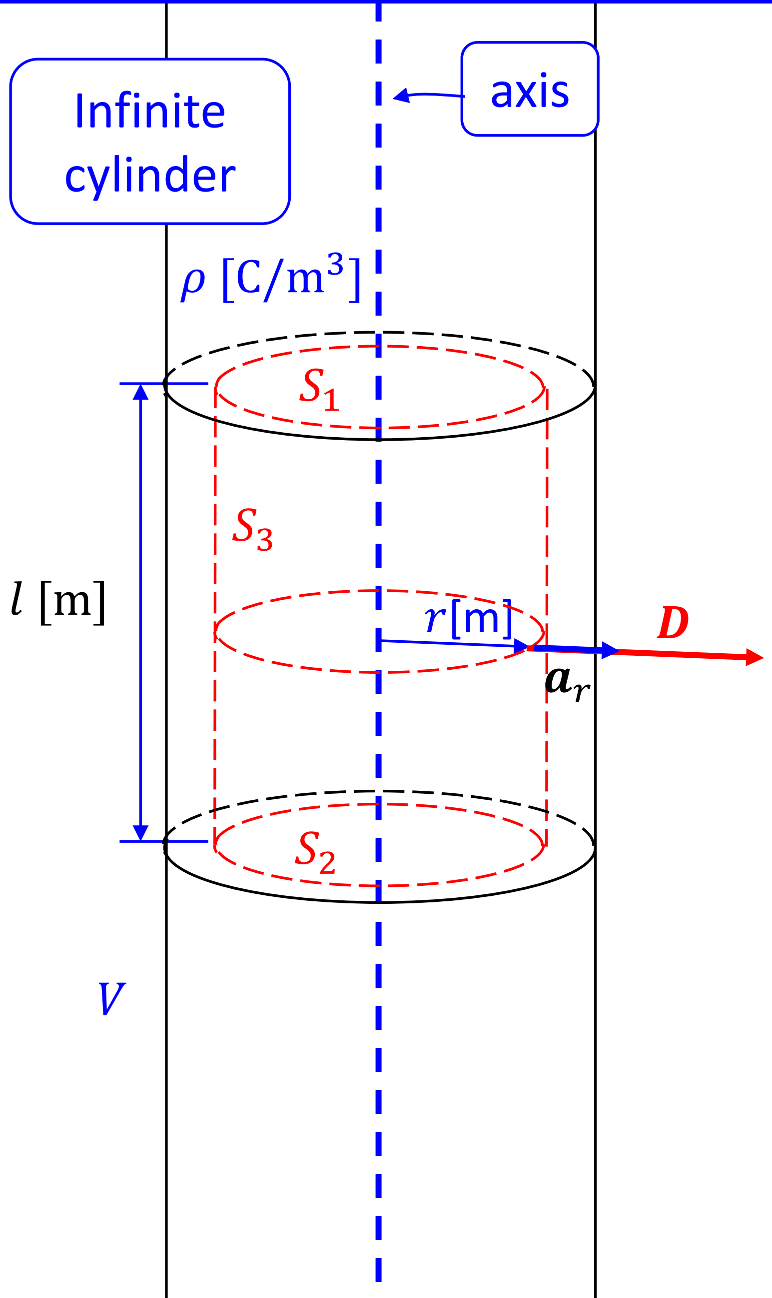


Charge is distributed with a constant density ρ $[C/m^3]$ throughout an infinite cylinder V of radius a [m]. Show that

$$\boldsymbol{D} = \begin{cases} \dfrac{r\rho}{2}\boldsymbol{a}_r & (r \le a) \\ \dfrac{a^2\rho}{2r}\boldsymbol{a}_r & (a \le r) \end{cases}$$

Exercise 2.3.2 (Homework)



(Hint) Assume a cylinder that shares the same axis as the infinite cylinder. Its radius is r [m] and its height is l [m]. The cylinder has three surfaces: S_1 (top surface), S_2 (bottom surface), and S_3 (side surface). According to the result of Exercise 1.6.1, the total electric field intensity $\boldsymbol{E}$ due to an infinite line charge is radial to the line. This means that there is no component of $\boldsymbol{D}$ along the axis. Thus, there is no electric flux crossing S_1 and S_2.

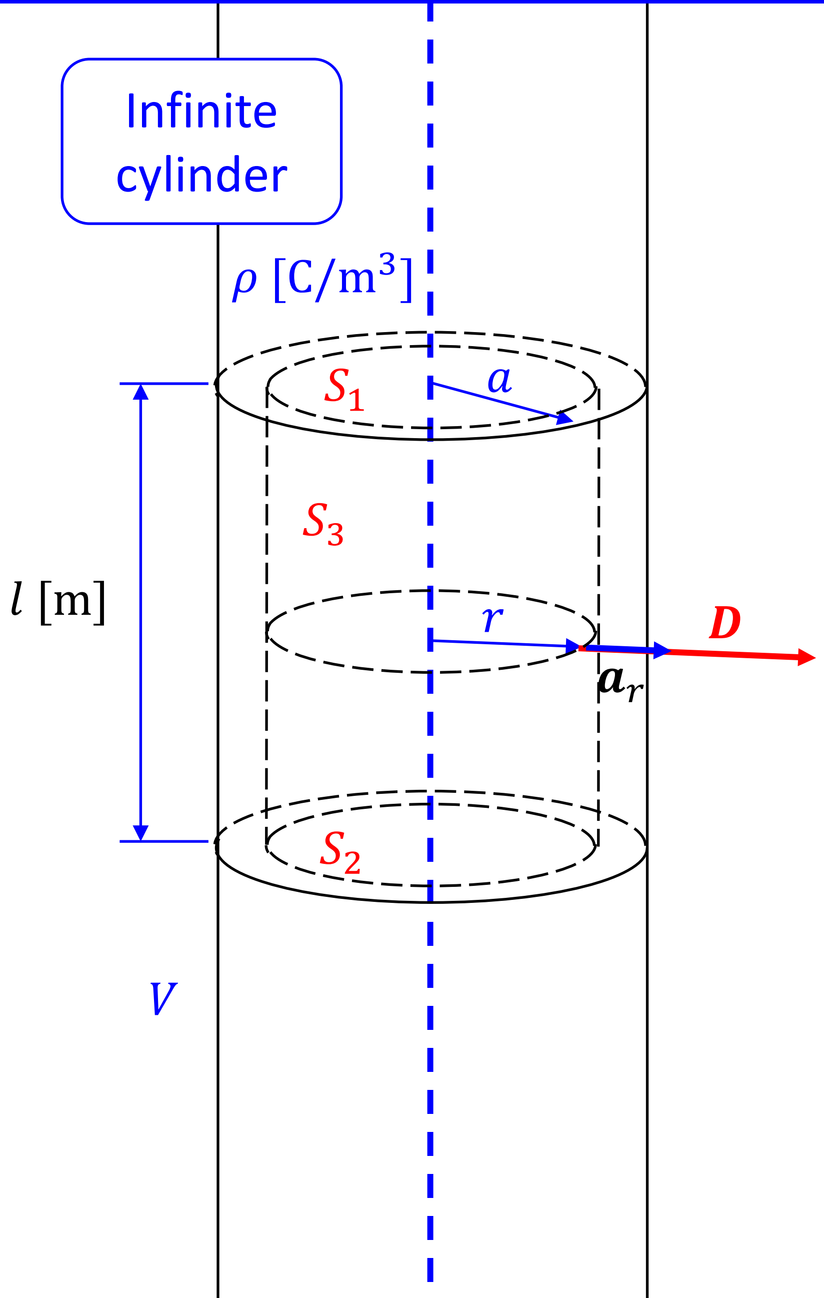
$$\int_{S_1+S_2+S_3} \boldsymbol{D}\cdot d\boldsymbol{S} = \int_{S_1} \boldsymbol{D}\cdot d\boldsymbol{S} + \int_{S_2} \boldsymbol{D}\cdot d\boldsymbol{S} + \int_{S_3} \boldsymbol{D}\cdot d\boldsymbol{S}$$

$$= 0 + 0 + \int_{S_3} \boldsymbol{D}\cdot d\boldsymbol{S}$$

$$\int_{S_3} \boldsymbol{D}\cdot d\boldsymbol{S} = Q_r$$

Q_r: charge inside the assumed cylinder

Exercise 2.3.2 (Answer)



If $r \leq a$, then the charge Q_r within the radius r [m], and the height is l [m].

$$Q_r = \pi r^2 l\rho \text{ [C]}$$

Thus, according to Gauss's Law, and the hint,

$$\int_{S_3} \boldsymbol{D} \cdot d\boldsymbol{S} = Q_r$$

From this surface integral,

$$2\pi r l D = Q_r.$$

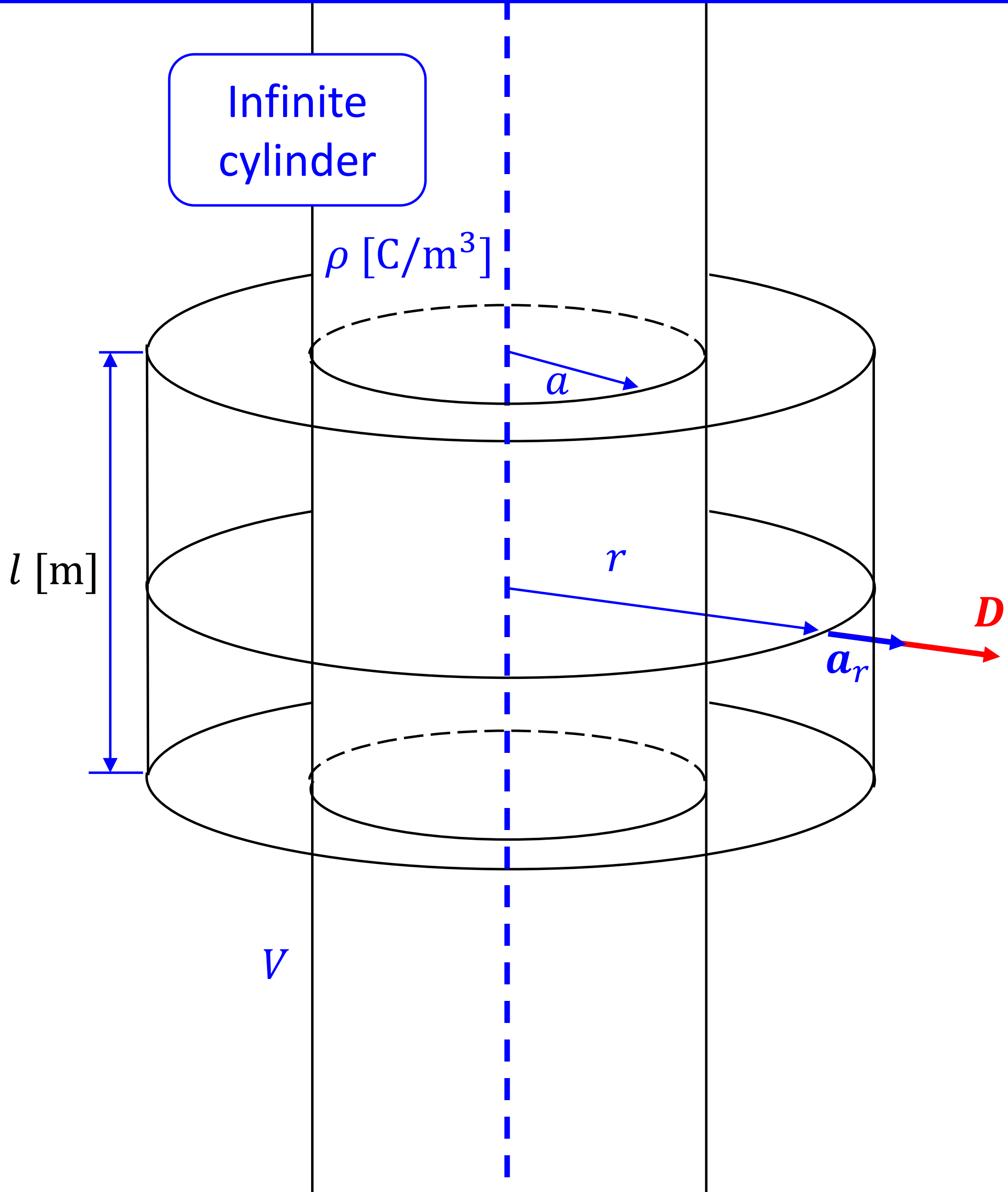
Then,

$$D = \frac{Q_r}{2\pi r l} = \frac{\pi r^2 l\rho}{2\pi r l} = \frac{r\rho}{2}.$$

Thus,

$$\boldsymbol{D} = \frac{r\rho}{2}\boldsymbol{a}_r.$$

Exercise 2.3.2 (Answer)



If $r \geq a$, then the charge Q_r within the radius r [m], and the height is l [m].

$$Q_r = \pi a^2 l\rho \text{ [C]}$$

Thus, according to Gauss's Law, and the hint,

$$\int_{S_3} \boldsymbol{D} \cdot d\boldsymbol{S} = Q_r$$

From this surface integral,

$$2\pi r l D = Q_r.$$

Then,

$$D = \frac{Q_r}{2\pi r l} = \frac{\pi a^2 l \rho}{2\pi r l} = \frac{a^2 \rho}{2r}.$$

Thus,

$$\boldsymbol{D} = \frac{a^2 \rho}{2r} \boldsymbol{a}_r.$$

Exercise 2.4 (Homework)

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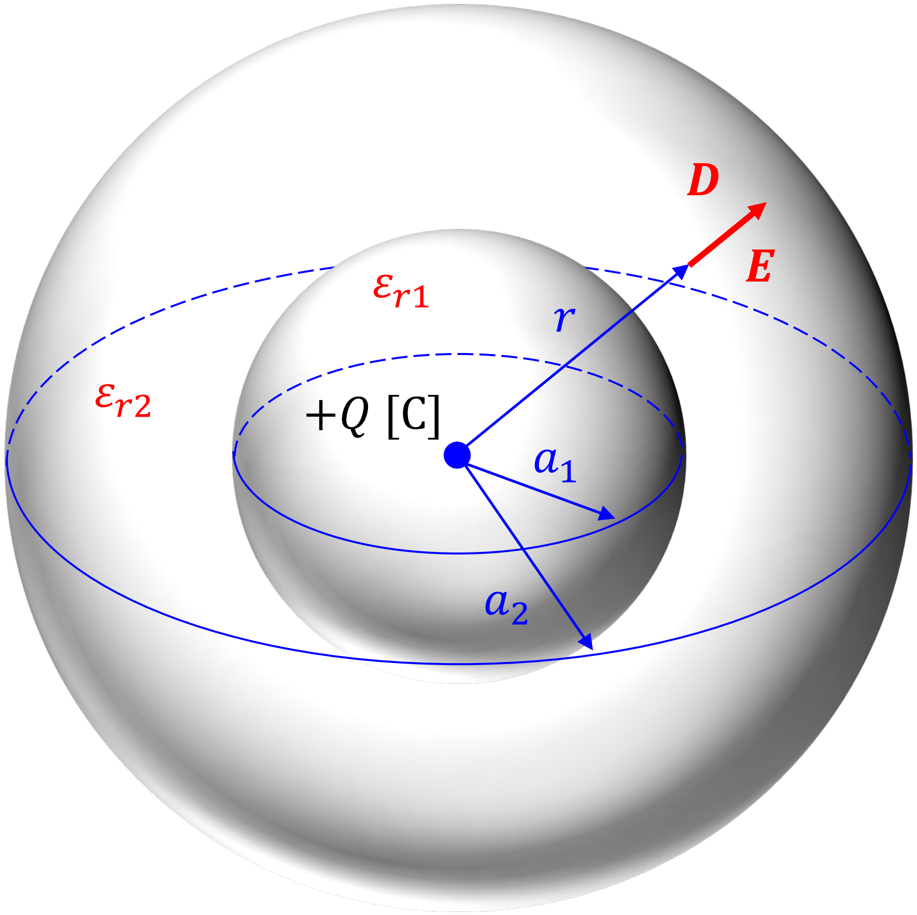


Suppose there is a sphere of radius a_2. A point charge $+Q$ [C] is located at the center of the sphere. The relative permittivity ε_r is

$$0 \leq r \leq a_1 \qquad \varepsilon_r = \varepsilon_{r1}$$

$$a_1 \leq r \leq a_2 \qquad \varepsilon_r = \varepsilon_{r2}$$

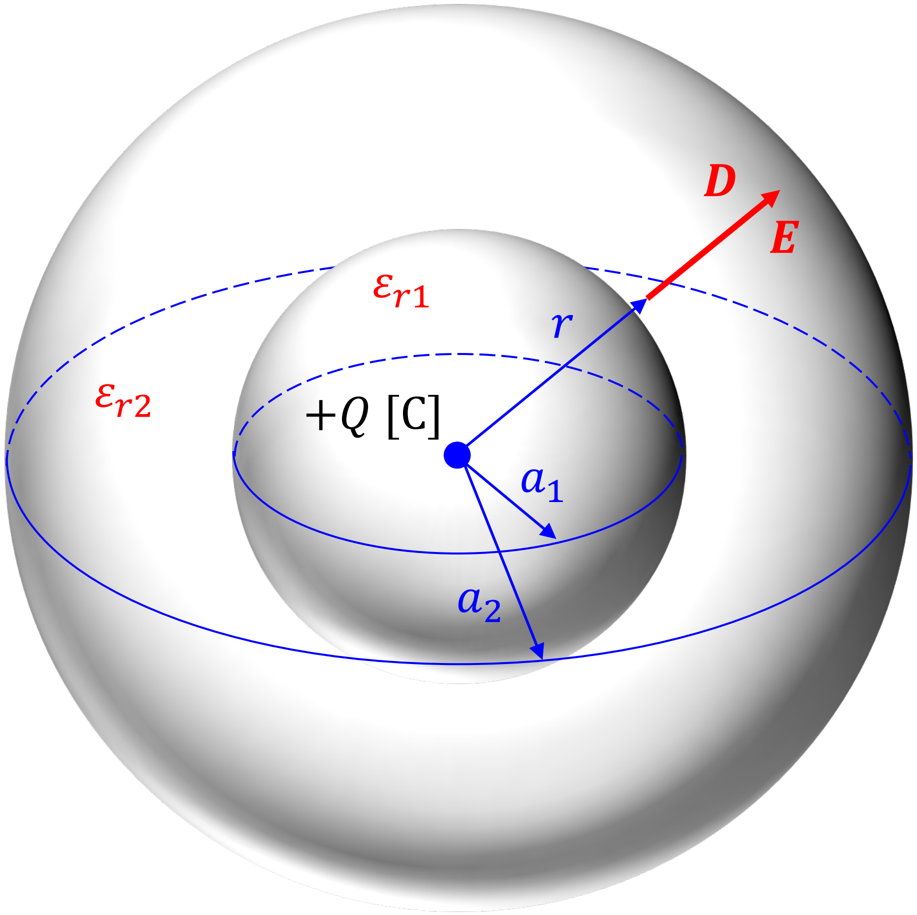
$$a_2 \leq r \qquad \varepsilon_r = 1$$

where r is the distance from the center of the sphere.

Find $\boldsymbol{D}$ and $\boldsymbol{E}$ in the three ranges where $0 < r \leq a_1$, $a_1 \leq r \leq a_2$, and $a_2 \leq r$.

Pause the video, and answer the problem. Find $\boldsymbol{D}$ and $\boldsymbol{E}$ in the three ranges.

Exercise 2.4 (Answer)



According to Exercise 2.3.1

$0 \leq r \leq a_1$

$$\boldsymbol{D} = \frac{r\rho}{3}\boldsymbol{a}_r \Rightarrow \boldsymbol{E} = \frac{r\rho}{3\varepsilon_{r1}\varepsilon_0}\boldsymbol{a}_r$$

$a_1 \leq r \leq a_2$

$$\boldsymbol{D} = \frac{r\rho}{3}\boldsymbol{a}_r \Rightarrow \boldsymbol{E} = \frac{r\rho}{3\varepsilon_{r2}\varepsilon_0}\boldsymbol{a}_r$$

$a_2 \leq r$

$$\boldsymbol{D} = \frac{a^3\rho}{3r^2}\boldsymbol{a}_r \Rightarrow \boldsymbol{E} = \frac{a^3\rho}{3\varepsilon_0 r^2}\boldsymbol{a}_r$$

Exercise 3.1.2 (Homework)

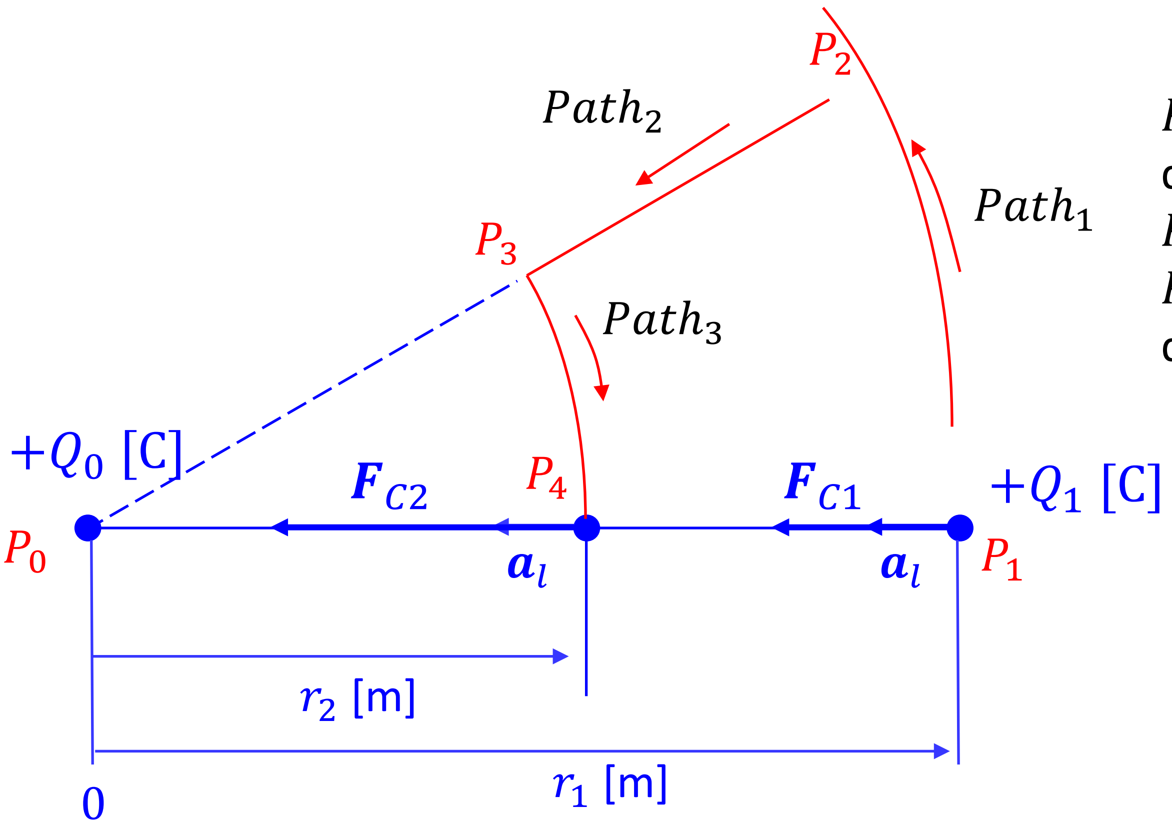
Exercise 3.1.2 (Homework)

Exercise 3.1.2. This is your homework.

Exercise 3.1.2 (Homework)

Suppose a point charge $+Q_0$ [C] is placed at P_0 and another point charge $+Q_1$ [C] is located at P_1. Show that the work W done in moving Q_1 along the path denoted by $Path_1, Path_2$, and $Path_3$ is given by:

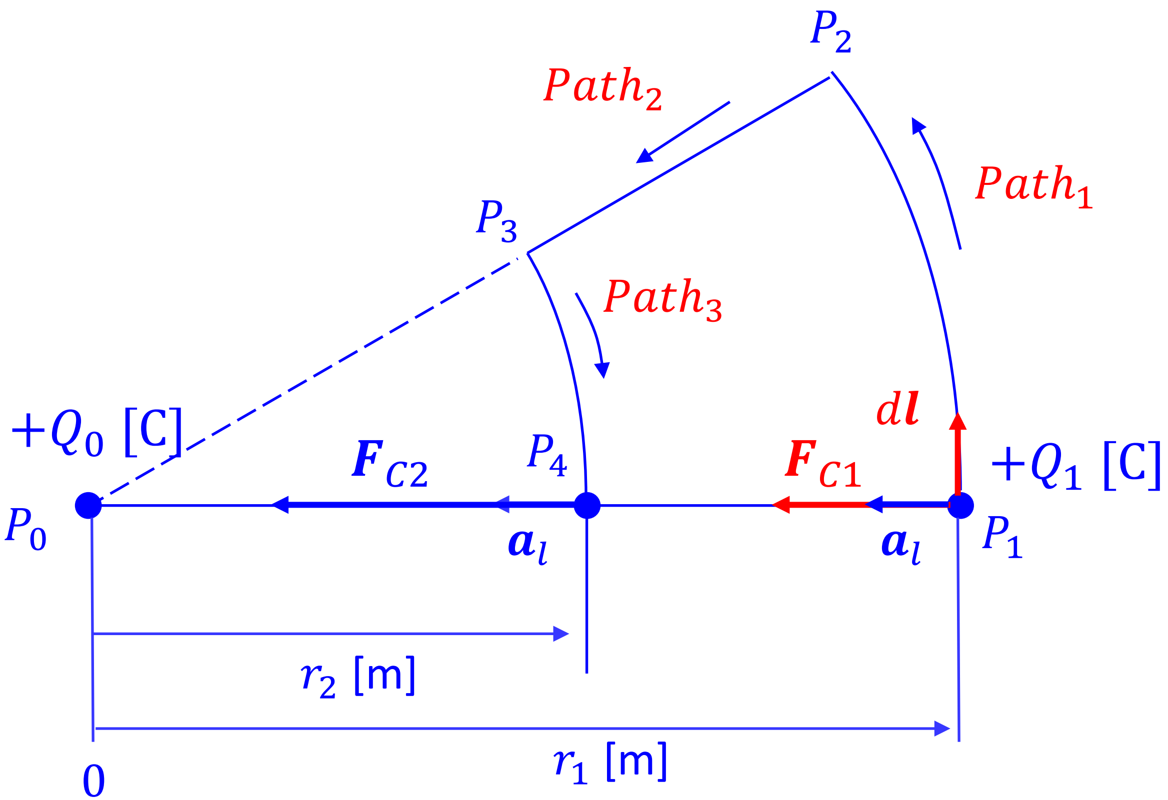
$$W = \frac{Q_1 Q_0}{4\pi\varepsilon}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$



$Path_1$ is equidistant from P_0. Its distance is r_1 [m].
$Path_2$ is on a radial line from P_0.
$Path_3$ is equidistant from P_0. Its distance is r_2 [m].

Exercise 3.1.2 (Homework)

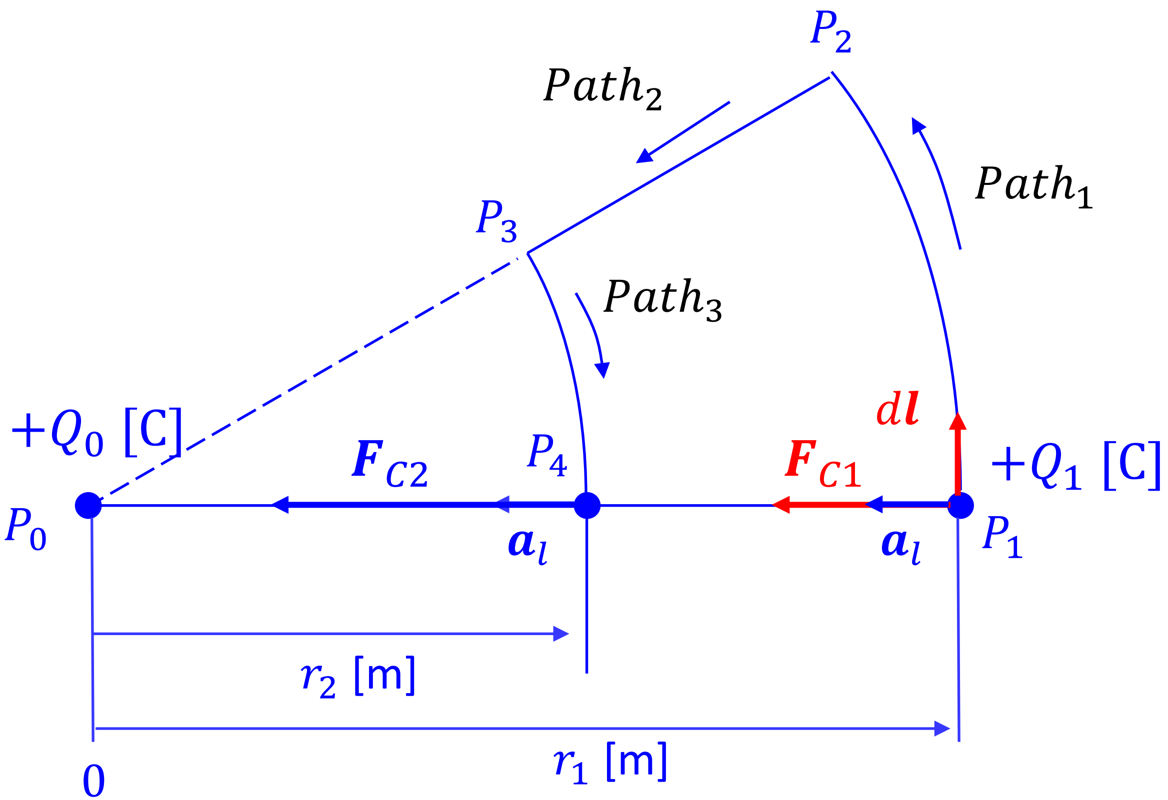
(Hint) The differential work dW is given by $dW = \boldsymbol{F}_c \cdot d\boldsymbol{l}$.
The work W is an integration of dW along $Path_1, Path_2,$ and $Path_3$.



$$W = \int_{Path_1} \boldsymbol{F}_c \cdot d\boldsymbol{l} + \int_{Path_2} \boldsymbol{F}_c \cdot d\boldsymbol{l} + \int_{Path_3} \boldsymbol{F}_c \cdot d\boldsymbol{l}$$

Exercise 3.1.2 (Answer)

On $Path_1$ and $Path_3$, the angle between $\boldsymbol{F}_c$ and $d\boldsymbol{l}$ is $90°$. Thus, $\boldsymbol{F}_c \cdot d\boldsymbol{l} = 0$.
The work on $Path_2$ is the same as that in straight moving from P_1 to P_4. Thus



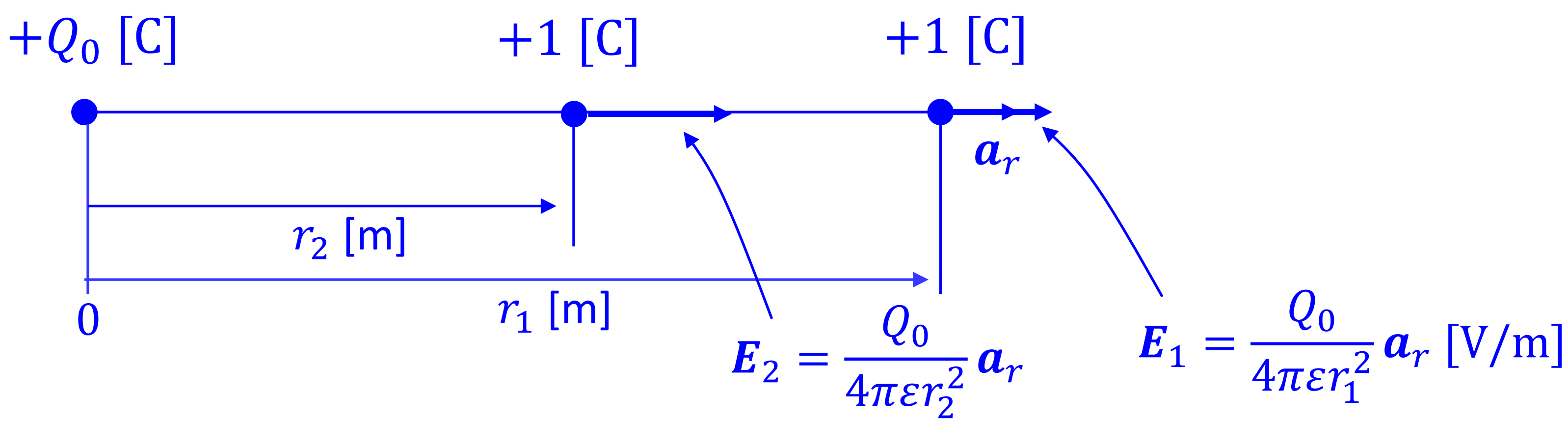
$$
\begin{aligned}
W &= \int_{Path_1} \boldsymbol{F}_c \cdot d\boldsymbol{l} \\
&\quad + \int_{Path_2} \boldsymbol{F}_c \cdot d\boldsymbol{l} \\
&\quad + \int_{Path_3} \boldsymbol{F}_c \cdot d\boldsymbol{l} \\
&= 0 + \frac{Q_1 Q_0}{4\pi\varepsilon}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) + 0 \\
&= \frac{Q_1 Q_0}{4\pi\varepsilon}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)
\end{aligned}
$$

Exercise 3.2.4 (Homework)

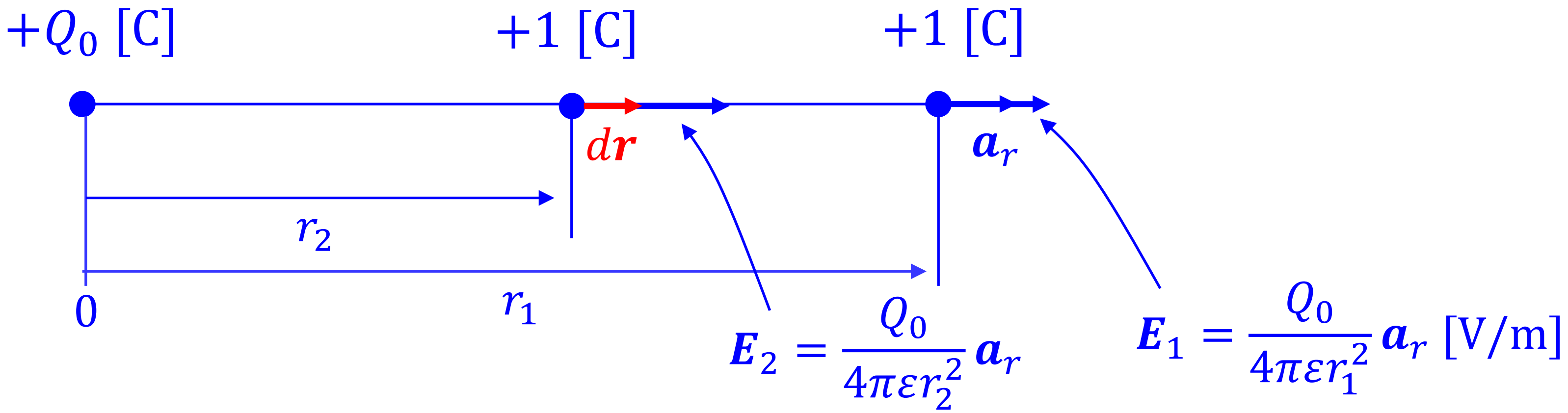
Exercise 3.2.4 (Homework)

Exercise 3.2.4 (Homework)

Find the work W in moving a unit charge from a point $r = r_2$ [m] to a point $r = r_1$ [m], pushed by the force exerted on the unit charge due to the point charge $+Q_0$ $[C]$.



Exercise 3.2.4 (Answer)



$$W = -\int_{P_{r2}}^{P_{r1}} \boldsymbol{E} \cdot d\boldsymbol{r} = -\int_{r_2}^{r_1} \frac{Q_0}{4\pi\varepsilon r^2}\boldsymbol{a}_r \cdot \boldsymbol{a}_r dr = \frac{Q_0}{4\pi\varepsilon}\left[\frac{1}{r}\right]_{r_2}^{r_1} = \frac{Q_0}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \text{ [J]}$$

Note: W is negative. This means that the electric field did the work.

Exercise 3.3 (Homework)

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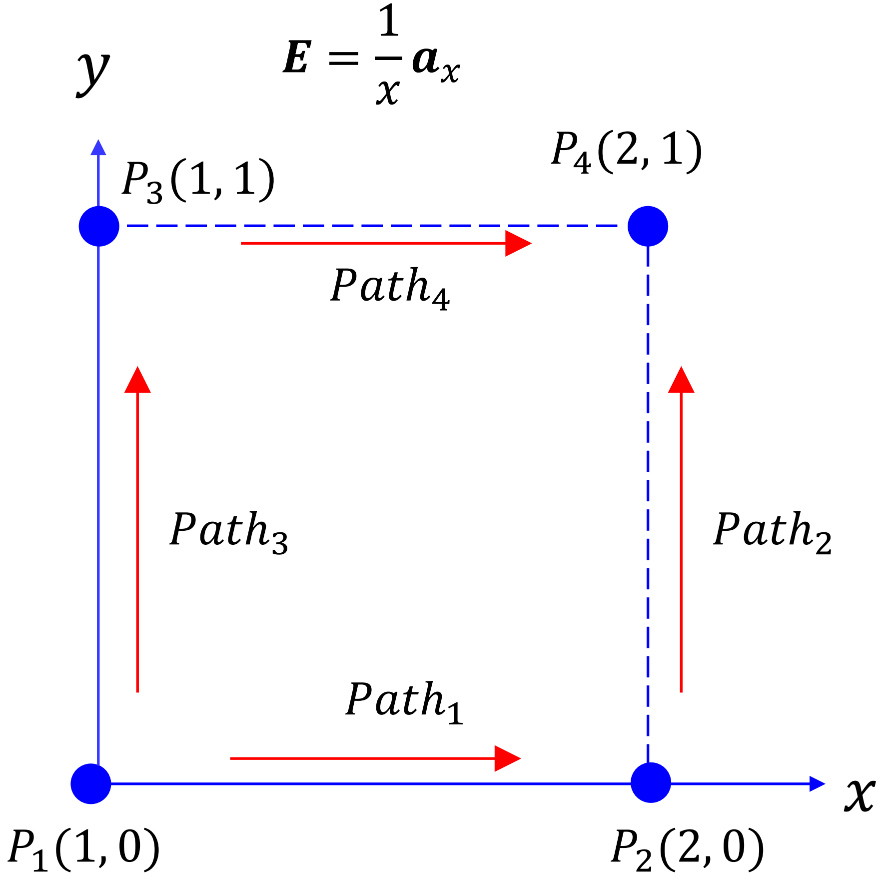


An electric field intensity is given by

$$\boldsymbol{E} = \frac{1}{x}\boldsymbol{a}_x \quad [\mathrm{V/m}].$$

Find the work done in moving a unit charge 1 [C]

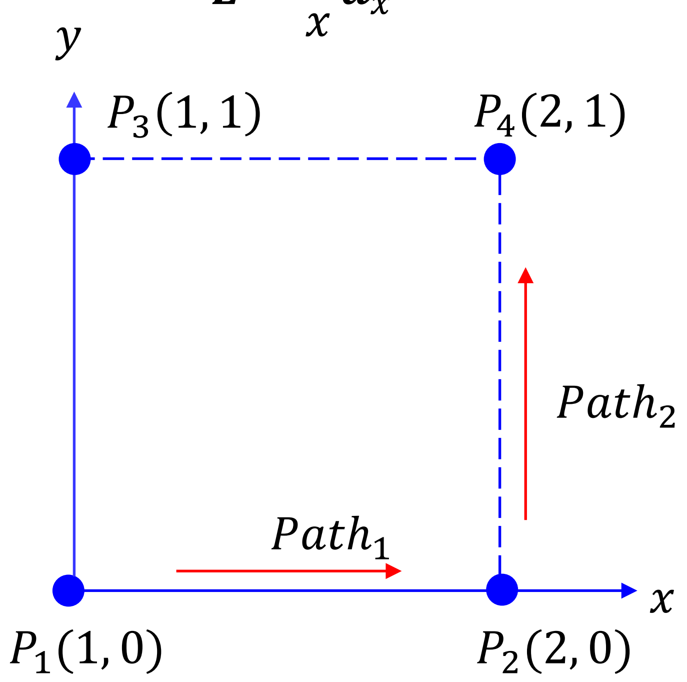
(a) from $P_1(1,0)$ [m] to $P_2(2,0)$ [m] along $Path_1$ and then to $P_4(2,1)$ [m] along $Path_2$,

(b) from $P_1(1,0)$ [m] to $P_3(1\,,1)$ [m] along $Path_3$ and then to $P_4(2,1)$ [m] along $Path_4$.

Find the potential at P_4 with respect to P_1.

Exercise 3.3 (Answer)

$$\boldsymbol{E} = \frac{1}{x}\boldsymbol{a}_x$$



$Path_1$

$$d\boldsymbol{l} = \boldsymbol{a}_x dl,\ x = 1 + l,\ y = 0$$

$$\boldsymbol{E} = \frac{1}{1+l}\boldsymbol{a}_x$$

$$W_{21} = -\int_{P_1}^{P_2} \boldsymbol{E} \cdot d\boldsymbol{l}$$

$$= -\int_0^1 \frac{1}{1+l}\boldsymbol{a}_x \cdot \boldsymbol{a}_x dl = -\int_0^1 \frac{1}{1+l} dl$$

$$= -[\ln(1+l)]_0^1 = -\ln 2$$

$Path_2$

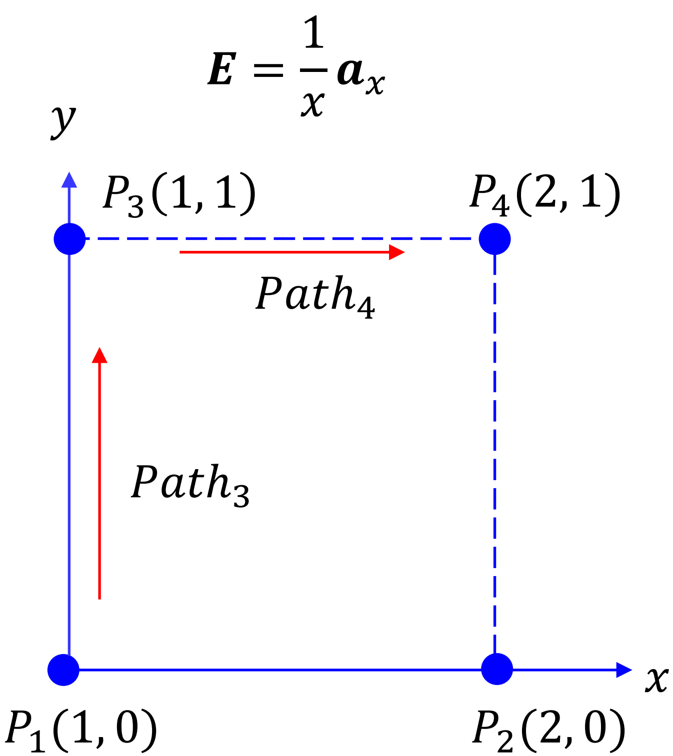
$$d\boldsymbol{l} = \boldsymbol{a}_y dl,\ x = 2,\ \ y = l$$

$$\boldsymbol{E} = \frac{1}{2}\boldsymbol{a}_x$$

$$W_{42} = -\int_0^1 \frac{1}{2}\boldsymbol{a}_x \cdot \boldsymbol{a}_y dl$$

$$= 0$$

Exercise 3.3 (Answer)



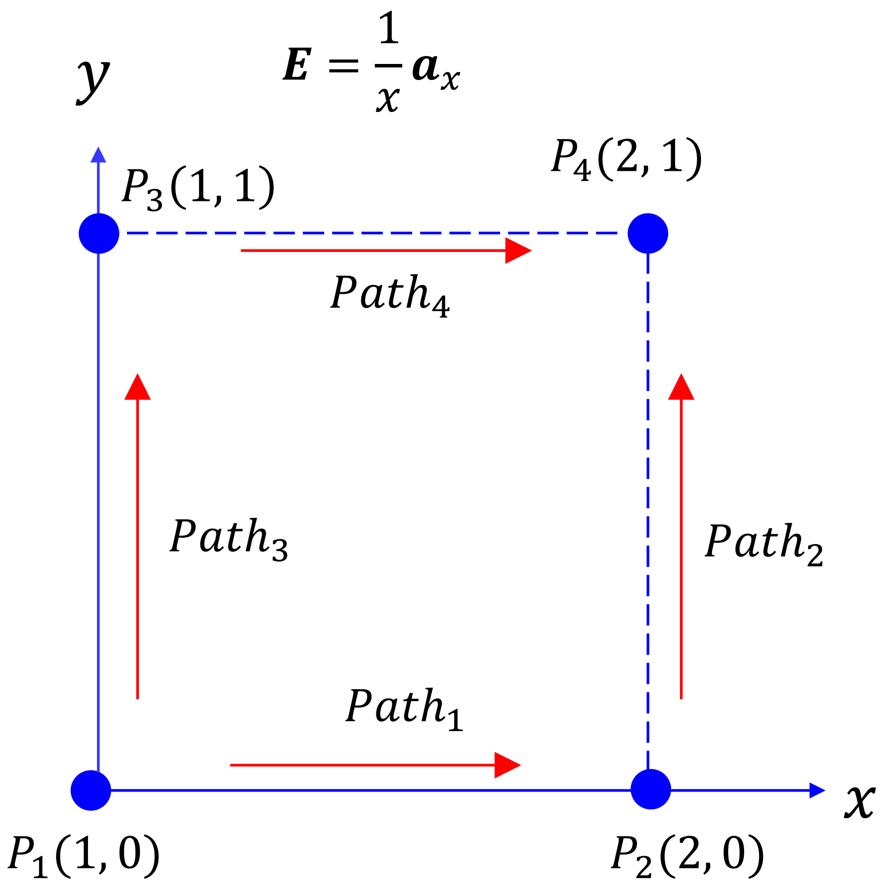
$Path_3$

$$d\boldsymbol{l} = \boldsymbol{a}_y dl,\ x = 1, y = l$$
$$\boldsymbol{E} = \boldsymbol{a}_x$$
$$W_{31} = -\int_{P_1}^{P_3} \boldsymbol{E} \cdot d\boldsymbol{l}$$
$$= -\int_0^1 \boldsymbol{a}_x \cdot \boldsymbol{a}_y dl$$
$$= 0$$

$Path_4$

$$d\boldsymbol{l} = \boldsymbol{a}_x dl,\ x = 1 + l, y = 1$$
$$\boldsymbol{E} = \frac{1}{1+l}\boldsymbol{a}_x$$
$$W_{43} = -\int_0^1 \frac{1}{1+l}\boldsymbol{a}_x \cdot \boldsymbol{a}_x dl = -\int_0^1 \frac{1}{1+l} dl$$
$$= -[\ln(1+l)]_0^1 = -\ln 2$$

Exercise 3.3 (Answer)



$Path_1 + Path_2$

$$W_{41} = W_{21} + W_{42} = \ln 2$$

$Path_3 + Path_4$

$$W_{41} = W_{31} + W_{43} = \ln 2$$

Exercise 3.4.4 (Homework)

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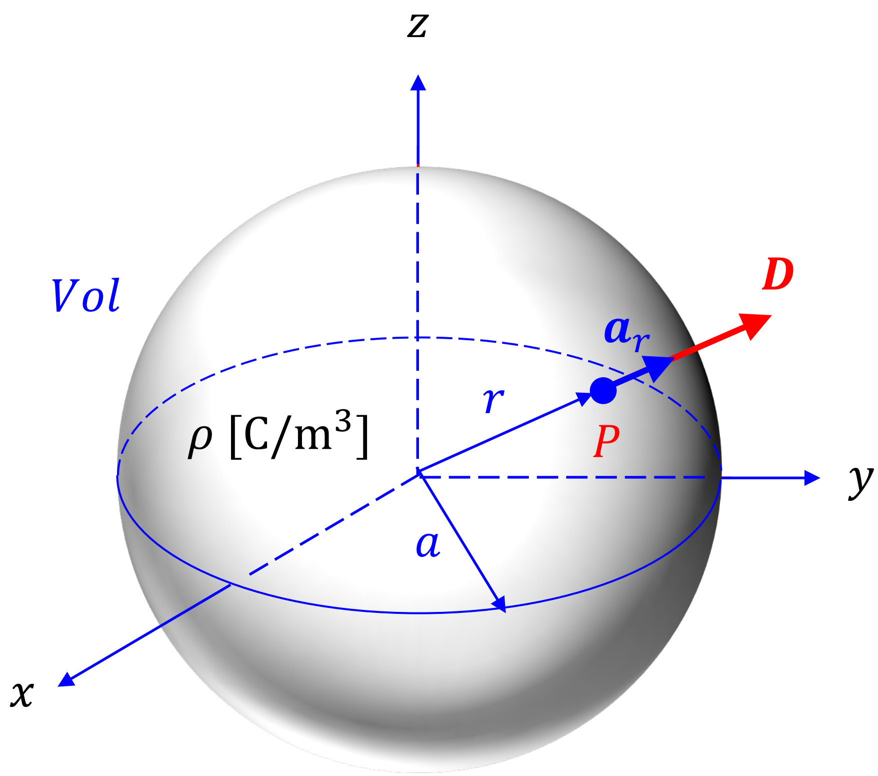
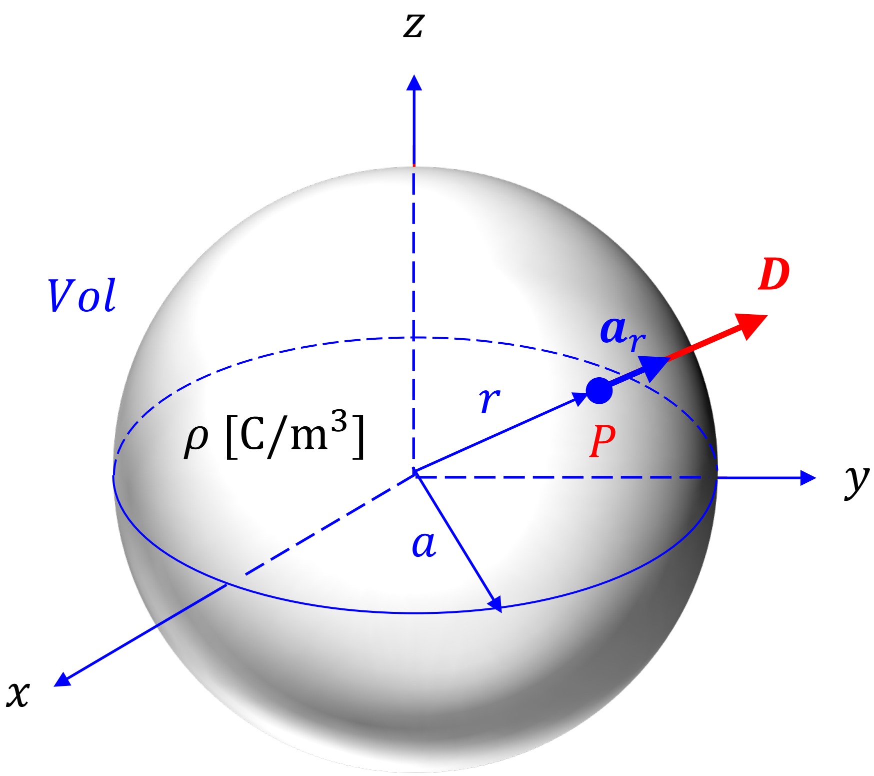


Charge is distributed with a constant density ρ $[C/m^3]$ throughout a spherical volume Vol of radius a [m]. The relative permittivity $\varepsilon_r \approx 1$.

Show that the potential V at P where $r \leq a$ is given by

$$V = \frac{\rho(a^2 - r^2)}{6\varepsilon_0} + \frac{\rho a^2}{3\varepsilon_0}.$$

Exercise 3.4.4 (Answer)



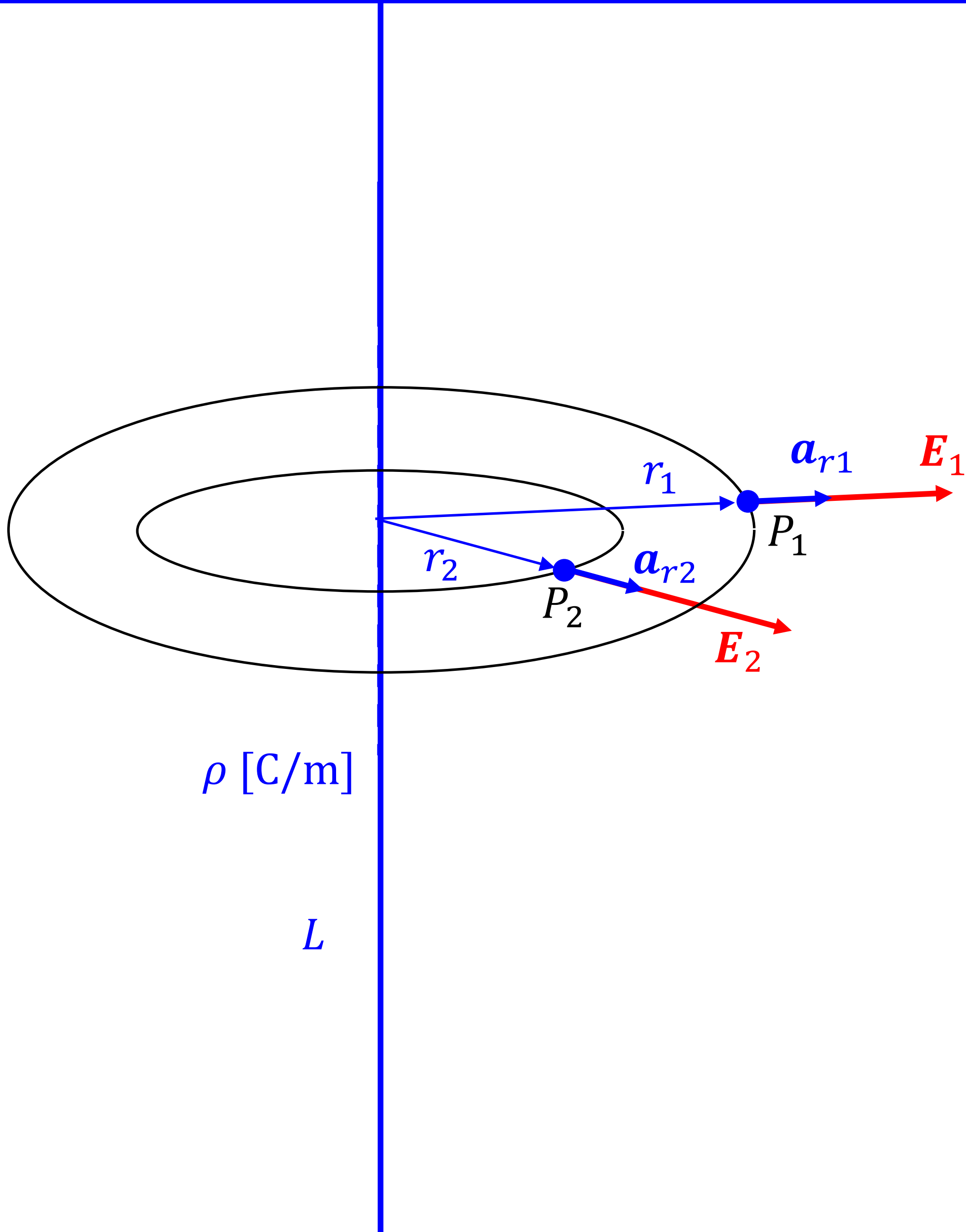
$r \le a$

$$V = -\int_{\infty}^{r} \boldsymbol{E}_r \cdot d\boldsymbol{r}$$
$$= -\int_{a}^{r} \boldsymbol{E}_r \cdot d\boldsymbol{r} + V_a$$
$$= -\int_{a}^{r} \frac{\rho r}{3\varepsilon_0} \boldsymbol{a}_r \cdot d\boldsymbol{r} + V_a$$
$$= -\int_{a}^{r} \frac{\rho r}{3\varepsilon_0} dr + V_a$$
$$= -\left[\frac{\rho r^2}{6\varepsilon_0}\right]_a^r + V_a$$
$$= \frac{\rho(a^2 - r^2)}{6\varepsilon_0} + V_a$$
$$= \frac{\rho(a^2 - r^2)}{6\varepsilon_0} + \frac{\rho a^2}{3\varepsilon_0}$$

Exercise 3.4.5 (Homework)

Exercise 3.4.5 (Homework)

Exercise 3.4.5 (Homework)

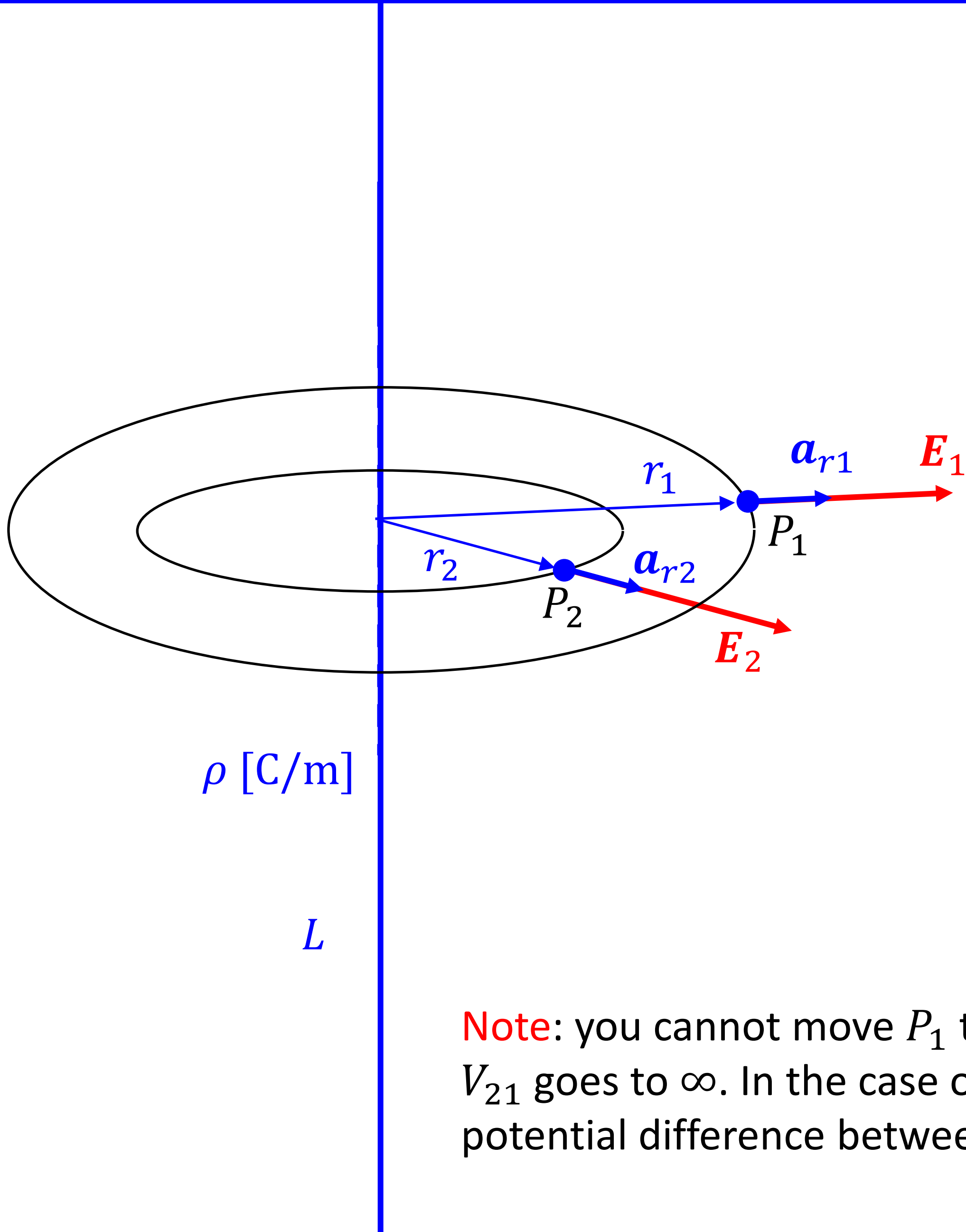


There is an infinite line charge along an infinite line L with zero thickness. The charge density is ρ [C/m].

Show that the potential V_{21} at P_2 with respect to P_1 is given by

$$V_{21} = \frac{\rho}{2\pi\varepsilon} \ln \frac{r_2}{r_1}.$$

Exercise 3.4.5 (Answer)



From Homework 2.2

$$\boldsymbol{D} = \frac{\rho}{2\pi r}\boldsymbol{a}_r.$$

From Section 2.4

$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon}.$$

Then

$$\boldsymbol{E} = \frac{\rho}{2\pi\varepsilon r}\boldsymbol{a}_r.$$

$\boldsymbol{E}$ is conservative. Thus

$$V_{21} = -\int_{r_1}^{r_2} \boldsymbol{E}\cdot d\boldsymbol{r} = -\int_{r_1}^{r_2} \frac{\rho}{2\pi\varepsilon r}dr$$
$$= \frac{\rho}{2\pi\varepsilon}\ln\frac{r_1}{r_2}.$$

Note: you cannot move P_1 to infinity, because the potential V_{21} goes to ∞. In the case of infinite line charge, only the potential difference between finite points can be determined.

Exercise 3.4.6 (Homework)

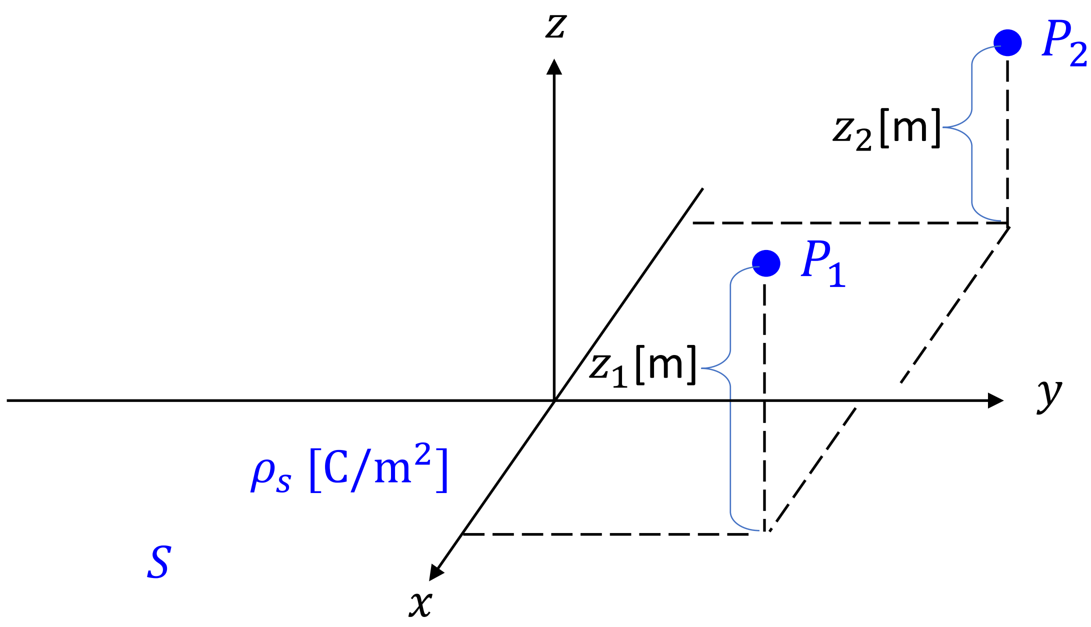
Exercise 3.4.6 (Homework)

Exercise 3.4.6 (Homework)

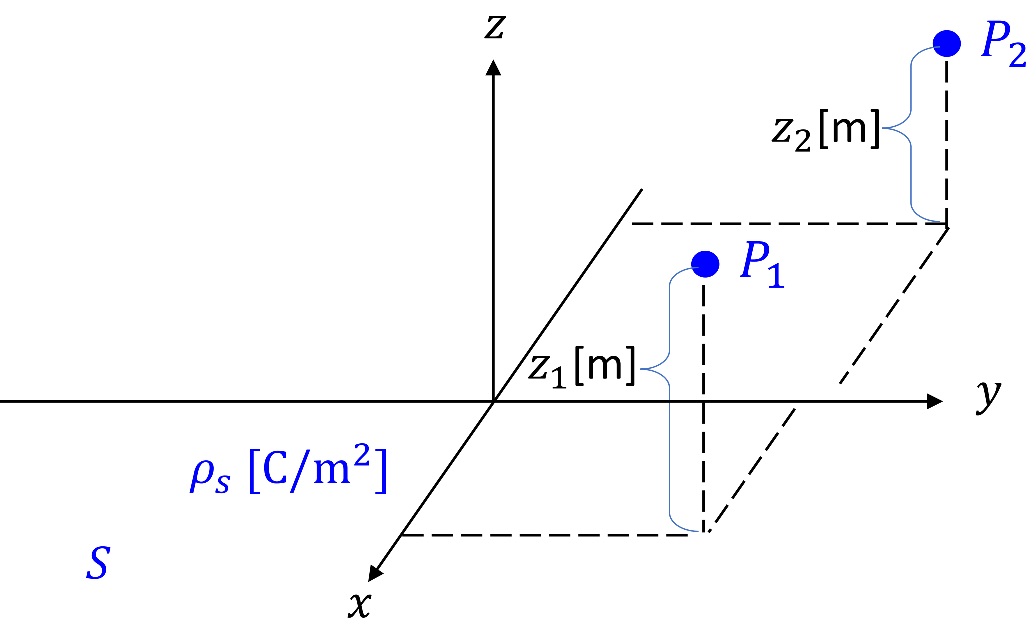
Suppose that charge is uniformly distributed over an infinite $x - y$ plane S. The charge density is ρ_s $[\mathrm{C/m^2}]$.

Show that the potential V_{21} at P_2 with respect to P_1 is given by

$$V_{21} = \frac{\rho_s}{2\epsilon}(z_1 - z_2).$$



Exercise 3.4.6 (Answer)



From Homework 1.6.3

$$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon}\boldsymbol{a}_z$$

$\boldsymbol{E}$ is conservative. Thus

$$V_{21} = -\int_{z_1}^{z_2} \boldsymbol{E} \cdot d\boldsymbol{z}$$
$$= -\int_{z_1}^{z_2} \frac{\rho_s}{2\varepsilon} dz$$
$$= \frac{\rho_s}{2\varepsilon}(z_1 - z_2)$$

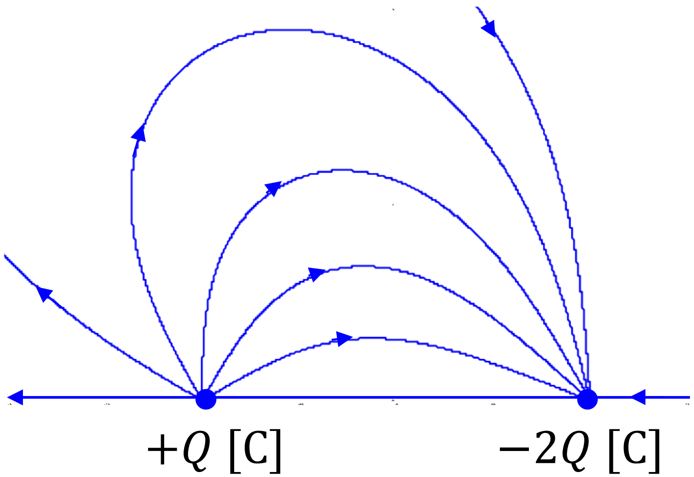
Note: you cannot move P_1 to infinity, because the potential V_{21} goes to ∞. In the case of infinite surface charge, only the potential difference between finite points can be determined.

Exercise 4.1 (Homework)

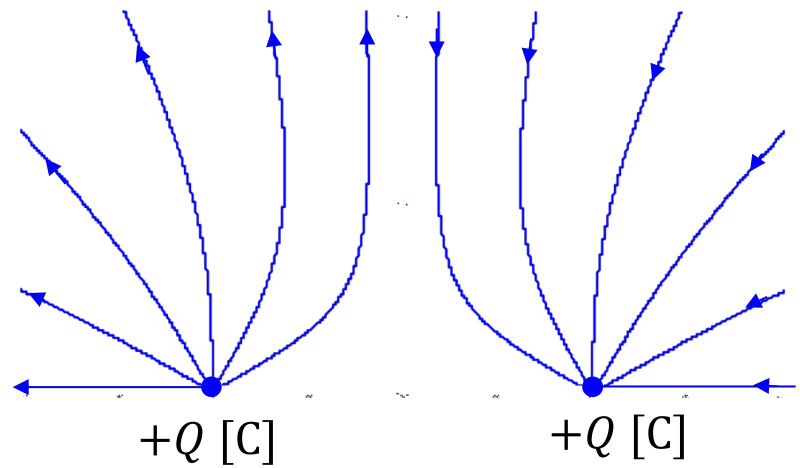
Exercise 4.1 (Homework)

Exercise 4.1 (Homework)

These are the electric field lines for the two cases (a) and (b). Only the upper halves of the electric field lines are drawn.
Draw some equipotential surfaces.

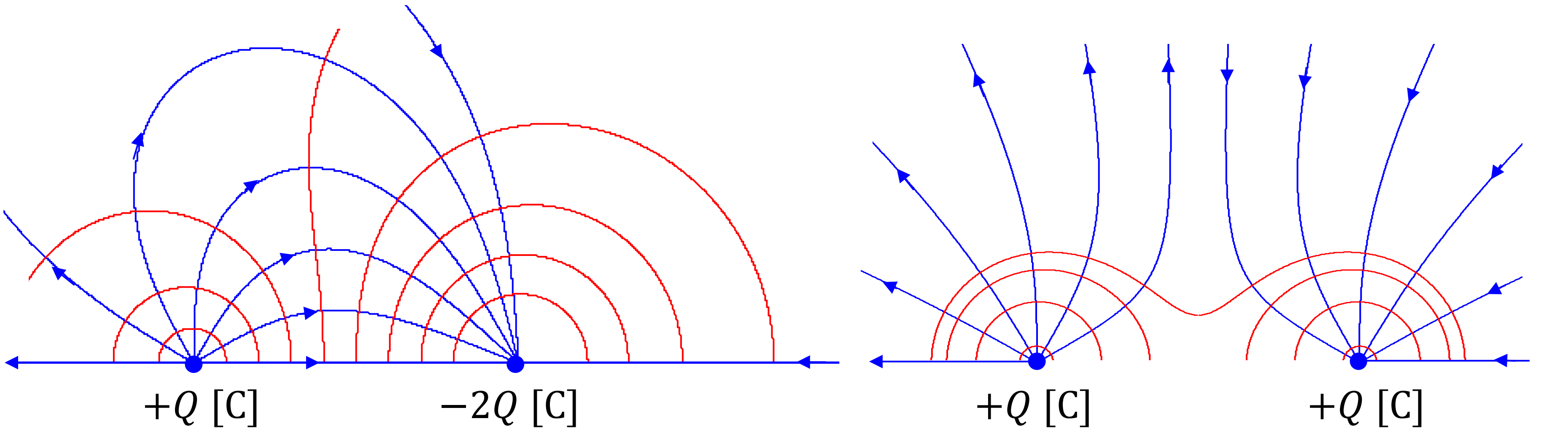


(a)



(b)

Exercise 4.1 (Answer)

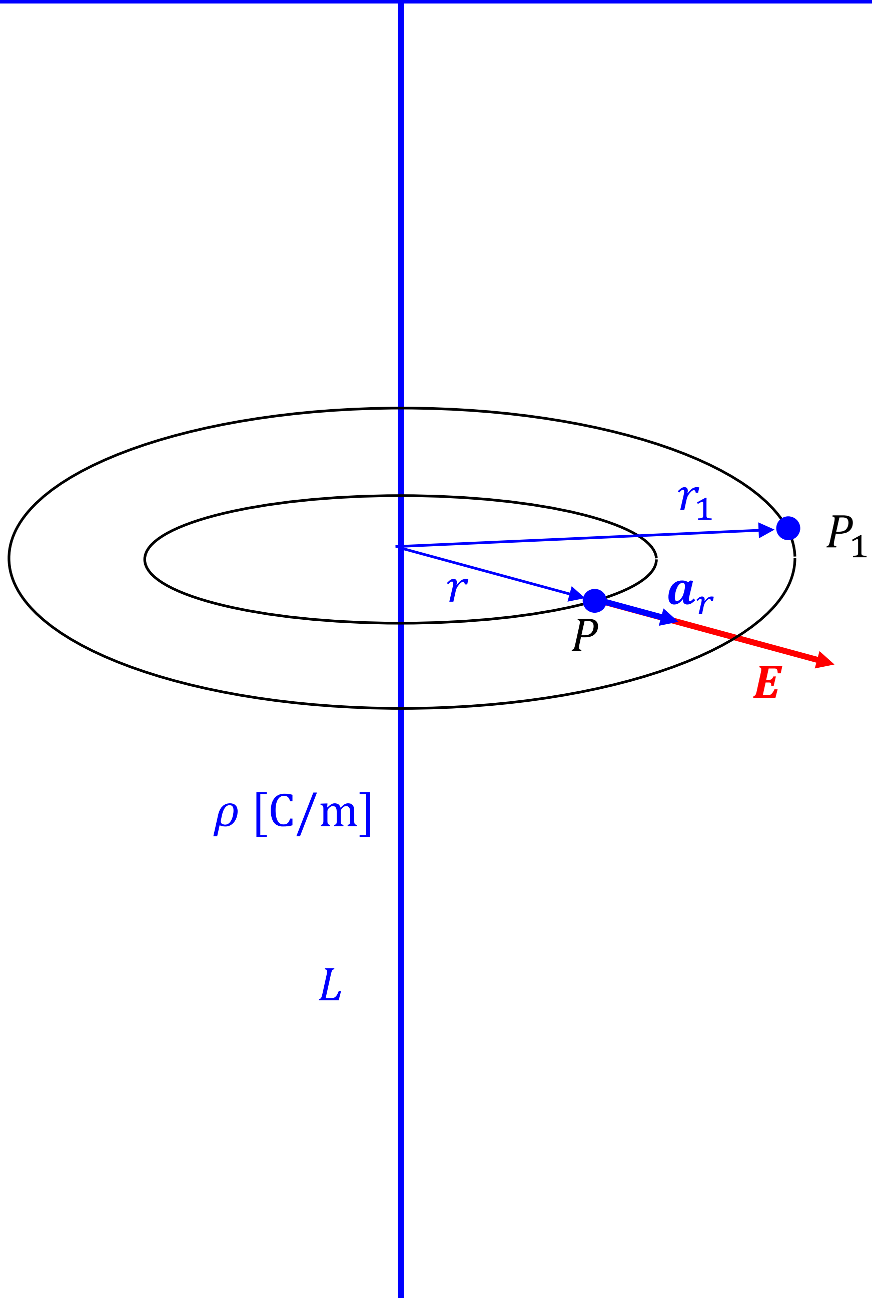


(a) (b)

Exercise 4.2.2 (Homework)

Exercise 4.2.2 (Homework)

Exercise 4.2.2 (Homework)



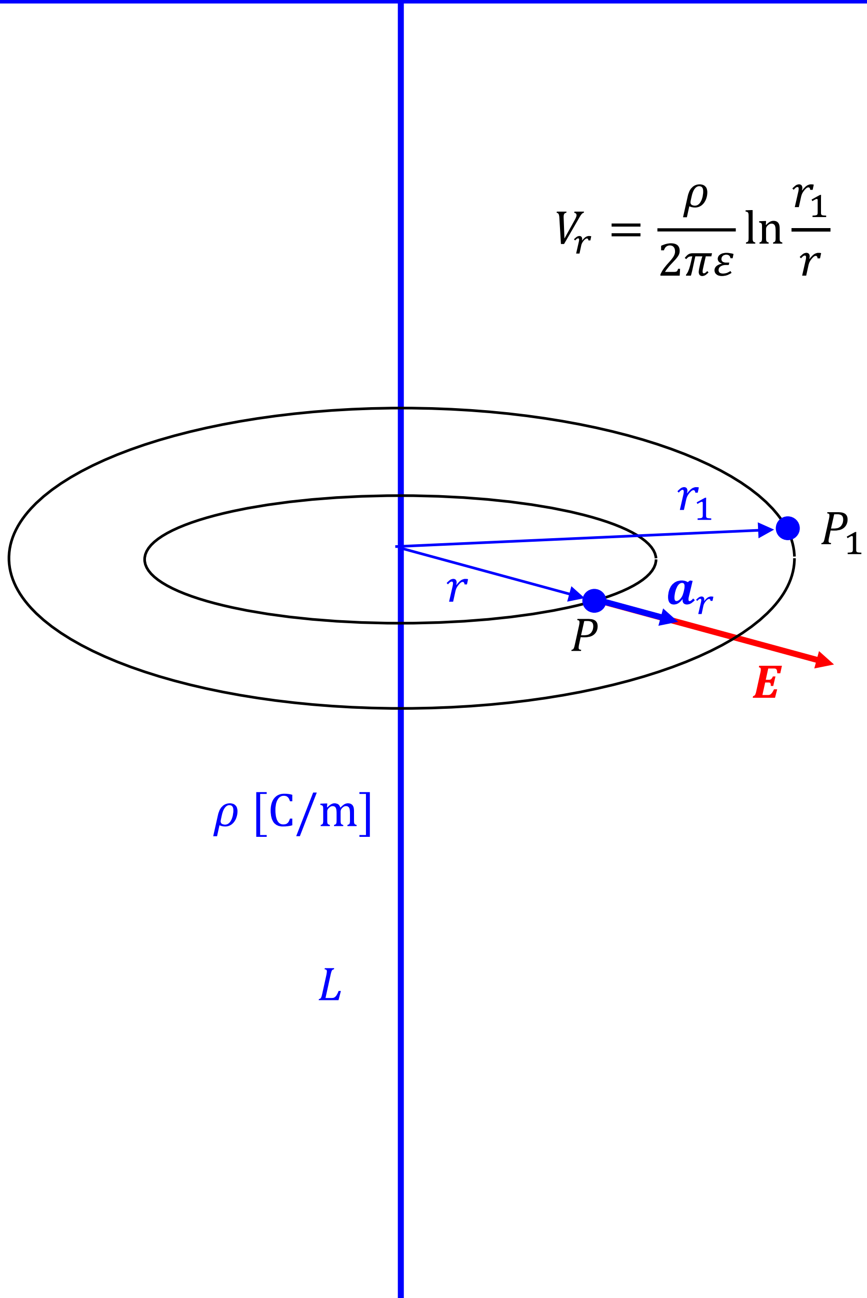
There is an infinite line charge along an infinite line L with zero thickness. The charge density is ρ [C/m]. In Homework 3.4.5, the potential V_r at point P with respect to P_1 was obtained as

$$V_r = \frac{\rho}{2\pi\varepsilon} \ln \frac{r_1}{r}.$$

Find the electric field intensity $\boldsymbol{E}$ at P by applying the following relationship:

$$\boldsymbol{E} = -\nabla V.$$

Homework 4.2.2 (Answer)



$$V_r = \frac{\rho}{2\pi\varepsilon} \ln \frac{r_1}{r}$$

$$\frac{\partial V_r}{\partial x} = \frac{\partial}{\partial x} \frac{\rho}{2\pi\varepsilon} (\ln r_1 - \ln r)$$
$$= -\frac{\rho}{2\pi\varepsilon r} \frac{x}{r}$$

$$\frac{\partial V_r}{\partial y} = -\frac{\rho}{2\pi\varepsilon r} \frac{y}{r}$$

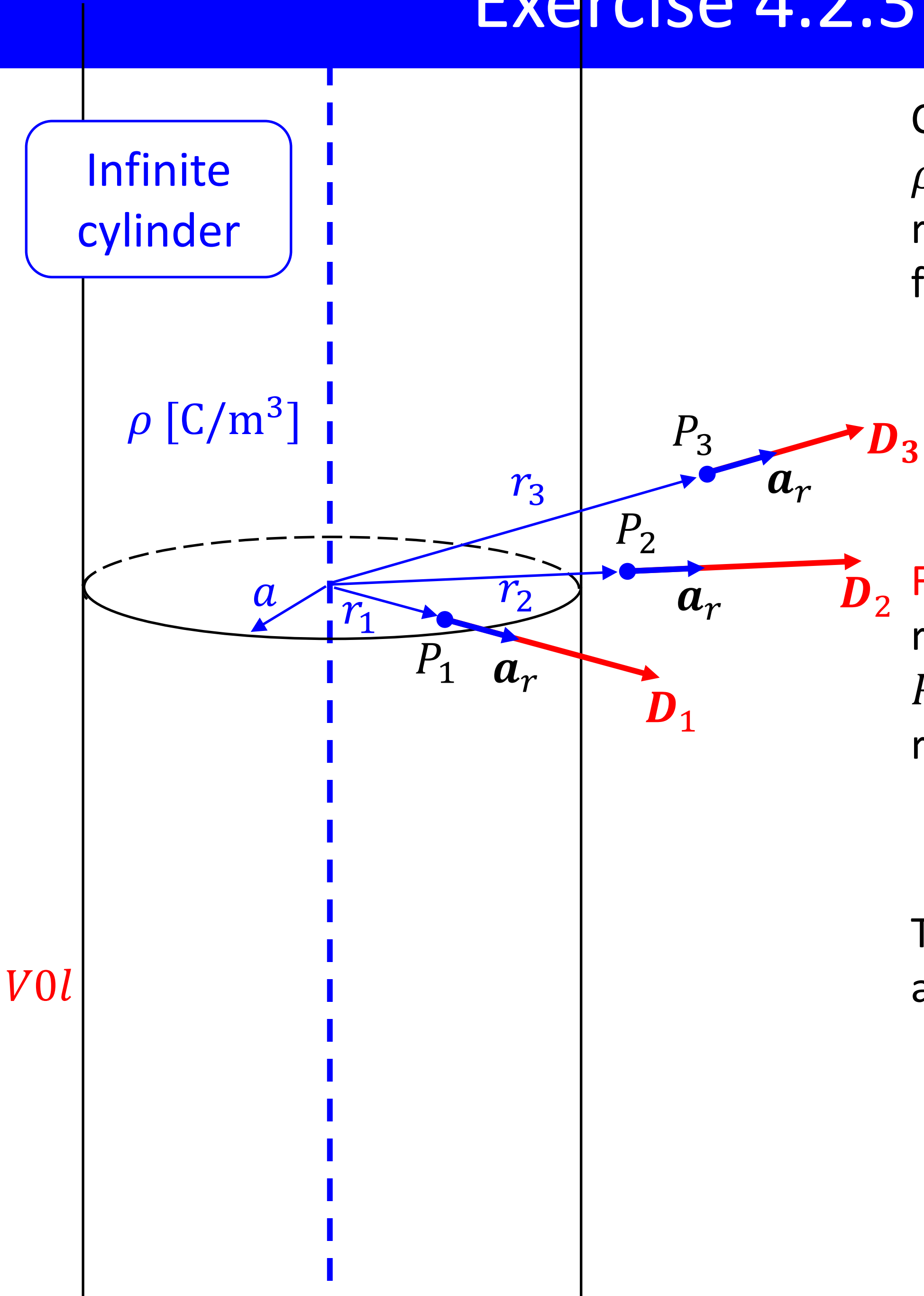
$$\frac{\partial V_r}{\partial z} = -\frac{\rho}{2\pi\varepsilon r} \frac{z}{r}$$

$$\boldsymbol{E} = -\nabla V_r$$
$$= -\frac{\partial V_r}{\partial x} \boldsymbol{a}_x - \frac{\partial V_r}{\partial y} \boldsymbol{a}_y - \frac{\partial V_r}{\partial z} \boldsymbol{a}_z$$
$$= \frac{\rho}{2\pi\varepsilon r} \left(\frac{x}{r} \boldsymbol{a}_x + \frac{y}{r} \boldsymbol{a}_y + \frac{z}{r} \boldsymbol{a}_z \right)$$
$$= \frac{\rho}{2\pi\varepsilon r} \frac{\boldsymbol{r}}{r}$$
$$= \frac{\rho}{2\pi\varepsilon r} \boldsymbol{a}_r$$

Exercise 4.2.3 (Homework)

Exercise 4.2.3 (Homework)

Exercise 4.2.3 (Homework)



Charge is distributed with a constant density $\rho\ [C/m^3]$ throughout an infinite cylinder Vol of radius a [m]. In Homework 2.3.2, the electric flux density was obtained as

$$\boldsymbol{D} = \begin{cases} \dfrac{\rho r}{2}\boldsymbol{a}_r & (r \le a) \\ \dfrac{\rho a^2}{2r}\boldsymbol{a}_r & (a \le r) \end{cases}$$

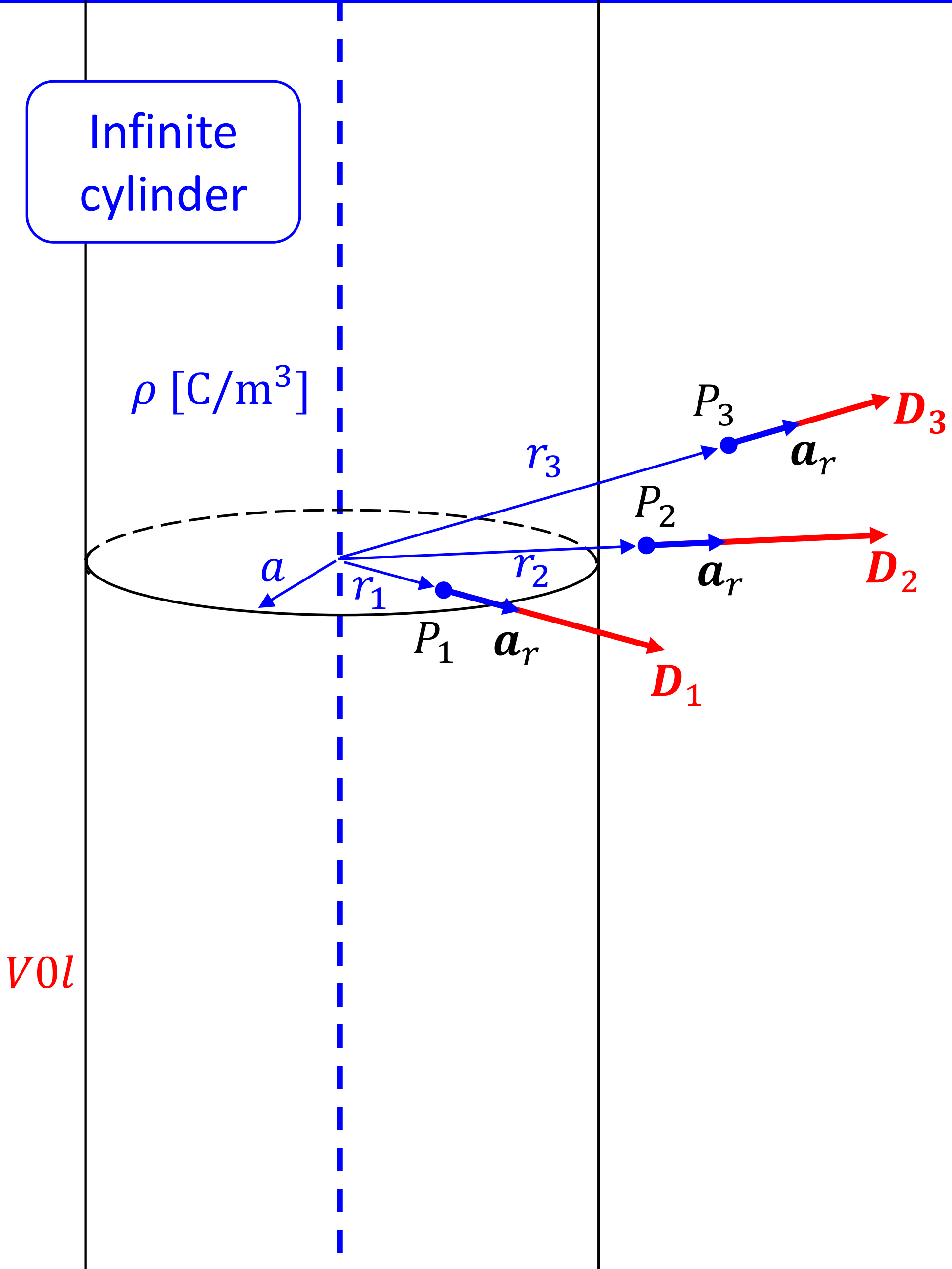
Find the potential V_{23} at P_2 $(r_2 \ge a)$ with respect to P_3 $(r_3 \ge a)$, and the potential V_{13} at P_1 $(r_1 \le a)$ with respect to P_3 by using the relationship

$$V_{i3} = -\int_{P_3}^{P_i} \boldsymbol{E} \cdot d\boldsymbol{r} \quad (i = 1, 2).$$

Then, Find the electric field intensity $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ at P_1 and P_2 by using

$$\boldsymbol{E}_i = -\nabla V_i \quad (i = 1, 2).$$

Exercise 4.2.3 (Answer)



$$V_{23} = -\int_{P_3}^{P_2} \boldsymbol{E}\cdot d\boldsymbol{r} = -\int_{r_3}^{r_2} \frac{\rho a^2}{2\varepsilon r}\boldsymbol{a}_r \cdot d\boldsymbol{r}$$

$$= \left[-\frac{\rho a^2}{2\varepsilon}\ln r\right]_{r_3}^{r_2} = \frac{\rho a^2}{2\varepsilon}\ln\frac{r_3}{r_2}$$

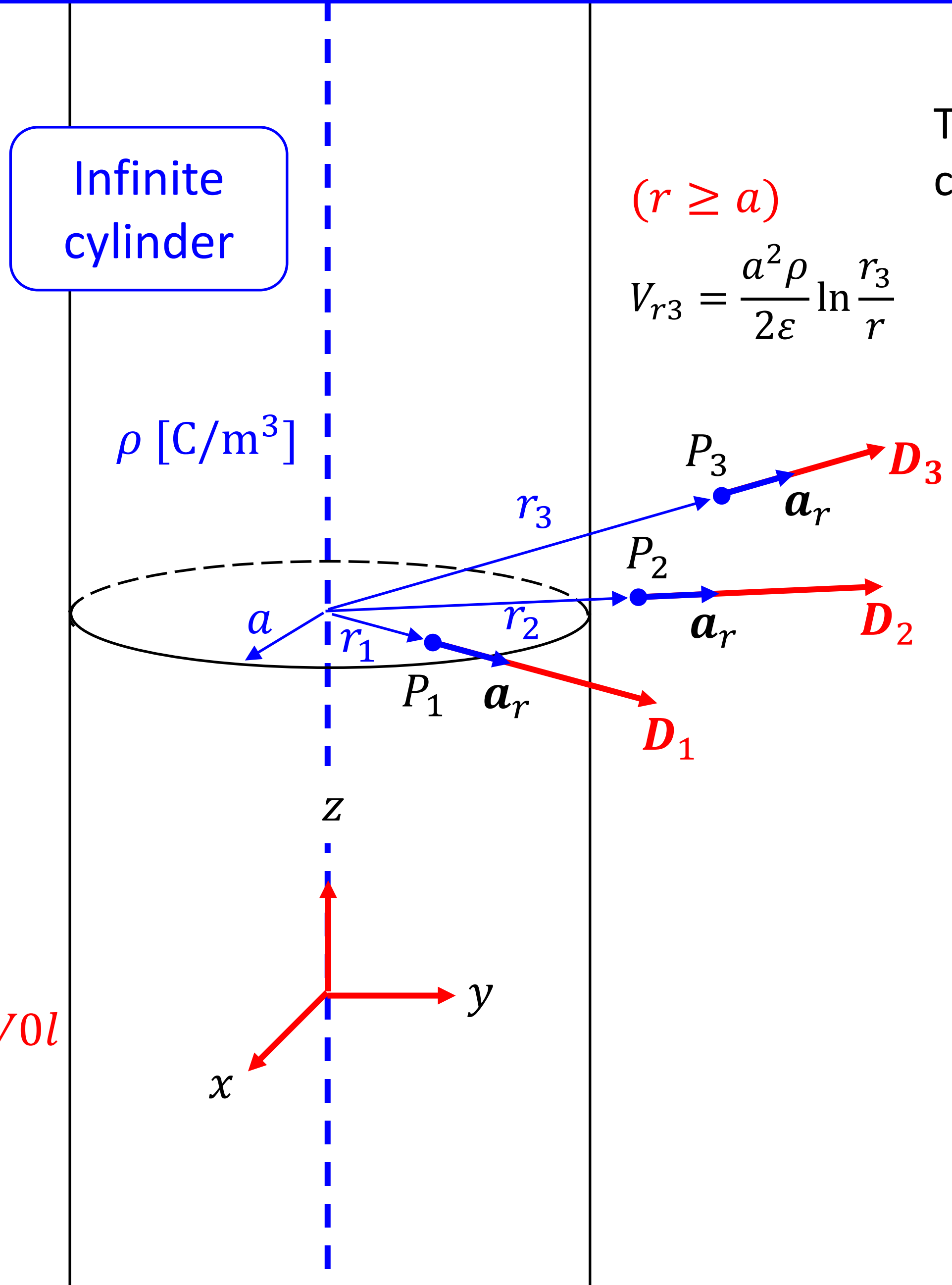
$$V_{13} = -\int_{P_3}^{P_1} \boldsymbol{E}\cdot d\boldsymbol{r}$$

$$= -\int_{a}^{r_1} \frac{\rho r}{2\varepsilon}\boldsymbol{a}_r \cdot d\boldsymbol{r} + \frac{\rho a^2}{2\varepsilon}\ln\frac{r_3}{a}$$

$$= \left[-\frac{\rho r^2}{4\varepsilon}\right]_{a}^{r_1} + \frac{\rho a^2}{2\varepsilon}\ln\frac{r_3}{a}$$

$$= \frac{\rho(a^2 - r_1^2)}{4\varepsilon} + \frac{\rho a^2}{2\varepsilon}\ln\frac{r_3}{a}$$

Exercise 4.2.3 (Answer)



V0l

The $z-$axis of Cartesian coordinate system is chosen as the cylinder axis.

$$r = \sqrt{x^2 + y^2}$$

$$\frac{\partial V_{r3}}{\partial x} = \frac{\partial}{\partial x}\frac{\rho a^2}{2\varepsilon}(\ln r_3 - \ln r) = -\frac{\rho a^2}{2\varepsilon r}\frac{x}{r}$$

$$\frac{\partial V_{r3}}{\partial y} = -\frac{\rho a^2}{2\varepsilon r}\frac{y}{r}$$

$$\frac{\partial V_{r3}}{\partial z} = 0$$

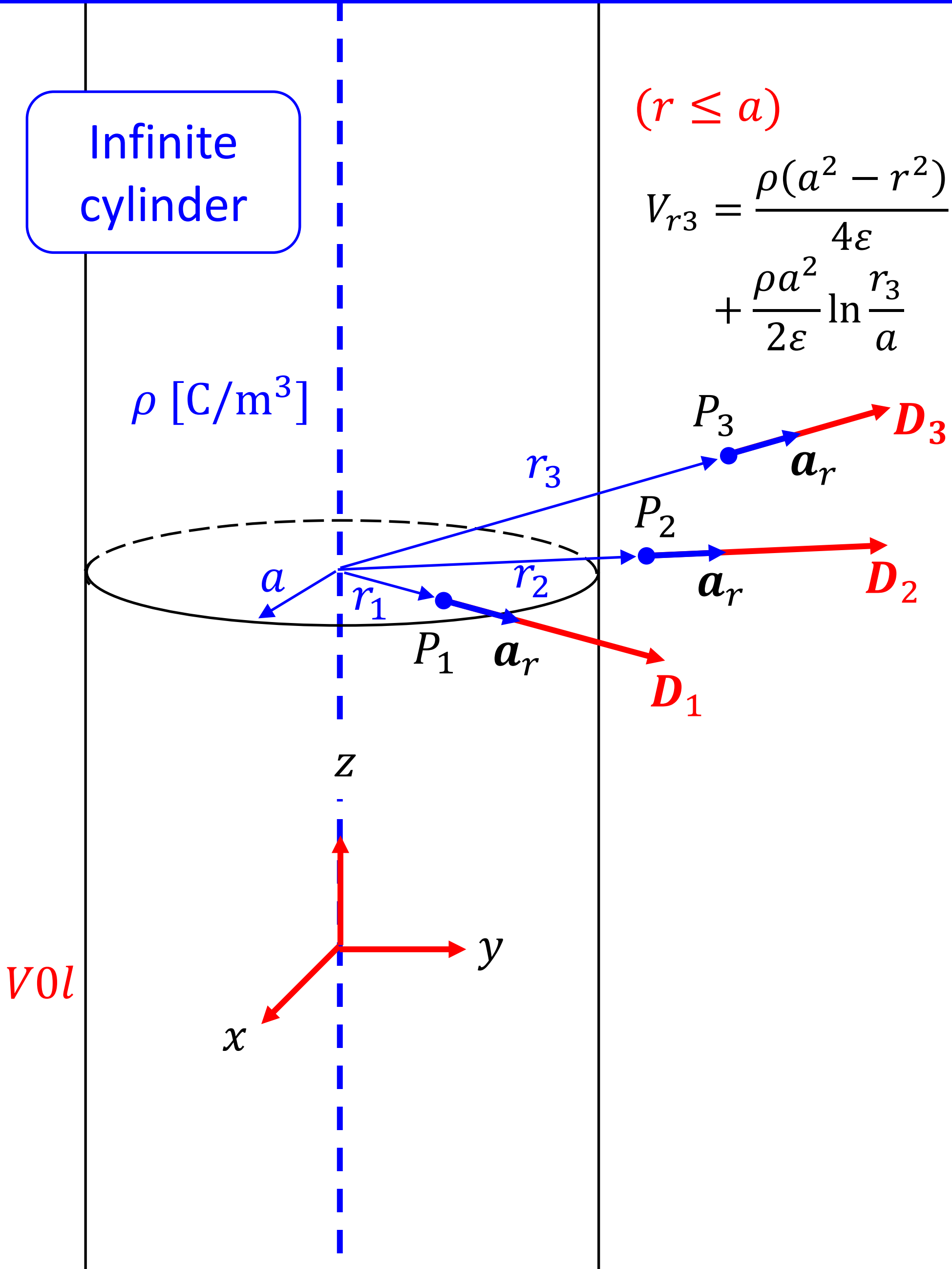
$$\boldsymbol{E} = -\nabla V_{r3}$$

$$= -\frac{\partial V_{r3}}{\partial x}\boldsymbol{a}_x - \frac{\partial V_{r3}}{\partial y}\boldsymbol{a}_y - \frac{\partial V_{r3}}{\partial z}\boldsymbol{a}_z$$

$$= \frac{\rho a^2}{2\varepsilon r}\left(\frac{x}{r}\boldsymbol{a}_x + \frac{y}{r}\boldsymbol{a}_y\right)$$

$$= \frac{\rho a^2}{2\varepsilon r}\frac{\boldsymbol{r}}{r} = \frac{\rho a^2}{2\varepsilon r}\boldsymbol{a}_r$$

Exercise 4.2.3 (Answer)



$(r \le a)$

$$V_{r3} = \frac{\rho(a^2 - r^2)}{4\varepsilon} + \frac{\rho a^2}{2\varepsilon} \ln \frac{r_3}{a}$$

$$\frac{\partial V_{r3}}{\partial x} = -\frac{\rho x}{2\varepsilon}$$

$$\frac{\partial V_{r2}}{\partial y} = -\frac{\rho y}{2\varepsilon}$$

$$\frac{\partial V_{r2}}{\partial z} = 0$$

$$\begin{aligned}\boldsymbol{E} &= -\nabla V_{r2} \\ &= -\frac{\partial V_{r2}}{\partial x}\boldsymbol{a}_x - \frac{\partial V_{r2}}{\partial y}\boldsymbol{a}_y \\ &= \frac{\rho}{2\varepsilon}(x\boldsymbol{a}_x + y\boldsymbol{a}_y) \\ &= \frac{\rho r}{2\varepsilon}\frac{\boldsymbol{r}}{r} = \frac{\rho r}{2\varepsilon}\boldsymbol{a}_r\end{aligned}$$

Exercise 4.2.4 (Homework)

Exercise 4.2.4 (Homework)

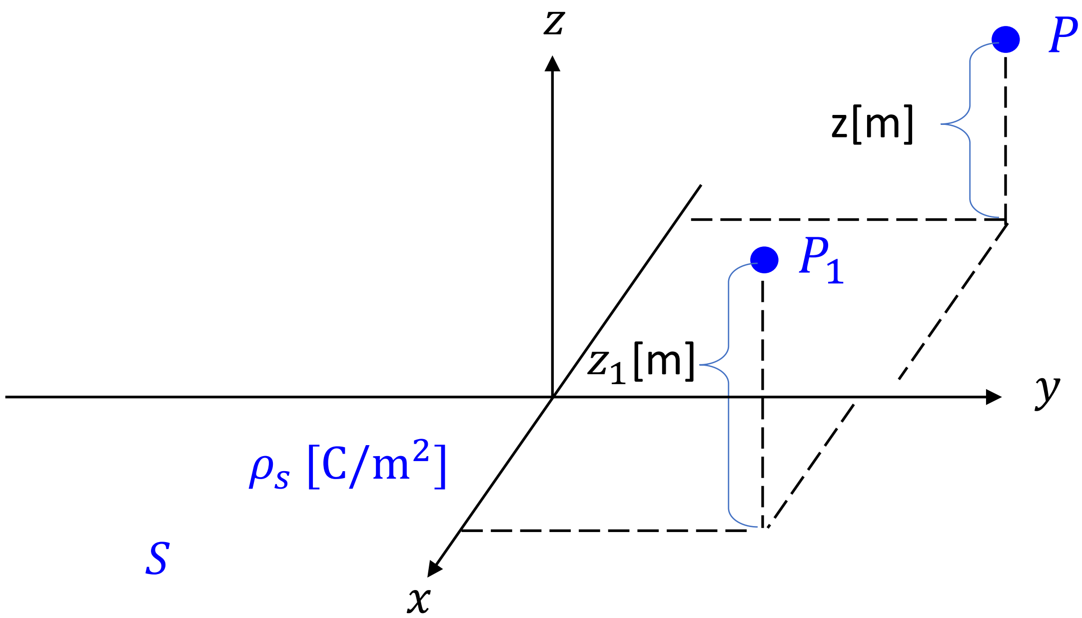
Exercise 4.2.4 (Homework)

Suppose that charge is uniformly distributed over an infinite $x - y$ plane S. The charge density is ρ_s $[\mathrm{C/m^2}]$. In Homework 3.4.6, the potential V_z at P with respect to P_1 was derived as

$$V_z = \frac{\rho_s}{2\epsilon}(z_1 - z).$$

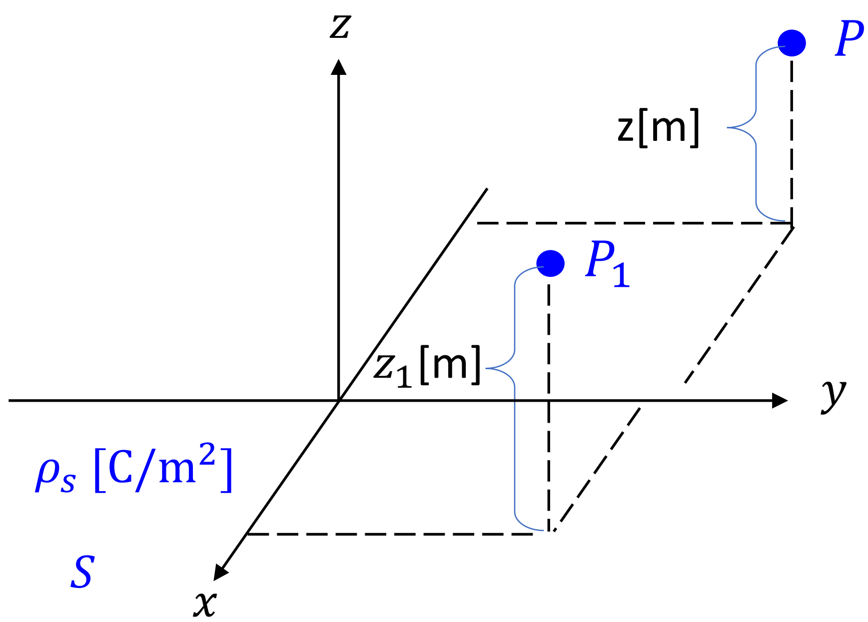
Find the electric field intensity $\boldsymbol{E}$ at P by applying the following relationship:

$$\boldsymbol{E} = -\nabla V.$$



Exercise 4.2.4 (Answer)

$$V_z = \frac{\rho_s}{2\epsilon}(z_1 - z).$$



$$\frac{\partial V_z}{\partial x} = 0$$

$$\frac{\partial V_z}{\partial y} = 0$$

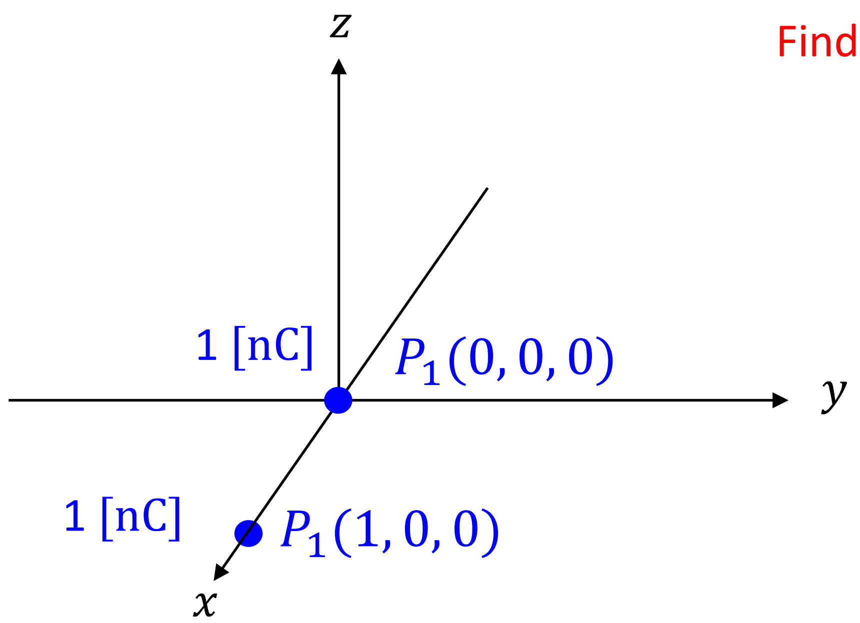
$$\frac{\partial V_z}{\partial z} = -\frac{\rho_s}{2\varepsilon}$$

$$\boldsymbol{E} = -\nabla V_z$$
$$= -\frac{\partial V_z}{\partial x}\boldsymbol{a}_x - \frac{\partial V_z}{\partial y}\boldsymbol{a}_y - \frac{\partial V_z}{\partial z}\boldsymbol{a}_z$$
$$= \frac{\rho_s}{2\varepsilon}\boldsymbol{a}_z$$

Exercise 4.3.2 (Homework)

Exercise 4.3.2 (Homework)

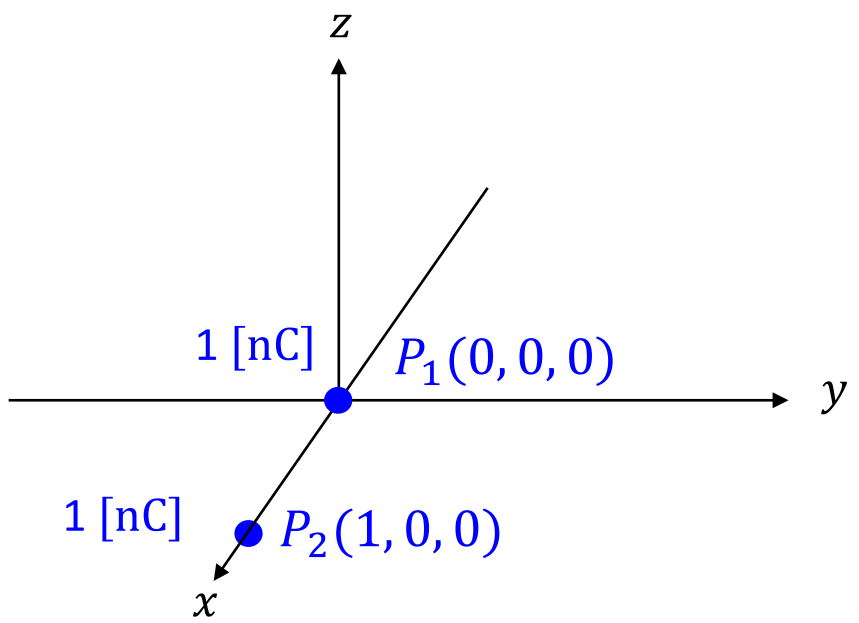
Exercise 4.3.2 (Homework)



Two identical point charges $Q = 1$ [nC] are placed at points $P_1(0,0,0)$ and $P_2(1,0,0)$ [m]. The relative permittivity $\varepsilon_r \approx 1$.

Find the energy stored in this electric field.

Exercise 4.3.2 (Answer)



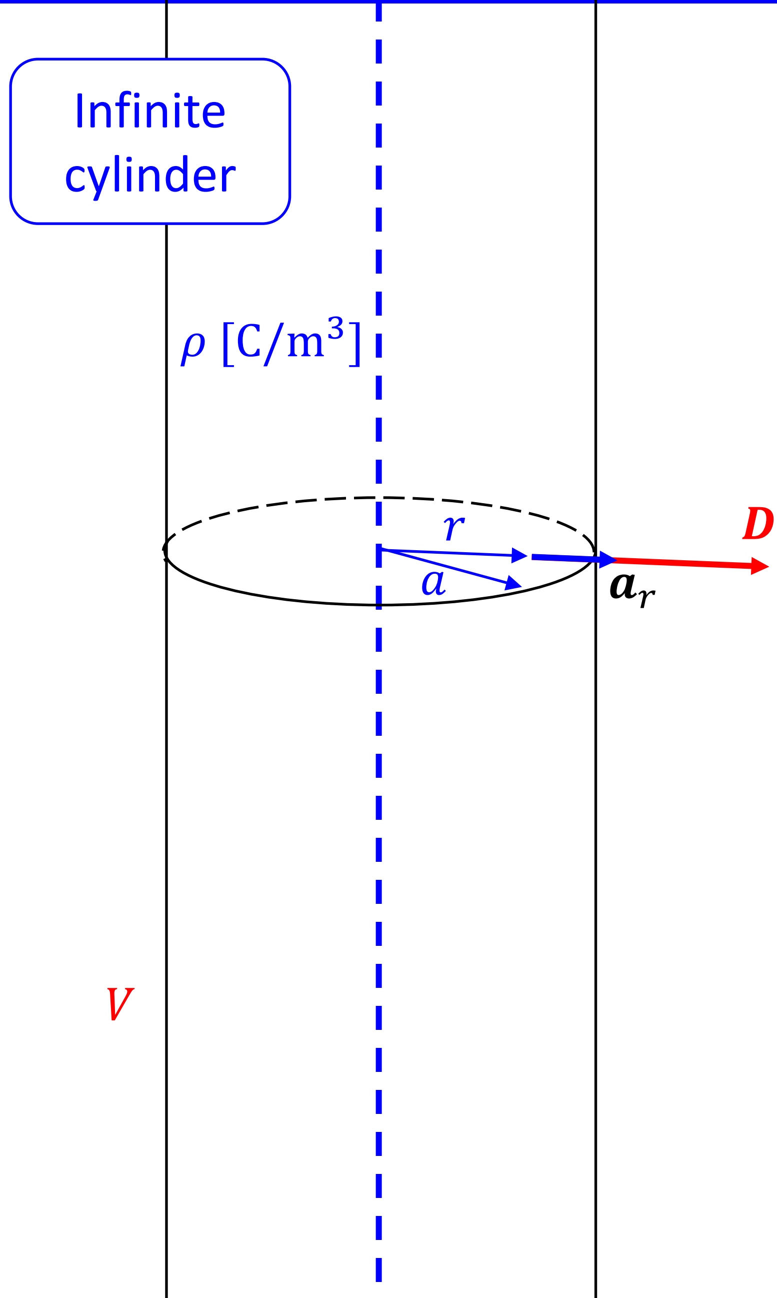
$$V_1 = V_{12} = \frac{Q}{4\pi\epsilon r_{12}}$$
$$= 9 \times 10^9 \times \frac{10^{-9}}{1} = 9\ [\mathrm{V}]$$
$$V_2 = V_{21} = \frac{Q}{4\pi\epsilon r_{21}}$$
$$= 9 \times 10^9 \times \frac{10^{-9}}{1} = 9\ [\mathrm{V}]$$

$$W_T = \frac{1}{2}\sum_{i=1}^{2} QV_i = 9\ [\mathrm{nJ}]$$

Exercise 5.1 (Homework)

Exercise 5.1 (Homework)

Exercise 5.1 (Homework)



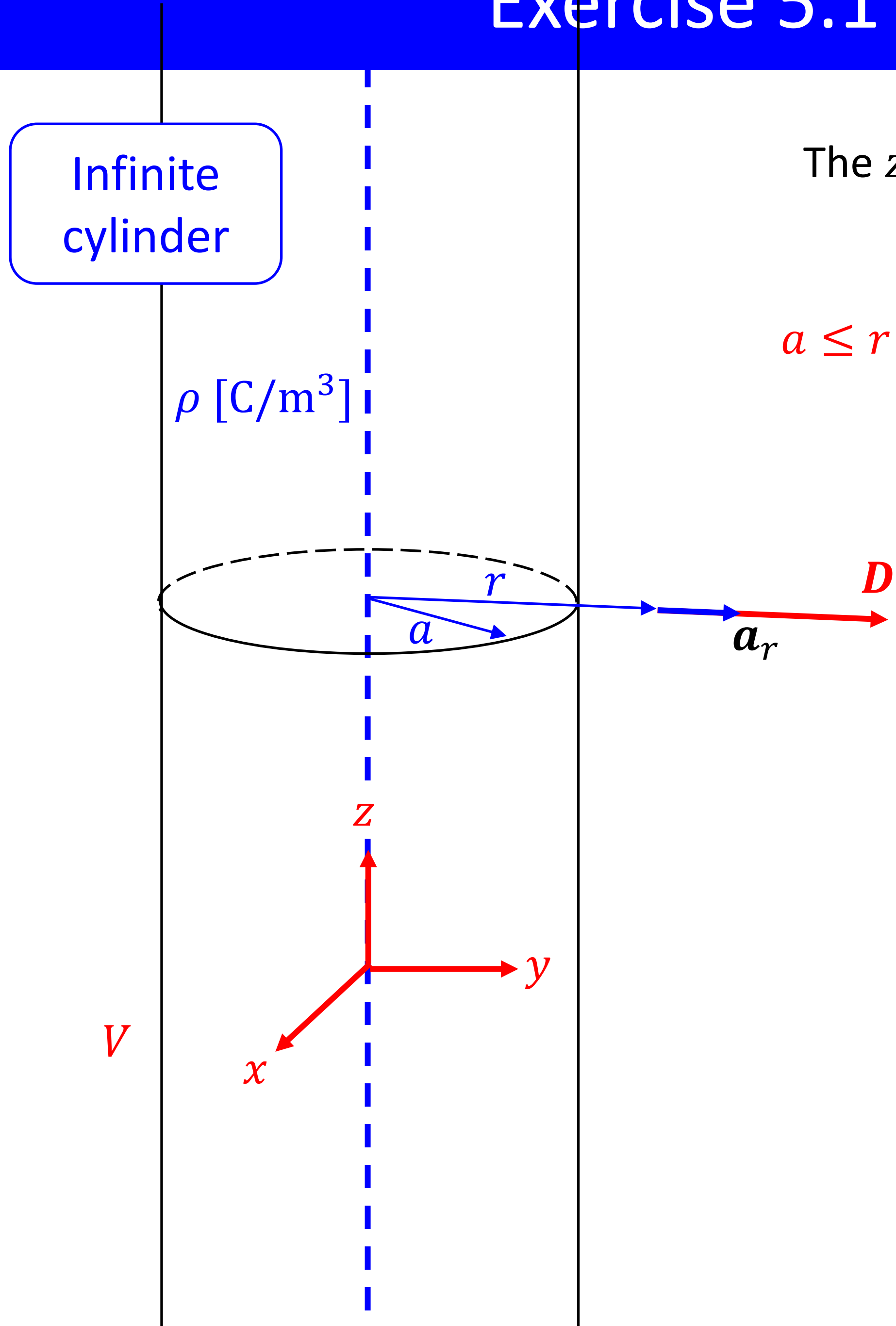
Charge is distributed with a constant density ρ $[\mathrm{C/m^3}]$ throughout an infinite cylinder V of radius a [m]. In Exercise 2.3.2, the flux density inside and outside the cylinder was obtained as follows:

$$\boldsymbol{D} = \begin{cases} \dfrac{r\rho}{2}\boldsymbol{a}_r & (r \leq a) \\ \dfrac{a^2\rho}{2r}\boldsymbol{a}_r & (a \leq r). \end{cases}$$

Find the charge inside and outside the cylinder using the following relationship:

$$\nabla \cdot \boldsymbol{D} = \rho.$$

Exercise 5.1 (Answer)



The $z-$axis is chosen as the axis of the cylinder.

$a \leq r$

$$\boldsymbol{r} = x\boldsymbol{a}_x + y\boldsymbol{a}_y$$

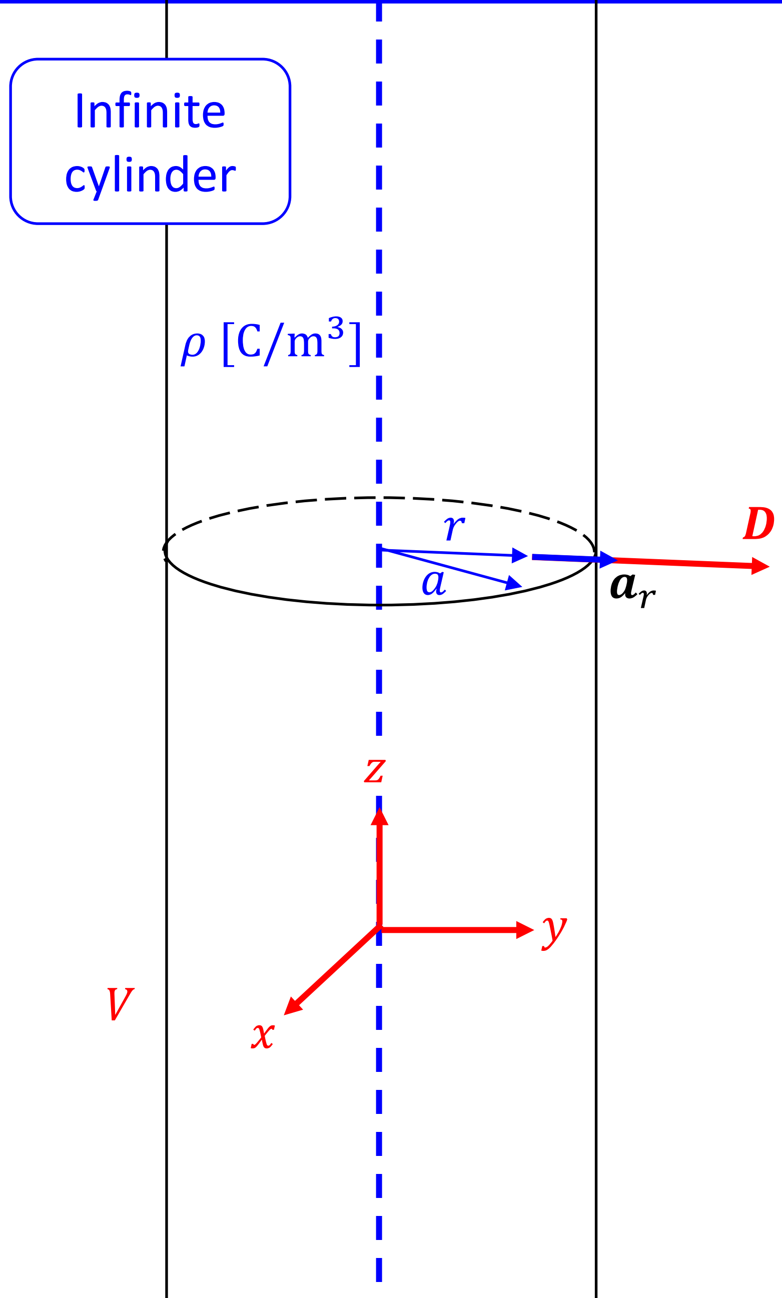
$$r = \sqrt{x^2 + y^2}$$

$$\boldsymbol{a}_r = \frac{\boldsymbol{r}}{r}$$

$$\boldsymbol{D} = \frac{r\rho}{2}\boldsymbol{a}_r = \frac{\rho}{2}(x\boldsymbol{a}_x + y\boldsymbol{a}_y)$$

$$\nabla \cdot \boldsymbol{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = \frac{\rho}{2}(1+1) = \rho$$

Exercise 5.1 (Answer)



$r \leq a$

$$\boldsymbol{r} = x\boldsymbol{a}_x + y\boldsymbol{a}_y$$

$$r = \sqrt{x^2 + y^2}$$

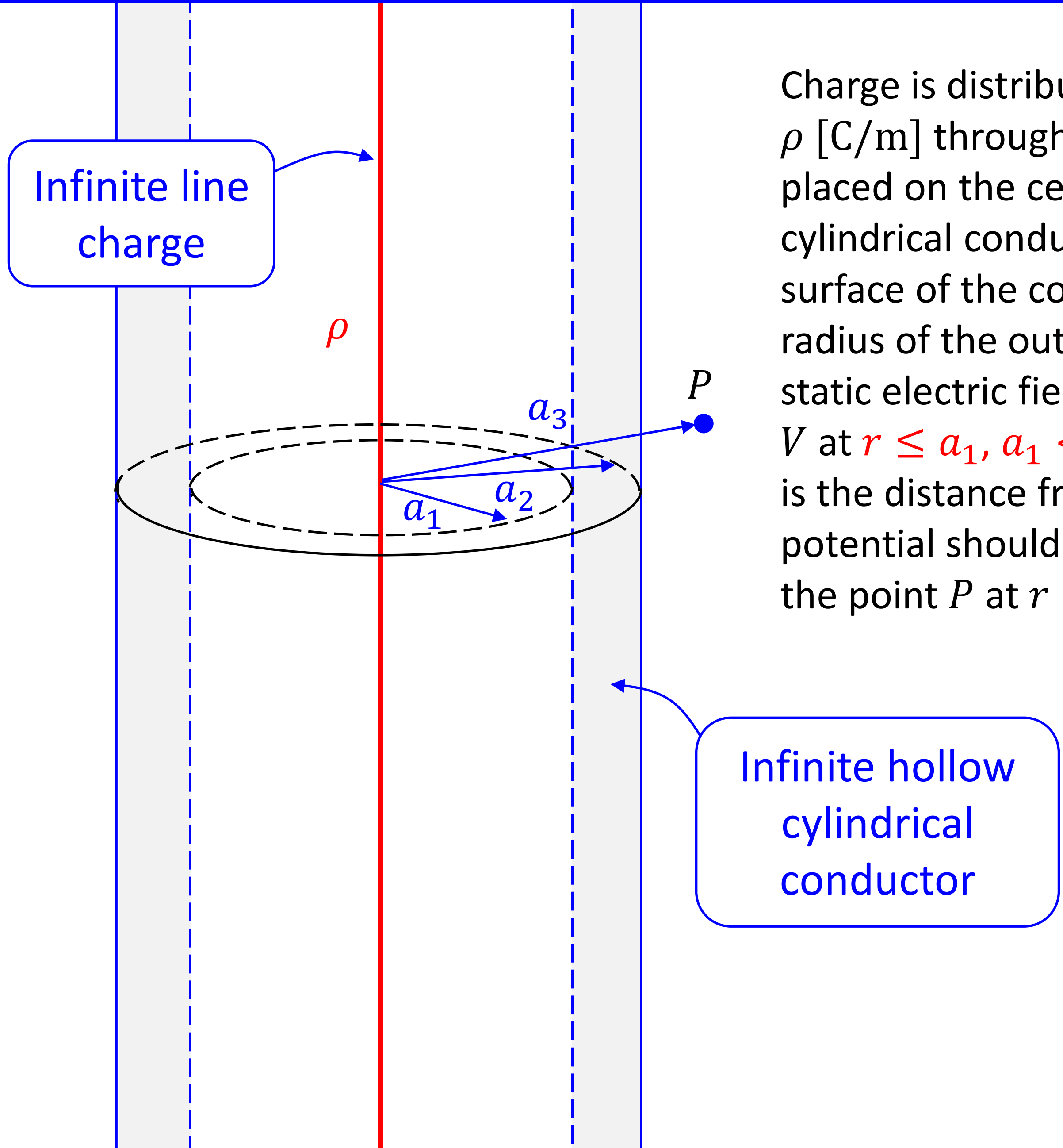
$$\boldsymbol{a}_r = \frac{\boldsymbol{r}}{r}$$

$$\boldsymbol{D} = \frac{a^2\rho}{2r}\boldsymbol{a}_r = \frac{\rho}{2r^2}\left(x\boldsymbol{a}_x + y\boldsymbol{a}_y\right)$$

$$\begin{aligned}\nabla \cdot \boldsymbol{D} &= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} \\ &= \frac{\rho}{2}\left(\frac{r^2 - 2x^2}{r^4} + \frac{r^2 - 2y^2}{r^4}\right) \\ &= \frac{\rho}{2}\left(\frac{2r^2 - 2(x^2 + y^2)}{r^4}\right) \\ &= \frac{\rho}{2}\left(\frac{2r^2 - 2r^2}{r^4}\right) \\ &= 0\end{aligned}$$

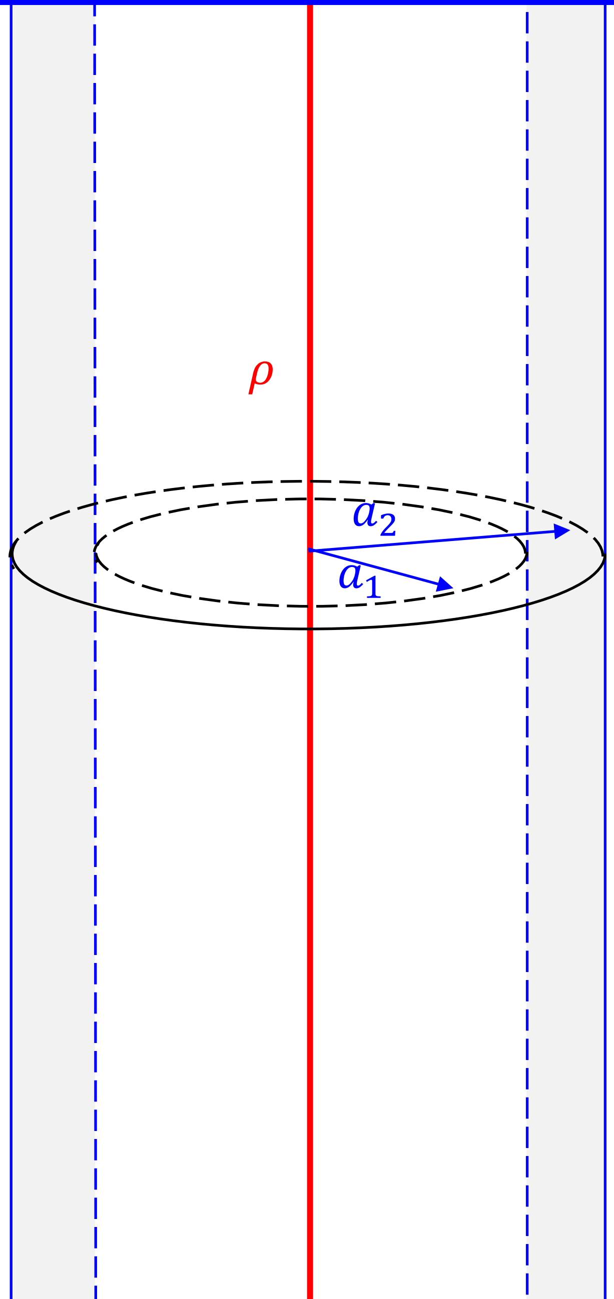
Exercise 5.2.3 (Homework)

Exercise 5.2.3 (Homework)



Charge is distributed with a constant density ρ [C/m] throughout an infinite line. This line is placed on the central axis of an infinite hollow cylindrical conductor. The radius of the inner surface of the conductor is a_1 [m], and the radius of the outer surface is a_2 [m]. Find the static electric field intensity $\boldsymbol{E}$ and the potential V at $r \leq a_1$, $a_1 < r < a_2$, and $a_2 \leq r$, where r is the distance from the central axis. The potential should be determined with respect to the point P at $r = a_3 > a_2$.

Exercise 5.2.3 (Answer)



$(a_2 \le r)$

$$\boldsymbol{E} = \frac{\rho}{2\pi\varepsilon r}\boldsymbol{a}_r$$
$$V = \frac{\rho}{2\pi\varepsilon}\ln\frac{r_3}{r}$$

$(a_1 < r < a_2)$

$$\boldsymbol{E} = 0$$
$$V = \frac{\rho}{2\pi\varepsilon}\ln\frac{r_3}{a_2}$$

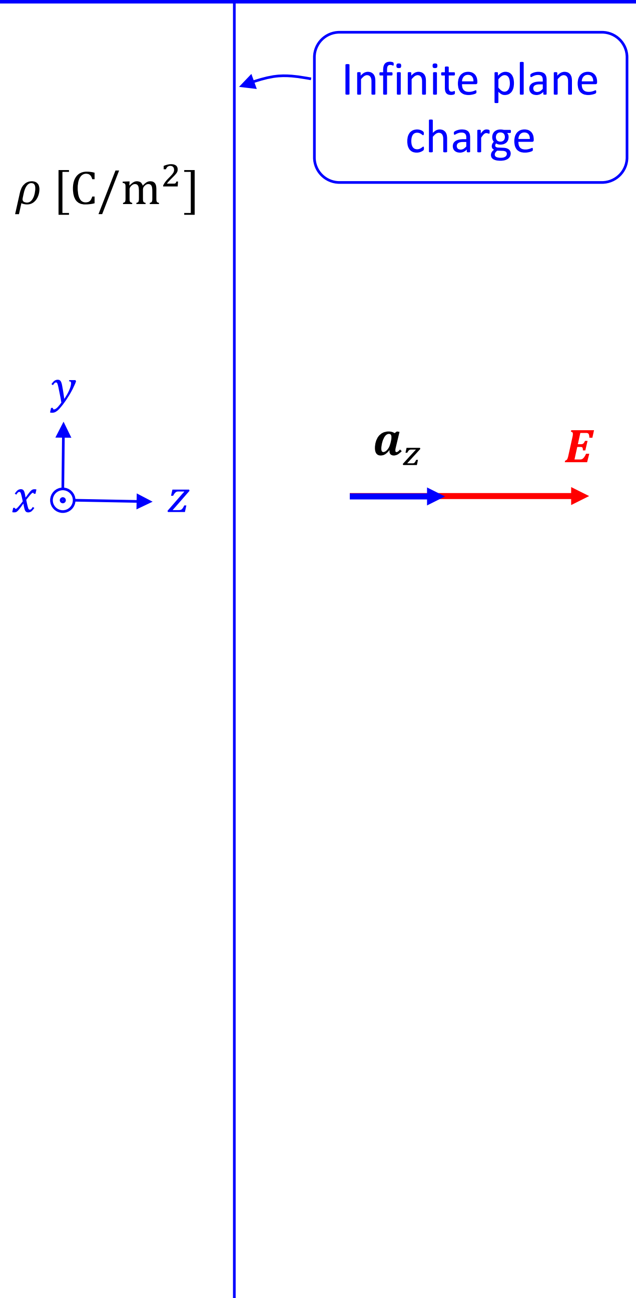
$(r \le a_1)$

$$\boldsymbol{E} = \frac{\rho}{2\pi\varepsilon r}\boldsymbol{a}_r$$
$$V = \frac{\rho}{2\pi\varepsilon}\ln\frac{a_1}{r} + \frac{\rho}{2\pi\varepsilon}\ln\frac{r_3}{a_2}$$

Exercise 5.3 (Homework)

Exercise 5.3 (Homework)

Exercise 5.3 (Homework)



According to the result of Exercise 1.6.3, the electric field intensity $\boldsymbol{E}$ due to an infinite plane charge on the $x-y$ plane with a constant $\rho\ [\mathrm{C/m^2}]$ is given by

$$\boldsymbol{E} = \frac{\rho}{2\epsilon}\boldsymbol{a}_z$$

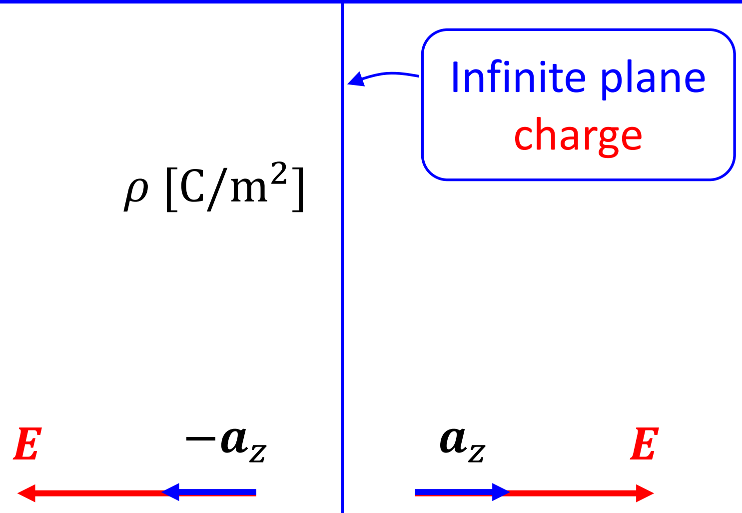
where $\boldsymbol{a}_z$ is a unit vector parallel to the z −axis and normal to the plane.

If the plane is made of a conductor and the surface charge density is $\rho\ [\mathrm{C/m^2}]$, then the electric field intensity $\boldsymbol{E}$ is

$$\boldsymbol{E} = \frac{\rho}{\epsilon}\boldsymbol{a}_z.$$

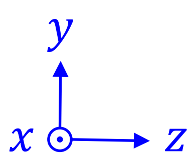
Explain the reason for the difference.

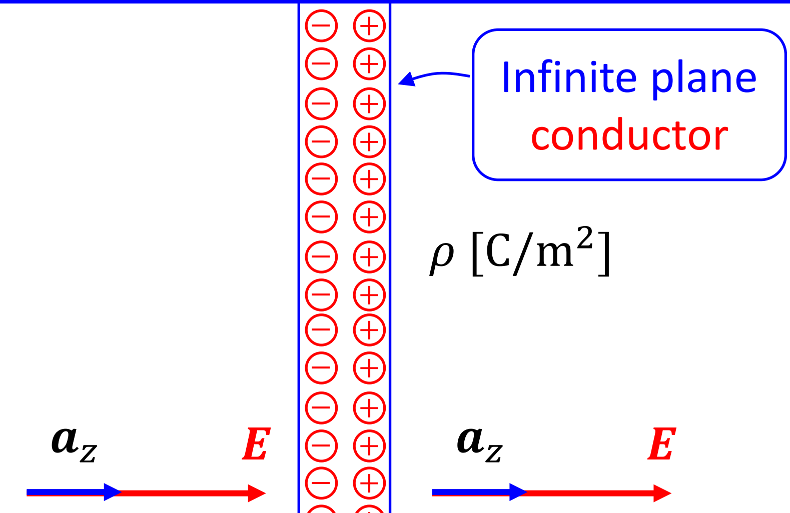
Exercise 5.3 (Answer)



Electric field lines from ρ go in both the $+z$ and $-z$ directions.
Thus

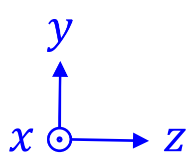
$$\boldsymbol{E} = \frac{\rho}{2\epsilon}\boldsymbol{a}_z$$





Electric field lines from ρ go only in the $+z$ direction.
Thus

$$\boldsymbol{E} = \frac{\rho}{\epsilon}\boldsymbol{a}_z$$

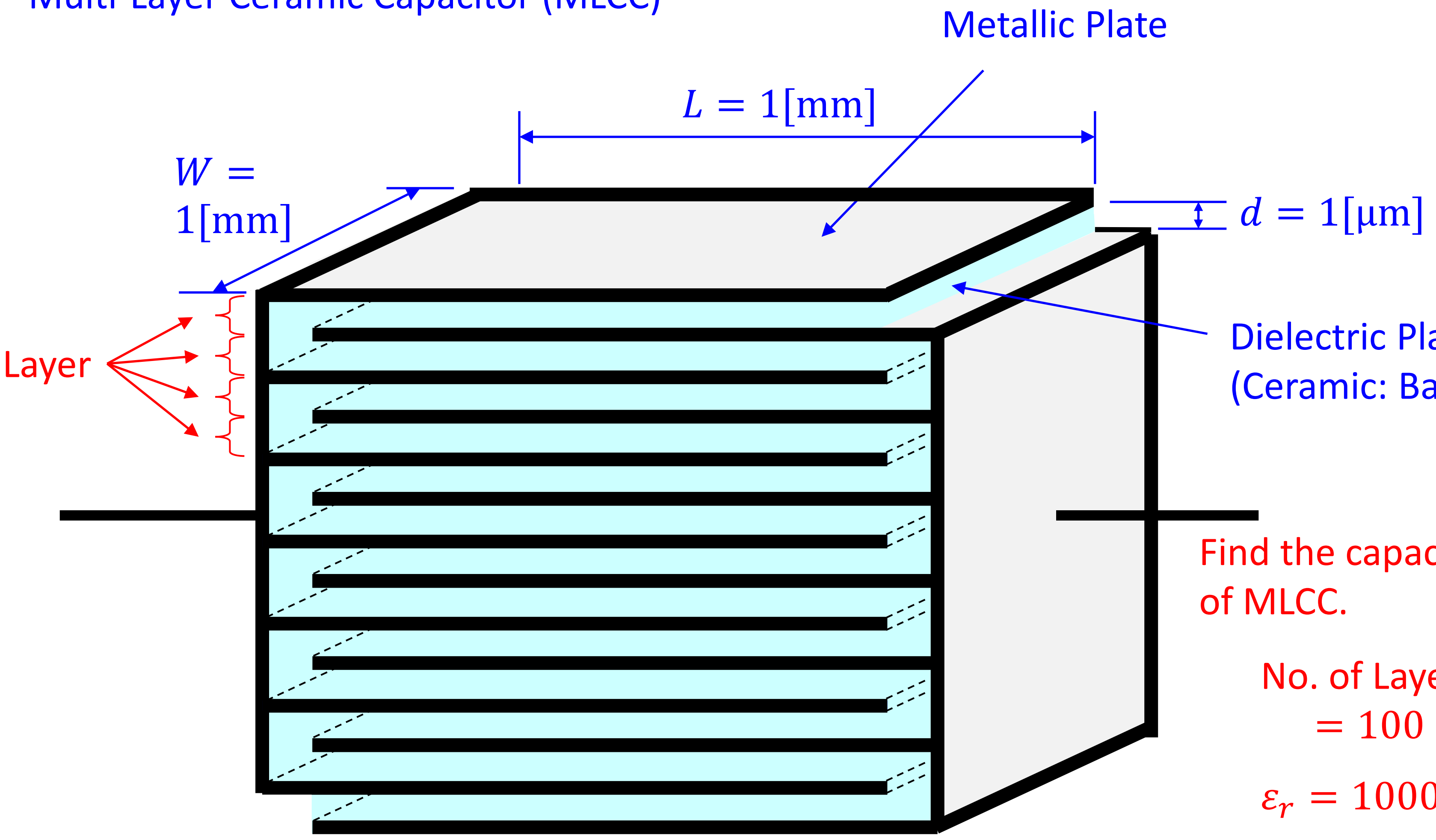


Exercise 6.1.2 (Homework)

Exercise 6.1.2 (Homework)

Exercise 6.1.2 (Homework)

Multi-Layer Ceramic Capacitor (MLCC)



Find the capacitance of MLCC.

No. of Layers $= 100$

$\varepsilon_r = 1000$

Exercise 6.1.2 (Answer)

Multi-Layer Ceramic Capacitor

Metallic plate

$$\left.\begin{array}{l}\text{Length } L = 1\ [\mathrm{mm}] \\ \text{Width } W = 1\ [\mathrm{mm}]\end{array}\right\} \quad A = 10^{-6}\ [\mathrm{m}^2]$$

Dielectrics

Thickness $d = 1\ [\mu\mathrm{m}]$

Relative Permittivity $\varepsilon_r = 1000$

No of Layers

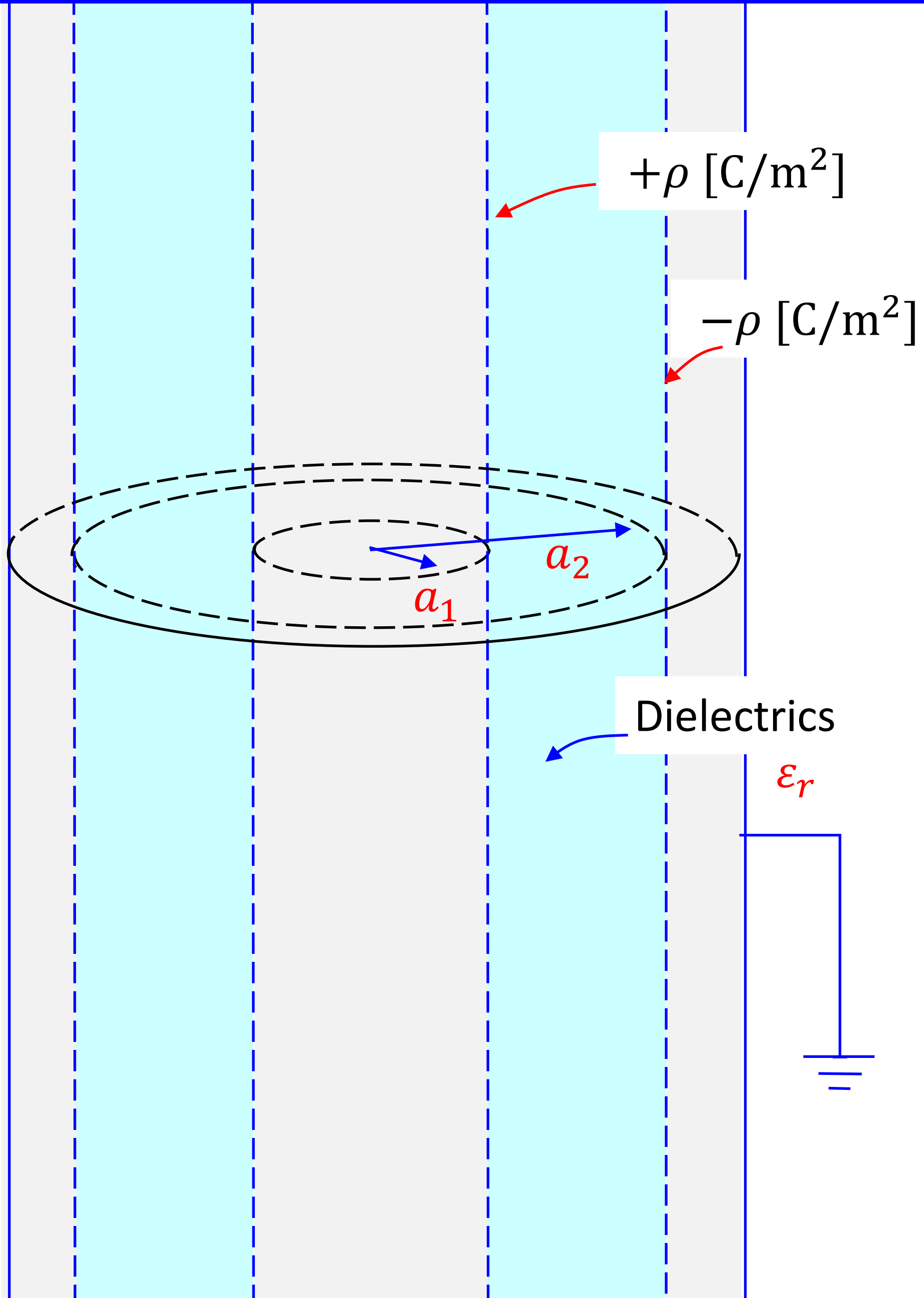
$n = 100$

$$C = n\frac{\varepsilon_r \varepsilon_0 A}{d} = 100 \times \frac{1000 \times 8.854 \times 10^{-12} \times 10^{-6}}{10^{-6}} \approx 0.89\ [\mu\mathrm{F}]$$

Exercise 6.1.3 (Homework)

Exercise 6.1.3 (Homework)

Exercise 6.1.3 (Homework)



This is a coaxial cable. The length of cable is infinite. There are two concentric cylindrical conductors inside the cable. One is the conductor of radius a_1 [m], and the other is the hollow cylinder of inner radius a_2 [m].

Show that If the charge on the surface of inner conductor is $+\rho$ $[\mathrm{C/m^2}]$, then

$$(a_1 \le r \le a_2)$$

$$V_{r2} = \frac{a_1\rho}{\varepsilon}\ln\frac{a_2}{r}.$$

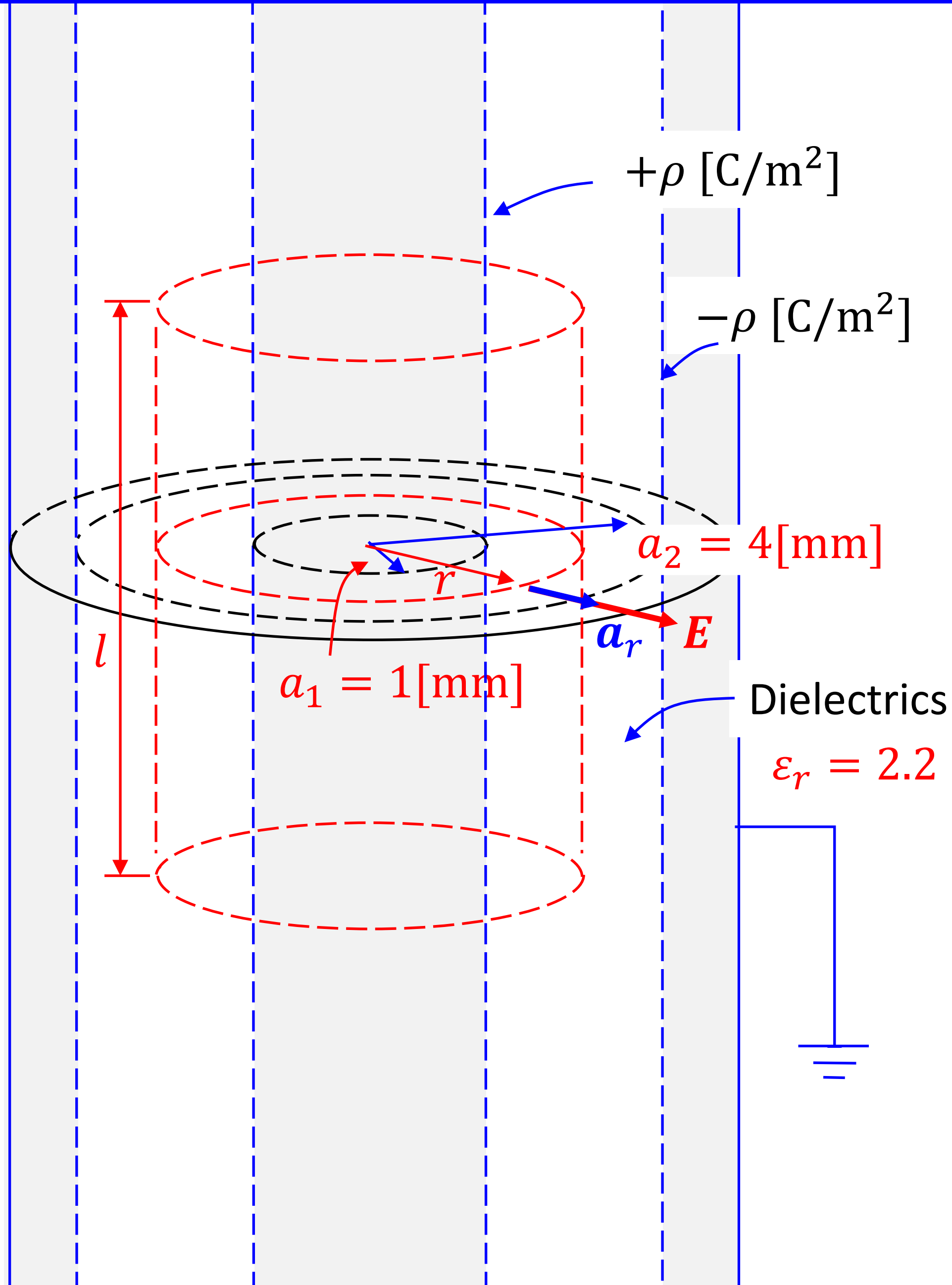
Find the capacitance per unit length.

The radii are

$$a_1 = 1[\mathrm{mm}], a_2 = 4[\mathrm{mm}].$$

The dielectrics is polyethylene. Its relative permittivity $\varepsilon_r = 2.2$.

Exercise 6.1.3 (Answer)



According to Gauss's Law, on the surface of cylinder of radius r, length l,

$$\int_{CS} \boldsymbol{\varepsilon E} \cdot d\boldsymbol{S} = \int_V \rho dv$$

$$2\pi r l \varepsilon E = 2\pi a_1 l \rho$$

$$\boldsymbol{E} = \frac{a_1 \rho}{\varepsilon r} \boldsymbol{a}_r$$

Then

$$V_{r2} = -\int_{a_2}^{r} \boldsymbol{E} \cdot d\boldsymbol{r} = \frac{a_1 \rho}{\varepsilon} \ln \frac{a_2}{r}$$

$$C = \frac{Q}{V_{12}} = \frac{2\pi a_1 l \rho}{\frac{a_1 \rho}{\varepsilon} \ln \frac{a_2}{a_1}} = \frac{2\pi \varepsilon_r \varepsilon_0 l}{\ln \frac{a_2}{a_1}}$$

$$= \frac{2\pi \times 2.2 \times 8.854 \times 10^{-12} \times 1}{\ln \frac{4}{1}}$$

$$= 88[\mathrm{pF/m}]$$

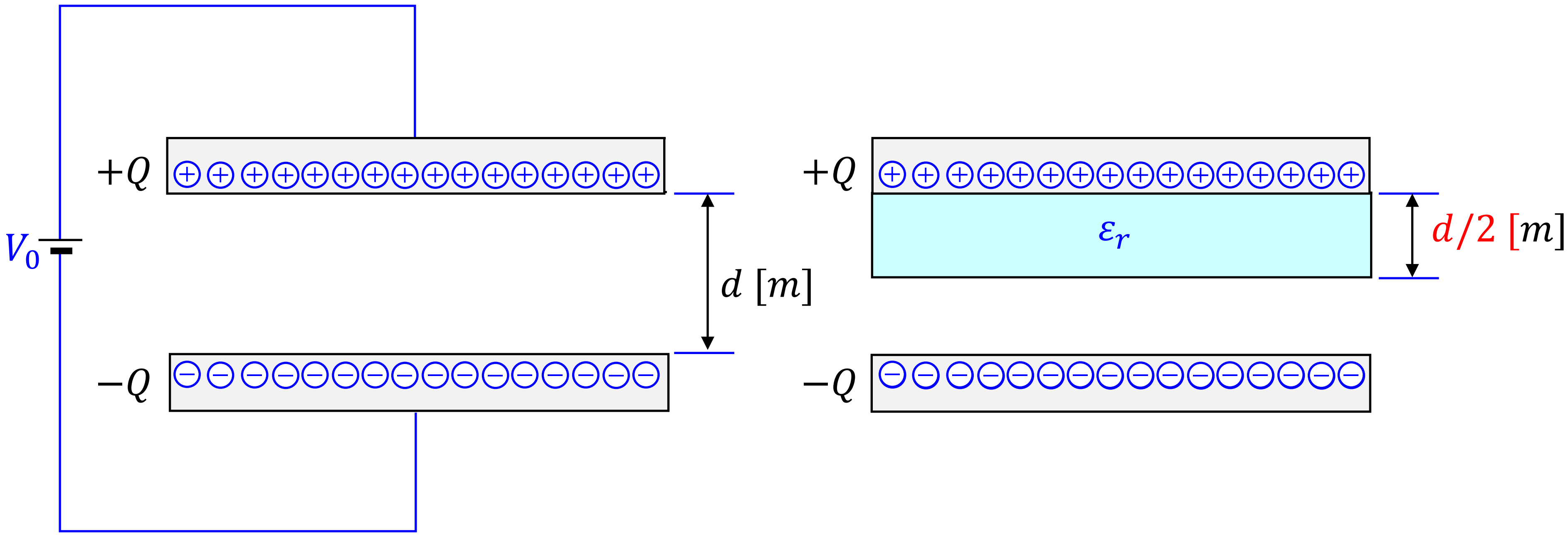
$(l = 1\ [\mathrm{m}])$

Exercise 6.2.2 (Homework)

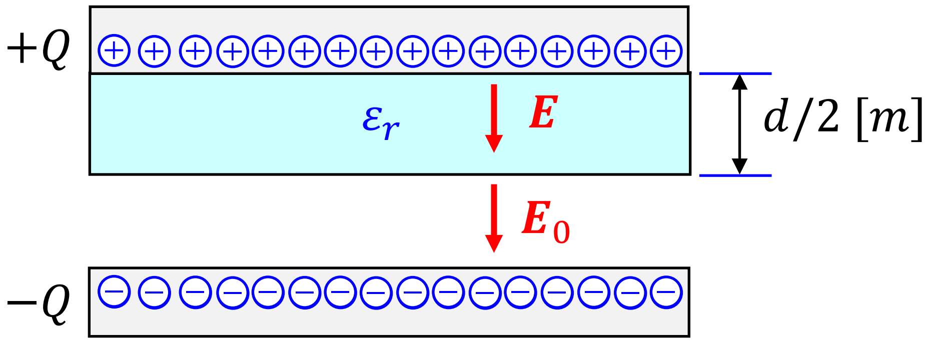
Exercise 6.2.2 (Homework)

Exercise 6.2.2 (Homework)

A voltage V_0 is applied to a capacitor with air as shown in the figure below on the left. The facing surface area is A $[\mathrm{m}^2]$, and the distance between the conductors is d $[\mathrm{m}]$. Charges of $\pm Q_0$ $[\mathrm{C}]$ are stored on the surfaces of the conductors. Find the capacitance if a dielectric material with permittivity ε_r and thickness $d/2$ $[\mathrm{m}]$ is inserted between the conductors as in the figure below on the right.



Exercise 6.2.2 (Answer)



$$E_{\mathbf{0}} = \frac{Q}{\varepsilon_0 A} \quad [\mathrm{V/m}]$$

$$E = \frac{Q}{\varepsilon_r \varepsilon_0 A} \quad [\mathrm{V/m}]$$

$$\begin{aligned} V &= E\frac{d}{2} + E_0\frac{d}{2} \\ &= \frac{Qd}{2\varepsilon_0 A}\left(1 + \frac{1}{\varepsilon_r}\right) \quad [\mathrm{V}] \end{aligned}$$

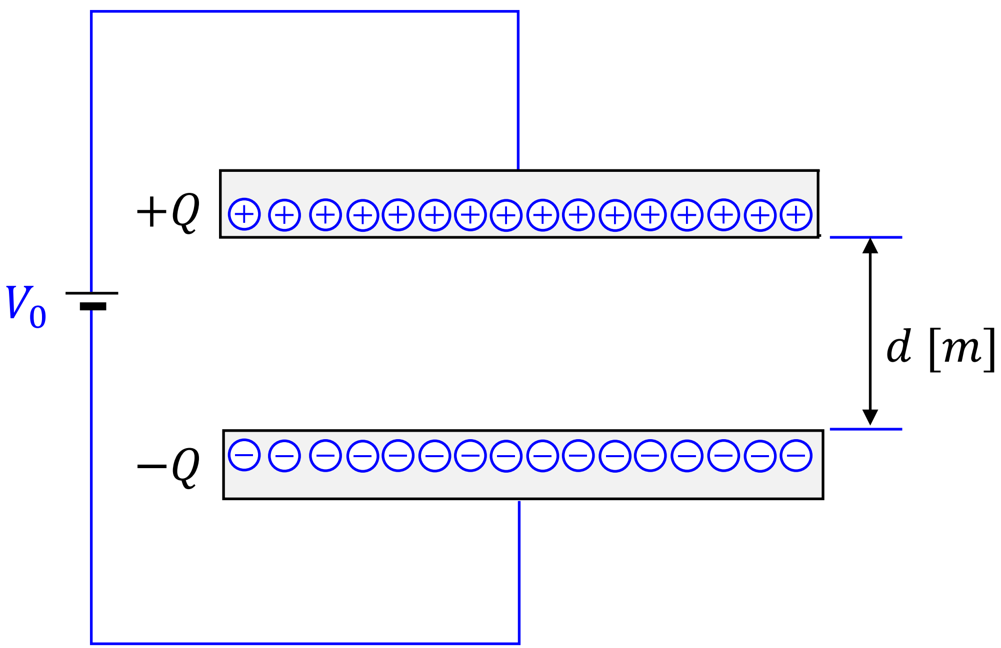
$$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{\varepsilon_0 A}{d}\frac{2}{1 + \frac{1}{\varepsilon_r}} \quad [\mathrm{F}] \end{aligned}$$

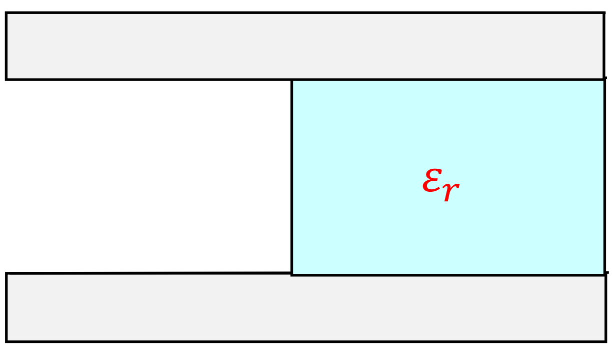
Exercise 6.2.3 (Homework)

Exercise 6.2.3 (Homework)

Exercise 6.2.3 (Homework)

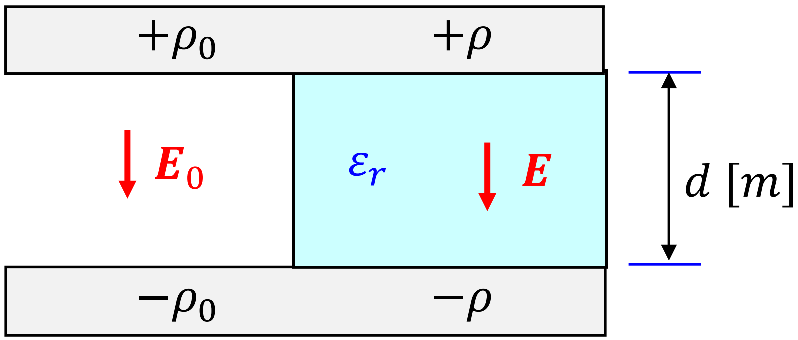
A voltage V_0 is applied to a capacitor with air as shown in the figure below on the left. The facing surface area is A $[\mathrm{m}^2]$, and the distance between the conductors is d $[\mathrm{m}]$. Charges of $\pm Q_0$ $[\mathrm{C}]$ are stored on the surfaces of the conductors. What will change when the voltage source of V_0 is removed, and then the dielectric material with permittivity ε_r and area $A/2$ $[\mathrm{m}^2]$ is inserted between the conductors as in the figure below on the right? Find the capacitance.





Exercise 6.2.4 (Answer)

ρ_0: charge density on the left half surface of conductor
ρ: charge density on the right half surface of conductor



$$E_{\mathbf{0}} = \frac{\rho_0}{\varepsilon_0} \quad [\mathrm{V/m}]$$

$$E = \frac{\rho}{\varepsilon_r \varepsilon_0} \quad [\mathrm{V/m}]$$

A conductor surface is equipotential.

$$V = E_0 d = Ed = \frac{\rho_0}{\varepsilon_0} d = \frac{\rho}{\varepsilon_r \varepsilon_0} d \; [\mathrm{V}].$$

Thus

$$\frac{\rho}{\rho_0} = \varepsilon_r.$$

Then

$$Q = \rho_0 \frac{A}{2} + \rho \frac{A}{2} = \left(\frac{\varepsilon_0 V}{d} + \frac{\varepsilon_r \varepsilon_0 V}{d} \right) \frac{A}{2} \; [\mathrm{C}].$$

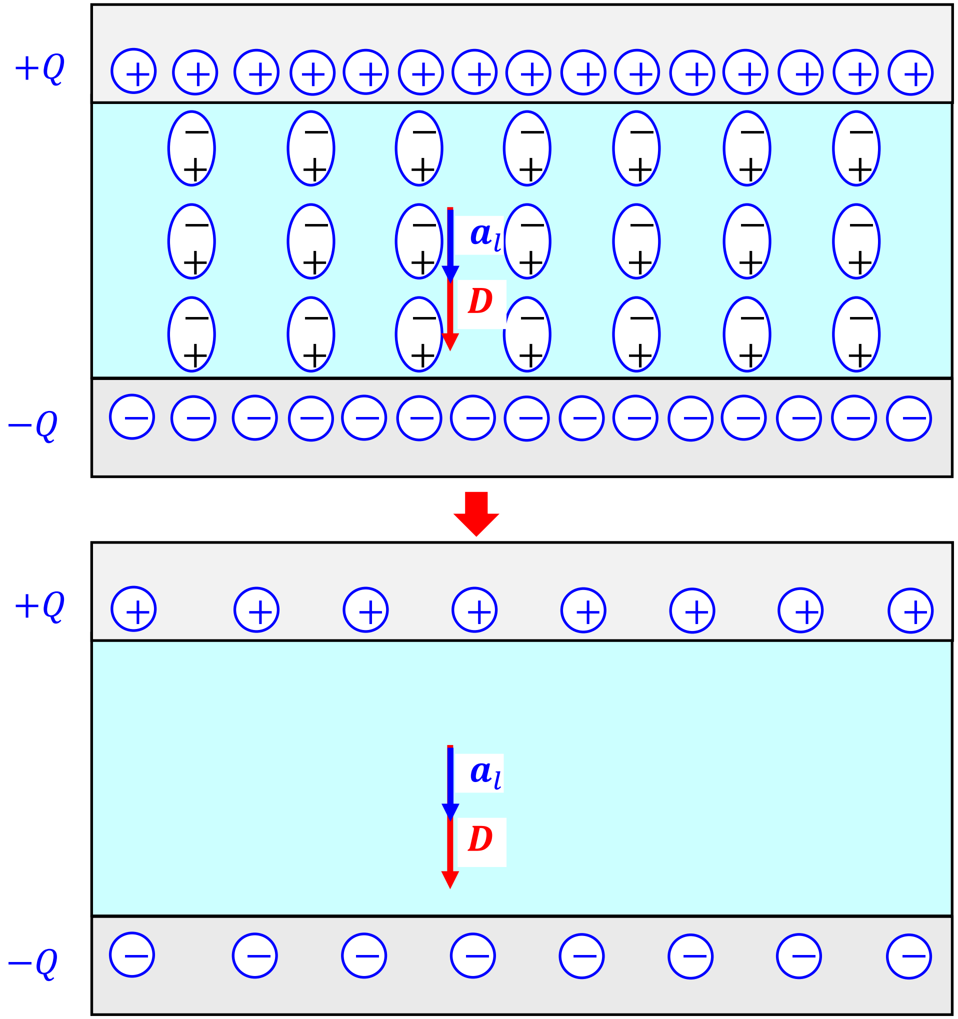
Thus

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d} \frac{1 + \varepsilon_r}{2} \quad [\mathrm{F}].$$

Exercise 6.3.2 (Homework)

Exercise 6.3.2 (Homework)

Exercise 6.3.2 (Homework)



In a dielectric material, the electric flux density is given by

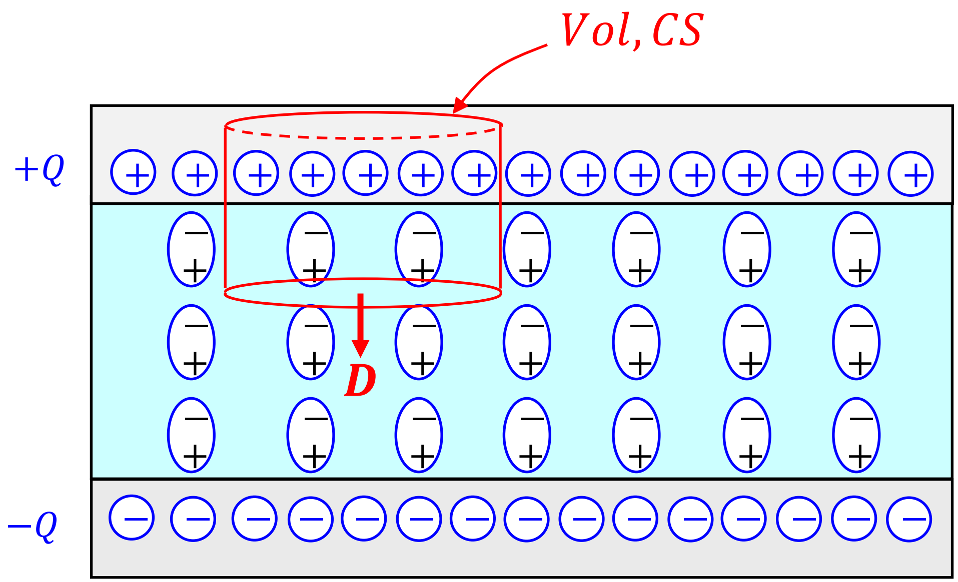
$$\boldsymbol{D} = \rho \boldsymbol{a}_l \ [\mathrm{C/m^2}]$$

where $\rho(= Q/A)$ is the charge density on the surface of the conductor. Q [C] is the charge on the surface, and A [m^2] is the surface area.

Due to the dipoles, some charges are canceled as shown in the bottom figure.

Show that even in this situation, $\boldsymbol{D} = \rho \boldsymbol{a}_l$ remains unchanged, by applying Gauss's law.

Exercise 6.3.2 (Homework)

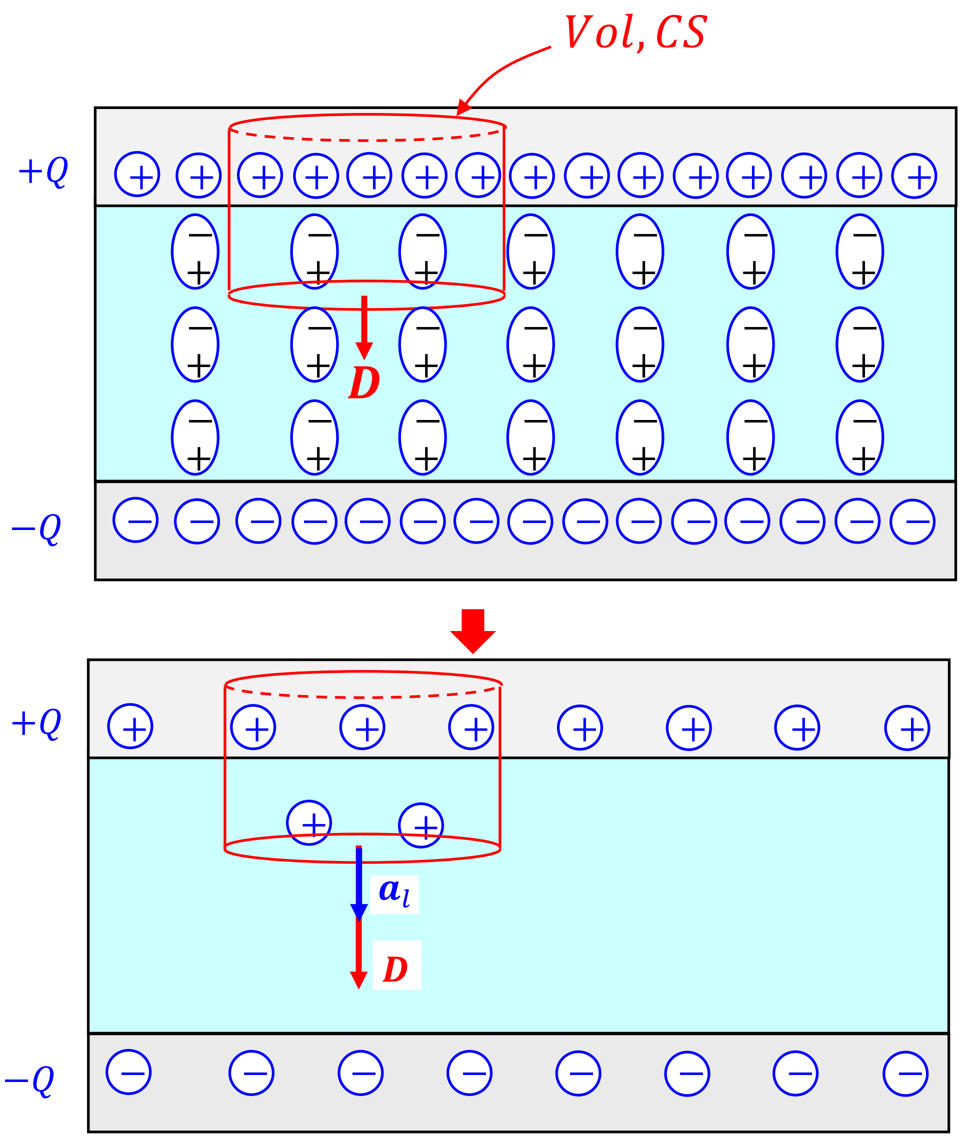


Hint: you can set a volume Vol, closed surface CS as in the left figure.

Gauss's Law

$$\int_{CS} D \cdot dS = \int_{Vol} \rho dv$$

Exercise 6.3.2 (Answer)



Gauss's Law

$$\int_{CS} D \cdot dS = \int_{Vol} \rho dv$$

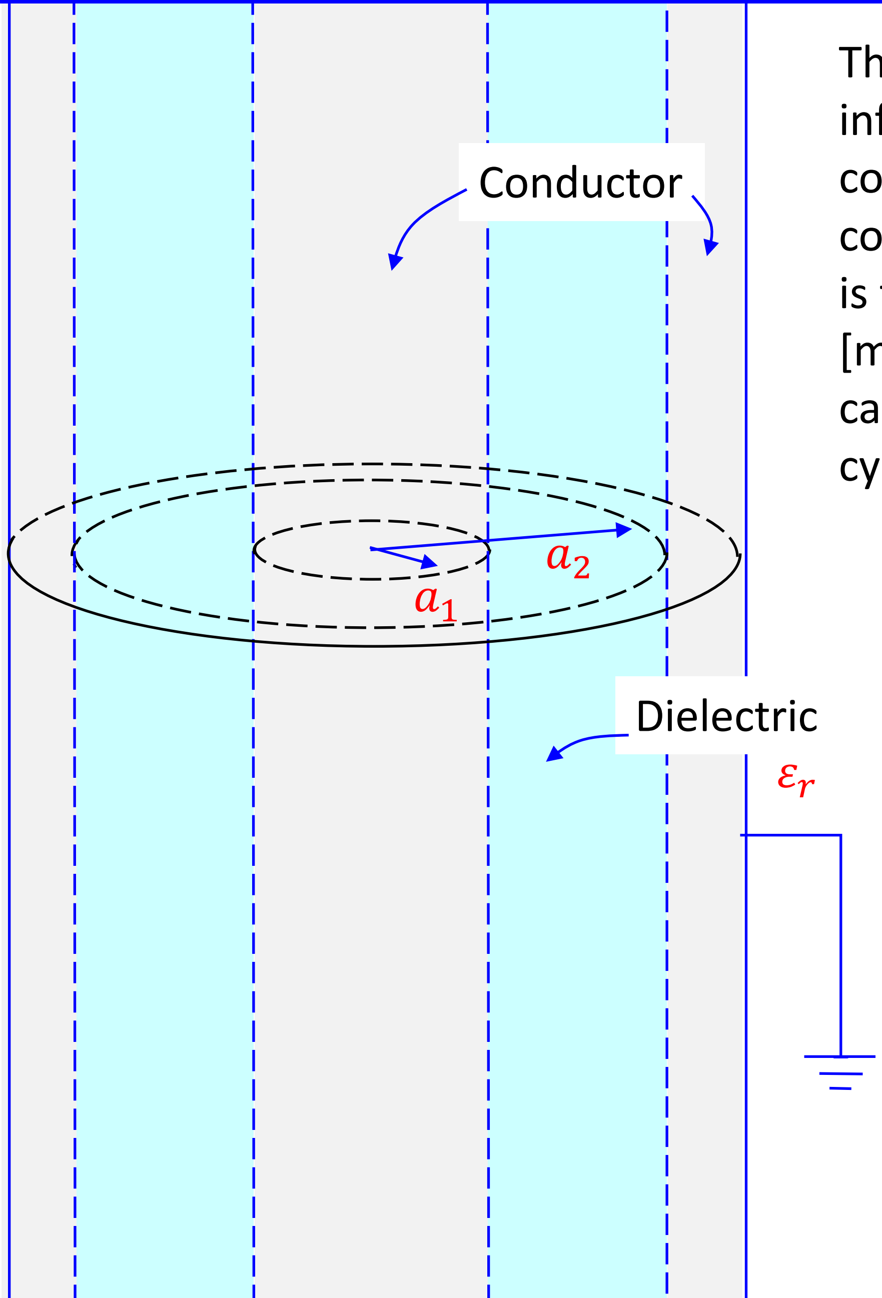
The volume for the integral can be set as in the top figure. Vol is a cylinder .

On the bottom surface of the cylinder, some positive charges of electric dipoles appear without being cancelled.

Exercise 6.4.3 (Homework)

Exercise 6.4.3 (Homework)

Exercise 6.4.3 (Homework)



This is a coaxial cable. The length of the cable is infinite. Inside the cable, there are two concentric cylindrical conductors. One is the conductor with a radius of a_1 [m], and the other is the hollow cylinder with an inner radius of a_2 [m]. From According to Exercise 6.1.3, the capacitance per unit length of this concentric cylindrical conductors is

$$C = \frac{2\pi\varepsilon}{\ln\frac{a_2}{a_1}} \quad [\mathrm{F/m}]$$

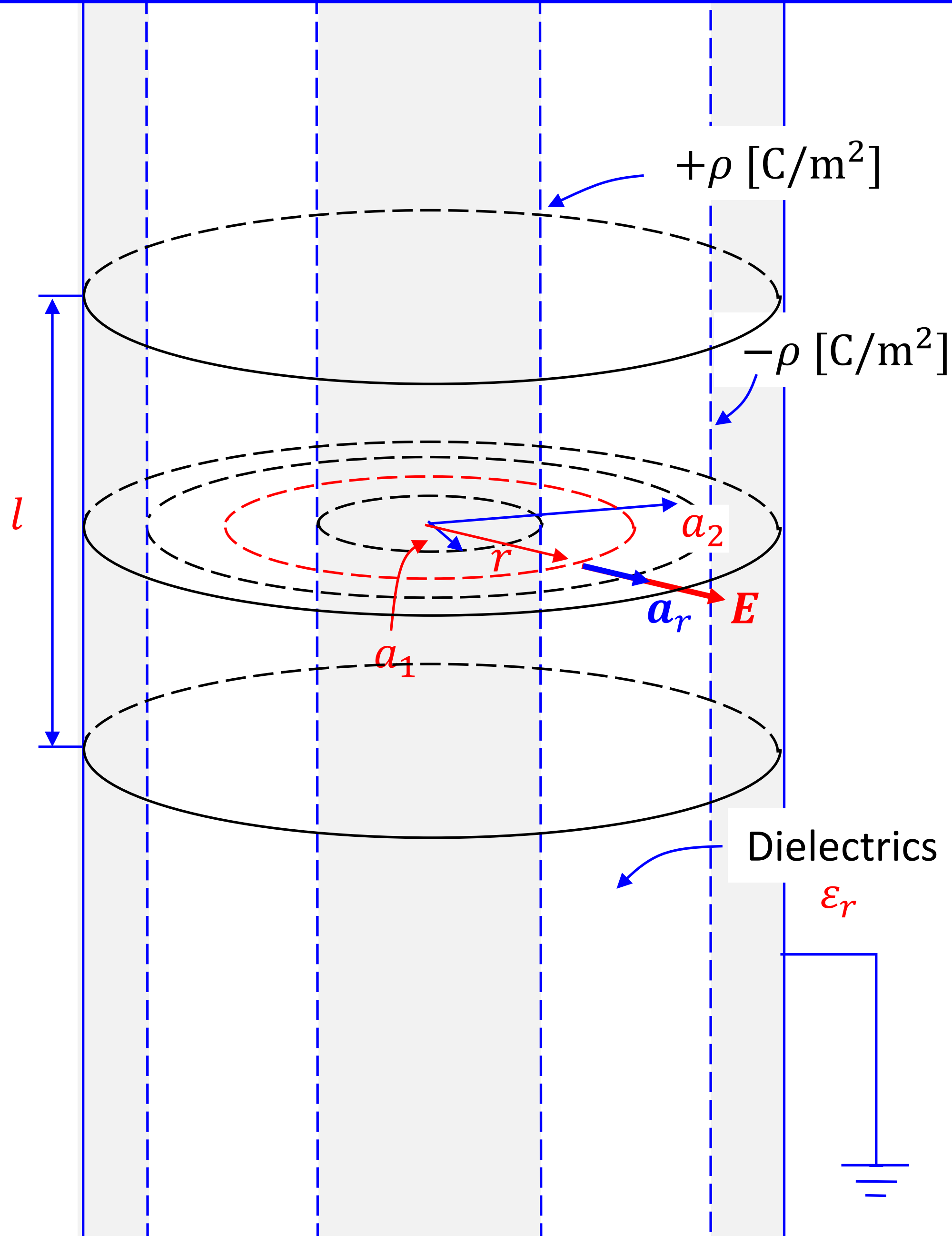
Show that the integral of

$$w_e = \frac{1}{2}\varepsilon E^2 \ \ [\mathrm{J/m^3}]$$

over the gap meets the energy

$$W_T = \frac{1}{2}CV^2 \quad [\mathrm{J/m}].$$

Exercise 6.4.3 (Answer)



Suppose that $+\rho$ [C/m^2] is charged on the surface of inner conductor. From Homework 6.1.3,

$$\boldsymbol{E} = \frac{a_1 \rho}{\varepsilon r} \boldsymbol{a}_r$$

and by definition

$$2\pi a_1 l \rho = ClV \qquad l\text{: length}$$

Then,

(Vol: gap region per unit length)

$$W_T = \int_{Vol} \frac{1}{2} \varepsilon E^2 dv$$

$$= \int_{a_1}^{a_2} \frac{1}{2} \varepsilon \left(\frac{CV}{2\pi\varepsilon r} \right)^2 2\pi r dr$$

$$= \frac{C^2 V^2}{4\pi\varepsilon} \ln \frac{a_2}{a_1}$$

$$= \frac{1}{2} C V^2 \quad [\mathrm{J/m}]$$

Exercise 7.1.2 (Homework)

Exercise 7.1.2 (Homework)

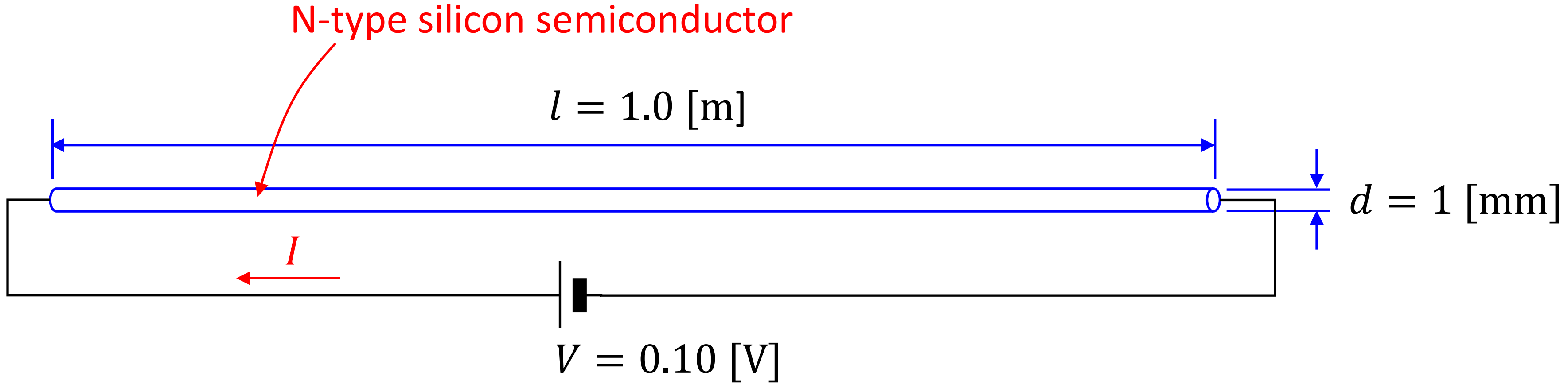
Exercise 7.1.2 (Homework)

Find the electric field intensity E and current density J, if a drift velocity

$$U = 1.5 \times 10^{-2} \quad [m/s]$$

in an n-type silicon semiconductor. For n-type silicon semiconductor , the conductivity $\sigma = 2.0 \times 10^{2}$ $[\mathrm{S/m}]$ and the mobility $\mu = 1.5 \times 10^{-1}$ $[\mathrm{m^2/Vs}]$.

Find the current I flowing through the semiconductor in the circuit below. You can neglect the voltage drop by the connecting wires.

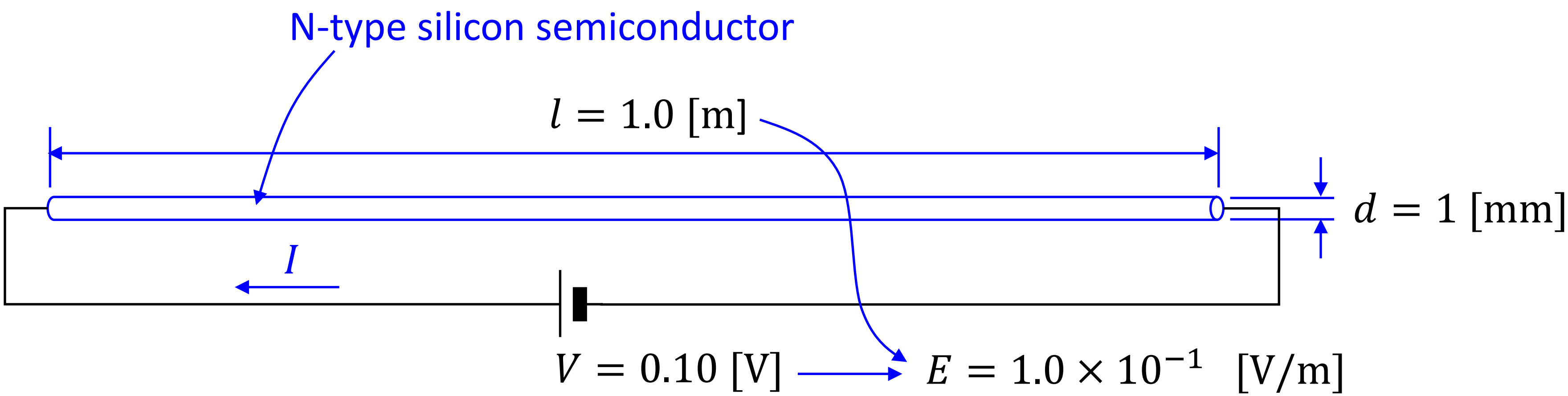


Pause the video and answer the problem.

Exercise 7.1.2 (Answer)

$$E = \frac{U}{\mu} = \frac{1.5 \times 10^{-2}}{1.5 \times 10^{-1}} = 1.0 \times 10^{-1} \quad [\mathrm{V/m}]$$

$$J = \sigma E = 2.0 \times 10^{2} \times 1.0 \times 10^{-1} = 2.0 \times 10^{1} \quad [\mathrm{A/m^2}]$$

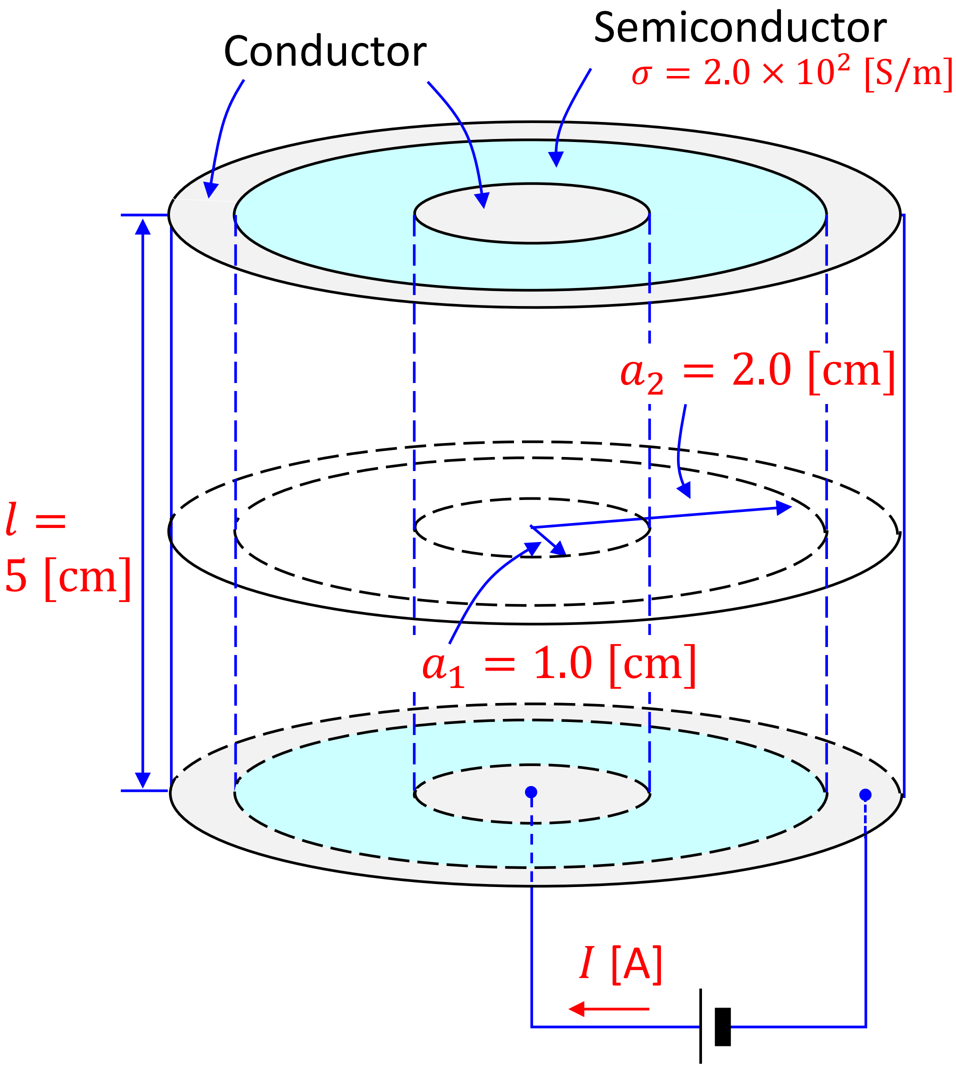


$$I = JS = 2.0 \times 10^{1} \times \pi \times (0.5 \times 10^{-3})^2 = 16 \; [\mu\mathrm{A}]$$

Exercise 7.2.2 (Homework)

Exercise 7.2.2 (Homework)

Exercise 7.2.2 (Homework)

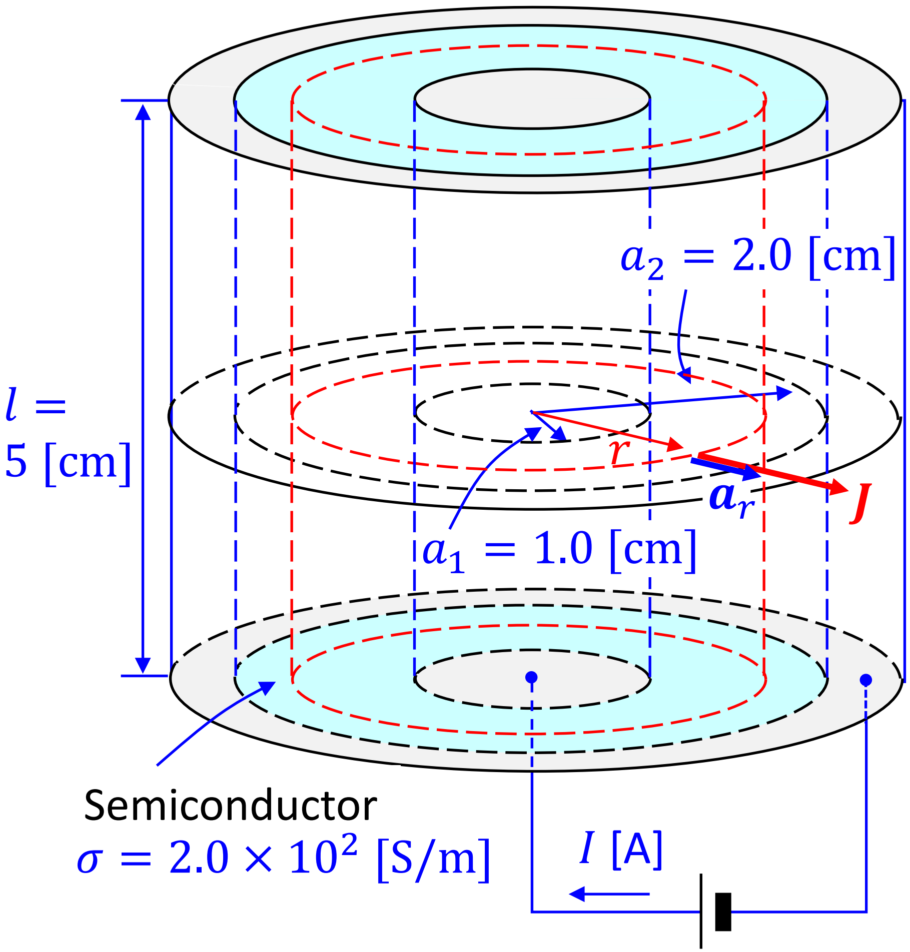


There are two concentric cylindrical conductors. One is a conductor of radius $a_1 = 1$ [mm], and the other is a hollow cylinder of inner radius $a_2 = 2$ [mm]. The length of cylinder $l = 5$ [cm]. The gap between the two conductors is filled with a resistor of conductivity $\sigma = 2.0 \times 10^2$ [S/m].

Find the resistance between the two conductors. You can neglect the resistances of the conductors.

Note: the conductance of the air is in the order of 10^{-15} [S/m]. No fringing effect exists.

Exercise 7.2.2 (Answer)



On the spherical surface of radius r,

$$\boldsymbol{J} = \frac{I}{2\pi r l}\boldsymbol{a}_r \ \ [\mathrm{A/m^2}]$$

From $\boldsymbol{J} = \sigma \boldsymbol{E}$,

$$\boldsymbol{E} = \frac{I}{2\pi\sigma r l}\boldsymbol{a}_r \ \ [\mathrm{V/m}]$$

Then

$$\boldsymbol{V} = -\int_{a_2}^{a_1} E \cdot d\boldsymbol{r} = \frac{I}{2\pi\sigma l}\ln\frac{a_2}{a_1} \ \ [\mathrm{V}]$$

Thus

$$\boldsymbol{R} = \frac{V}{I} = \frac{1}{2\pi\sigma l}\ln\frac{a_2}{a_1}$$

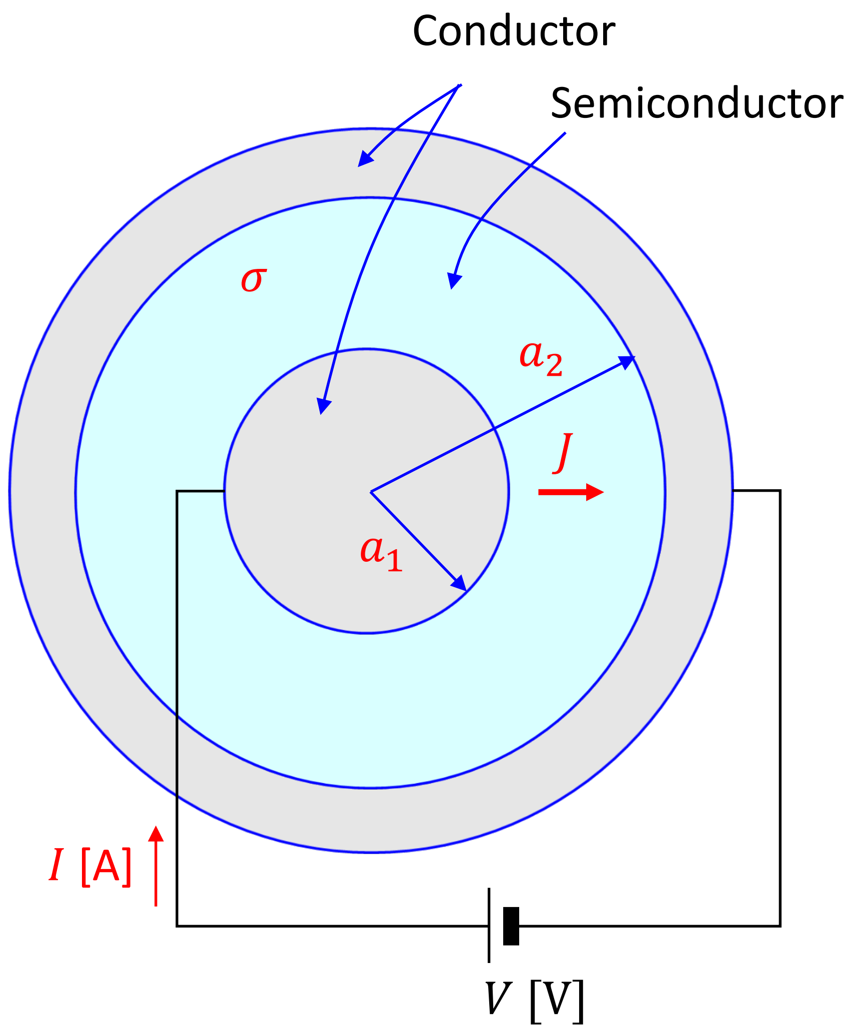
$$= \frac{1}{2\pi \times 2.0 \times 10^2 \times 5 \times 10^{-2}}\ln\frac{2.0 \times 10^{-2}}{1.0 \times 10^{-2}}$$

$$= 1.1 \times 10^{-2} \ [\Omega]$$

Exercise 7.3.3 (Homework)

Exercise 7.3.3 (Homework)

Exercise 7.3.3 (Homework)



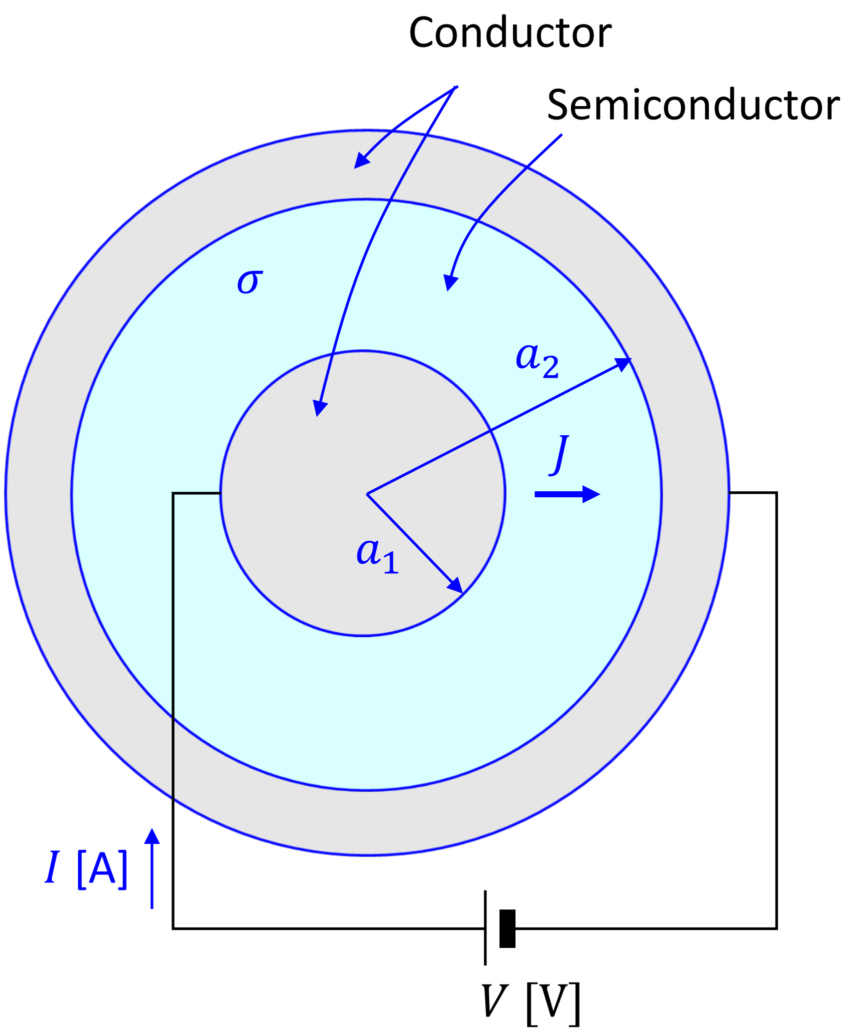
There are two concentric spherical conductors. One is a ball of radius a_1, and the other is a shell of inner radius a_2 surrounding the ball. The gap between the two conductors is filled with a semiconductor of conductivity σ.

Show that

$$\nabla \cdot \boldsymbol{J} = 0$$

where $\boldsymbol{J}$ is a current density in the semiconductor.

Exercise 7.3.3 (Answer)



From Exercise 7.2.2

$$\boldsymbol{J} = \sigma \boldsymbol{E}, \qquad \boldsymbol{E} = \frac{I}{4\pi\sigma r^2}\boldsymbol{a}_r \ \ [\mathrm{V/m}]$$

Then

$$\nabla \cdot \boldsymbol{J} = \nabla \cdot \left(\sigma \frac{I}{4\pi\sigma r^2}\boldsymbol{a}_r\right)$$

From Section 5.1

$$\nabla \cdot \left(\frac{1}{r^2}\boldsymbol{a}_r\right) = \nabla \cdot \left(\frac{1}{r^2}\frac{x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z}{r}\right)$$

$$= \frac{r^2 - 3x^2}{r^5} + \frac{r^2 - 3y^2}{r^5} + \frac{r^2 - 3z^2}{r^5}$$

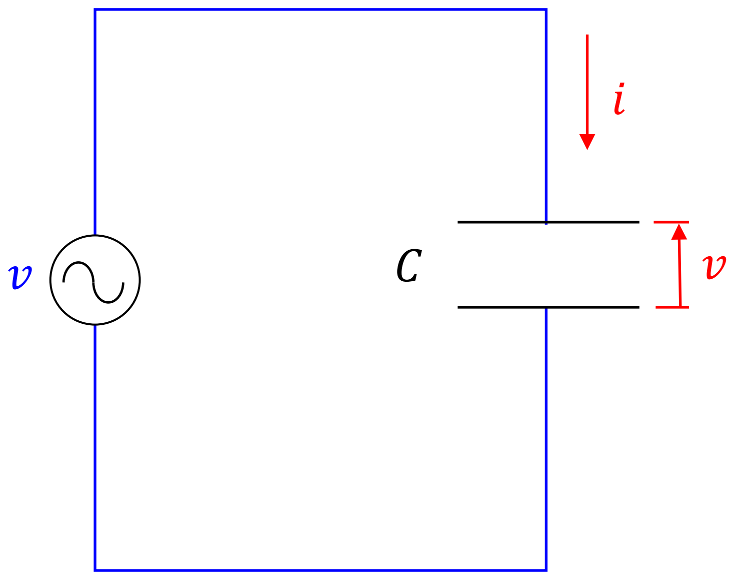
$$= \frac{3r^2 - 3(x^2 + y^2 + z^2)}{r^5}$$

$$= \frac{3r^2 - 3r^2}{r^5}$$

$$= 0$$

Exercise 7.4 (Homework)

Exercise 7.4 (Homework)



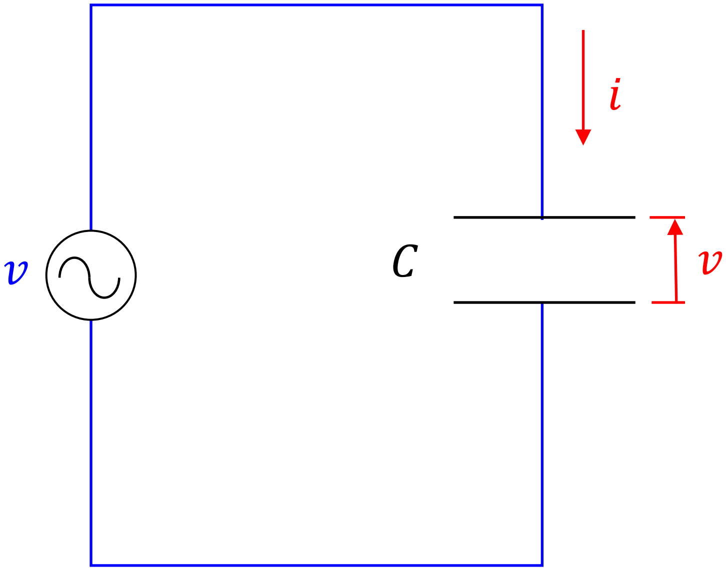
The source voltage is sinusoidal, expressed as follows:

$$v = V_m \sin \omega t \quad [\mathrm{V}]$$

where V_m is the amplitude, and ω is the angular frequency. The capacitance of the capacitor is C [F]. Find the current i.

Note that the current i and voltage v are time-varying variables, so they are denoted by lowercase letters.

Exercise 7.4 (Answer)



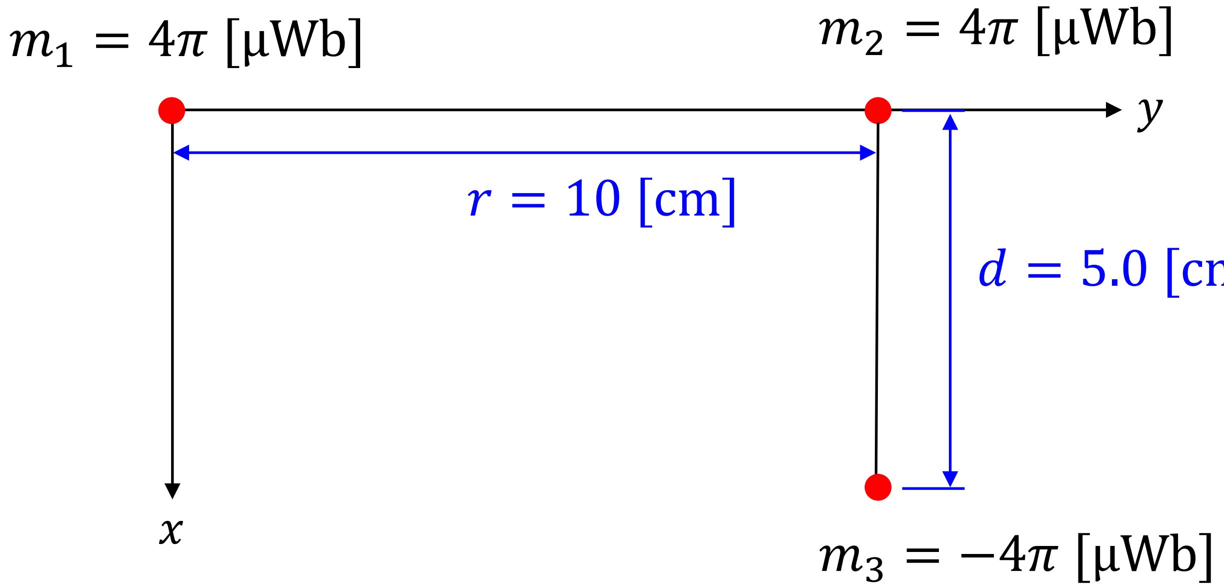
$$
\begin{aligned}
i &= C\frac{dv}{dt} \\
&= C\frac{dV_m \sin \omega t}{dt} \\
&= \omega C V_m \cos \omega t \quad \text{[A]}
\end{aligned}
$$

Exercise 8.2.2 (Homework)

Exercise 8.2.2 (Homework)

Exercise 8.2.2 (Homework)

A magnetic point charge m_1 is placed near a magnet of m_2 and m_3 as below. $m_1 = m_2 = 4.0\,\pi$ [μWb] and $m_3 = -4.0\,\pi$ [μWb]. $\mu_r \approx 1.0$. $r = 1.0 \times 10^1$ [cm] and $d = 5.0$ [cm]. Find the force $\boldsymbol{F}$ on m_1 due to m_2 and m_3.

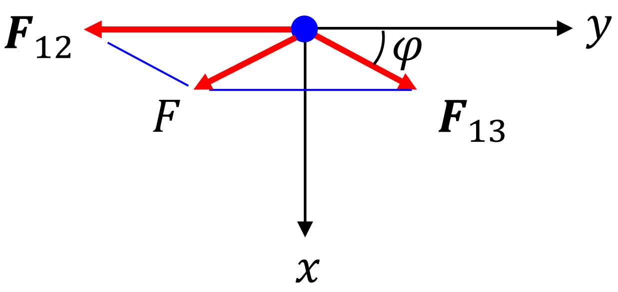


Exercise 8.2.2 (Answer)

$$F_{12} = \frac{m_1 m_2}{4\pi\mu_0 r^2} = \frac{4\pi \times 10^{-6} \times 4\pi \times 10^{-6}}{4\pi \times 4\pi \times 10^{-7} \times (10 \times 10^{-2})^2} = 1.0 \text{ [mN]}$$

$$\boldsymbol{F}_{12} = -1.0\, \boldsymbol{a}_y \text{ [mN]}$$

$$m_1 = 4\pi \text{ [μWb]}$$



$$F_{13} = \frac{m_1 m_3}{4\pi\mu_0 (r^2 + d^2)} = \frac{4\pi \times 10^{-6} \times 4\pi \times 10^{-6}}{4\pi \times 4\pi \times 10^{-7} \times (0.01 + 0.0025)} = 0.80 \text{ [mN]}$$

$$\boldsymbol{F}_{13} = F_{13} \sin\varphi\ \boldsymbol{a}_x + F_{13}\cos\varphi\ \boldsymbol{a}_y = F_{13}\frac{d}{\sqrt{r^2+d^2}}\boldsymbol{a}_x + F_{13}\frac{r}{\sqrt{r^2+d^2}}\boldsymbol{a}_y = 0.36\boldsymbol{a}_x + 0.72\boldsymbol{a}_y \text{ [mN]}$$

$$\boldsymbol{F} = \boldsymbol{F}_{12} + \boldsymbol{F}_{13} = 0.36\boldsymbol{a}_x - 0.28\boldsymbol{a}_y \text{ [mN]}$$

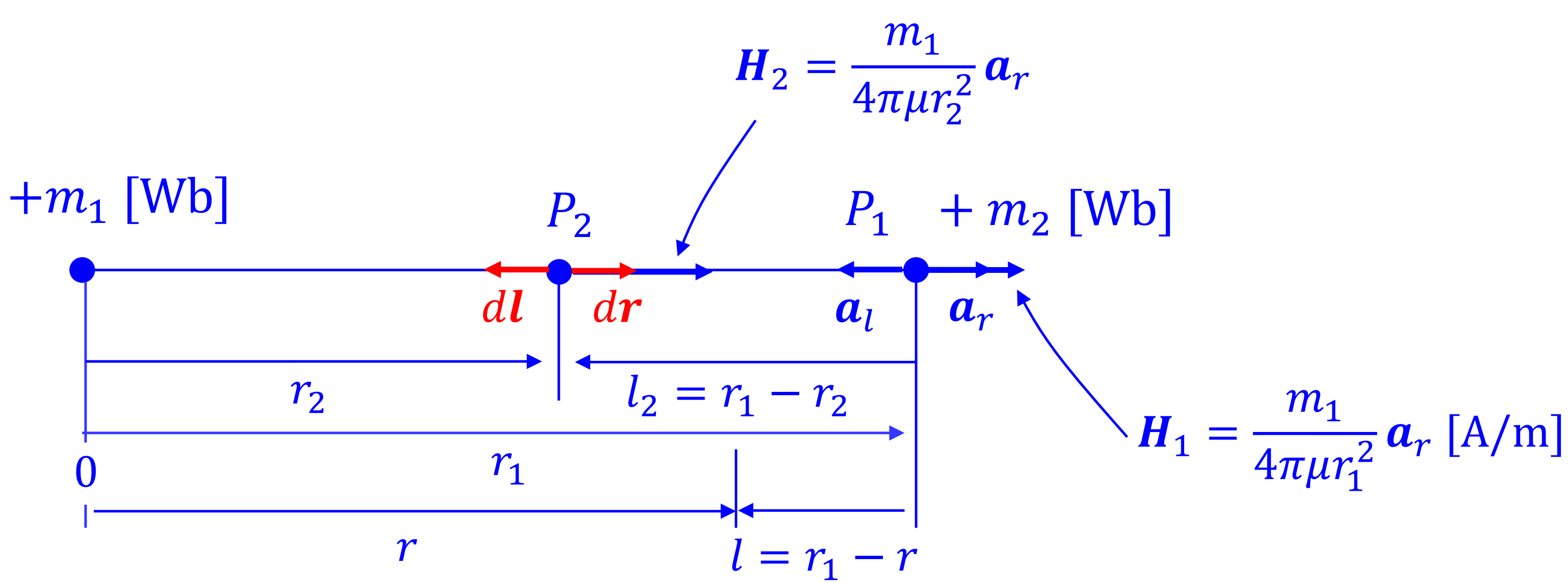
Exercise 8.5.2 (Homework)

Exercise 8.5.2 (Homework)

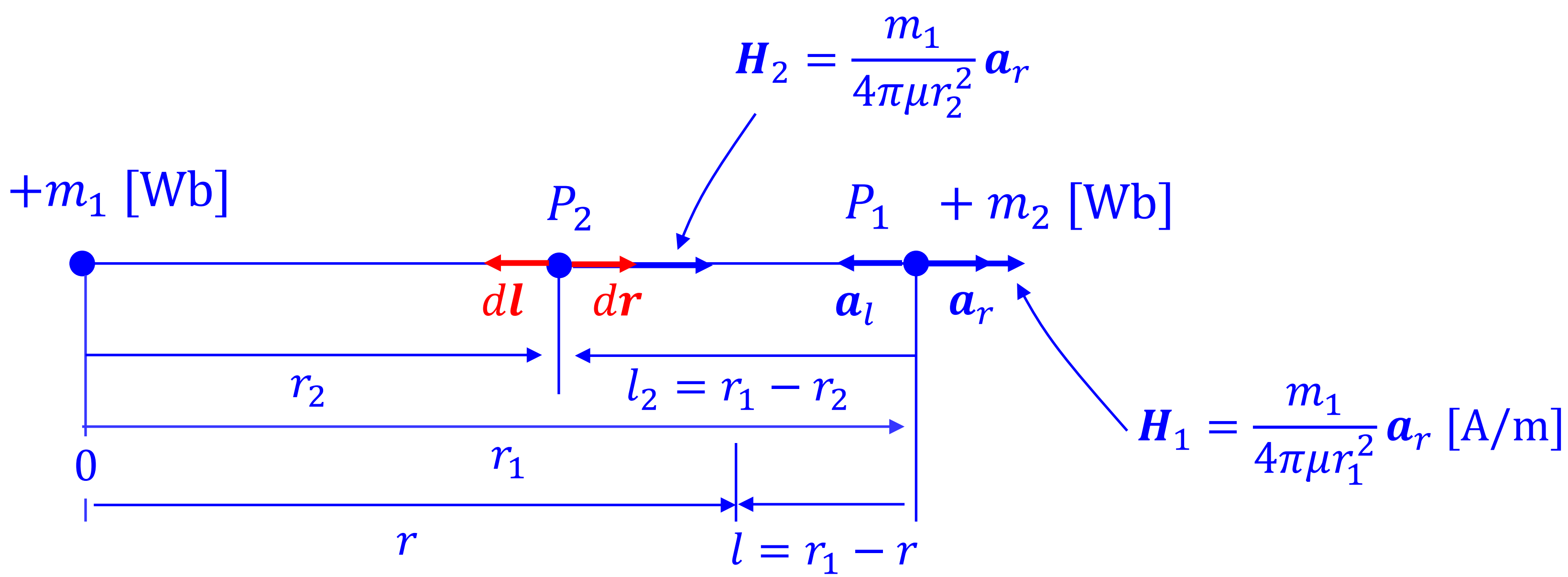
Exercise 8.5.2 (Homework)

Show that The potential V_{21} at P_2 with respect to P_1 is given by the line integral of $-\boldsymbol{H}\cdot d\boldsymbol{r}$ from r_1 to r_2 as below. Hint: Section 3.2

$$V_{21} = -\int_0^{l_2} \boldsymbol{H}\cdot d\boldsymbol{l} = -\int_{r_1}^{r_2} \boldsymbol{H}\cdot d\boldsymbol{r}$$



Homework 8.5.2 (Answer)



$$V_{21} = -\int_0^{l_2} \boldsymbol{H}\cdot d\boldsymbol{l} = \int_0^{l_2} \frac{m_1}{4\pi\mu(r_1 - l)^2}\,dl = -\int_{r_1}^{r_2} \frac{m_1}{4\pi\mu r^2}\,dr \quad = -\int_{r_1}^{r_2} \frac{m_1}{4\pi\mu r^2}\boldsymbol{a}_r\cdot d\boldsymbol{r}$$

$$= -\int_{r_1}^{r_2} \boldsymbol{H}\cdot d\boldsymbol{r}$$

Change of variable

$l = r_1 - r$

$dl = -dr$, If $l = 0$ then $r = r_1$

If $l = l_2$ then $r = r_2$

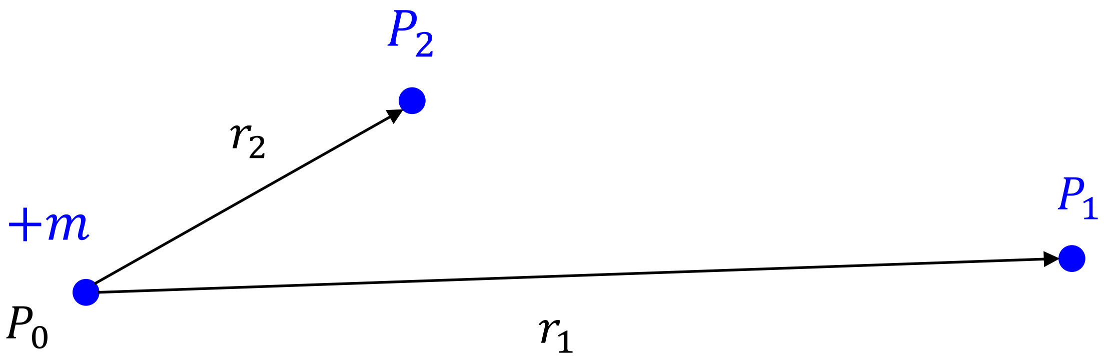
Exercise 8.5.3 (Homework)

Exercise 8.5.3 (Homework)

Exercise 8.5.3 (Homework)

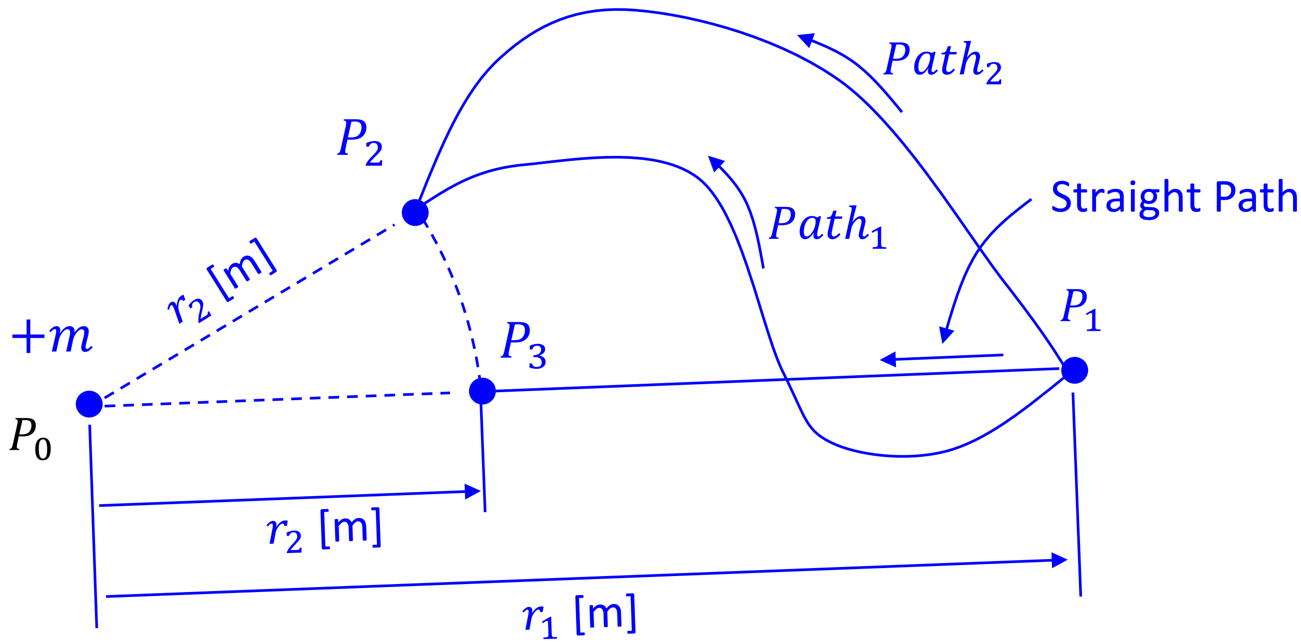
Show that the magnetic potential V_{21} at P_2 with respect to P_1 due to a magnetic point charge $+m$ at P_0 is conservative. Hint: Section 3.3

$$V_{21} = -\int_{r_1}^{r_2} \boldsymbol{H} \cdot d\boldsymbol{r}$$



Exercise 8.5.3 (Answer)

V_{21} does not depend on the path. ➡ The field is conservative.

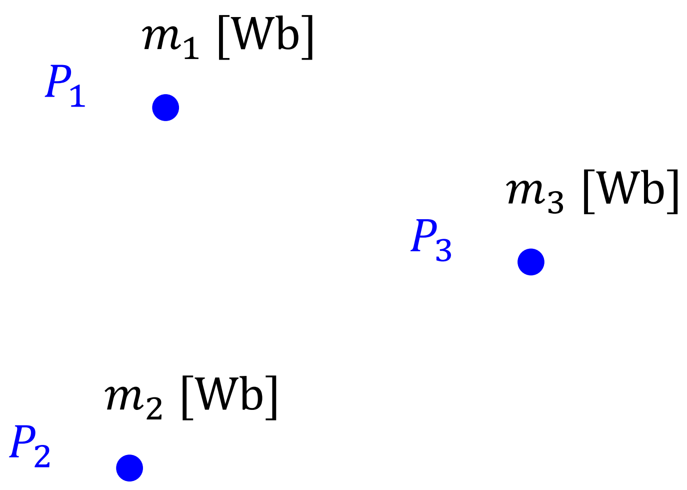


$$V_{21} = -\int_{Path_1} \boldsymbol{H} \cdot d\boldsymbol{r} = -\int_{Path_2} \boldsymbol{H} \cdot d\boldsymbol{r} = -\int_{Straight\ Path} \boldsymbol{H} \cdot d\boldsymbol{r} = \frac{m_1}{4\pi\mu}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad [\mathrm{V}]$$

Exercise 9.2.1 (Homework)

Exercise 9.2.1 (Homework)

Exercise 9.2.1 (Homework)



Find the work W_T necessary to bring the magnetic point charges in reverse order: m_3, m_2, and m_1.

Then show that the energy W_T stored in the magnetic field generated by m1, m2, and m3 is given by

$$W_T = \frac{1}{2}\sum_{i=1}^{n} m_i V_i$$

Where $V_1, V_2,$ and V_3 are the potential at $m_1, m_2,$ and m_3, respectively.

Exercise 9.2.1 (Answer)

The work W_T necessary to bring the magnetic point charges in reverse order: m_3, m_2, and m_1.

$$W_3 = 0, \qquad W_2 = m_2 V_{23}. \qquad W_1 = m_1 V_{12} + m_1 V_{13}$$

$$\begin{aligned} W_T &= W_1 + W_2 + W_3 \\ &= m_1 V_{12} + m_1 V_{13} + m_2 V_{23} \end{aligned}$$

The work W_T necessary to bring the charges in the order: m_1, m_2, and m_3.

$$W_T = m_2 V_{21} + m_3 V_{31} + m_3 V_{32}$$

Then

$$2W_T = \underbrace{m_1(V_{12} + V_{13})}_{m_1 V_1} + \underbrace{m_2(V_{21} + V_{23})}_{m_2 V_2} + \underbrace{m_3(V_{31} + V_{32})}_{m_3 V_3}$$

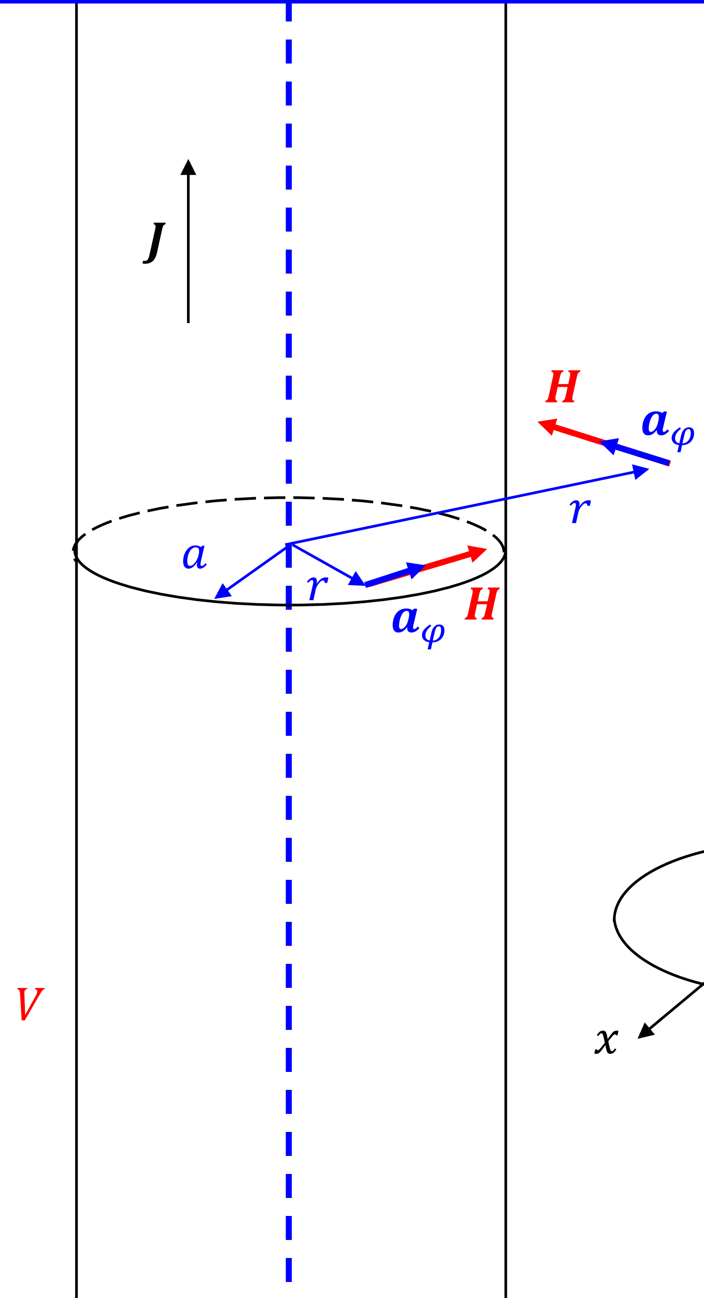
the work done against the fields of m_2 and m_3

$$W_T = \frac{1}{2} \sum_{i=1}^{n} m_i V_i$$

Exercise 9.5.2 (Homework)

Exercise 9.5.2 (Homework)

Exercise 9.5.2 (Homework)



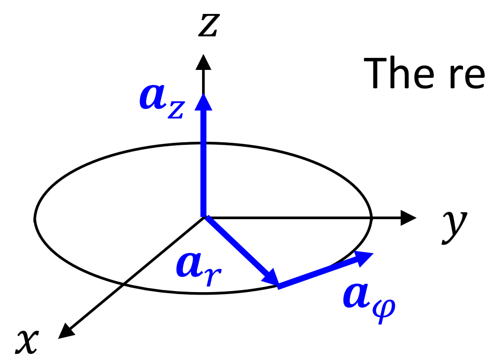
A current density $\boldsymbol{J} = J\boldsymbol{a}_z$ $[\mathrm{A/m^2}]$ flows throughout an infinite cylinder V of radius a [m]. According to Exercise 10.2.2,

$$\boldsymbol{H} = \begin{cases} \dfrac{rJ}{2}\boldsymbol{a}_\varphi & (r \leq a) \\ \dfrac{a^2 J}{2r}\boldsymbol{a}_\varphi & (a \leq r). \end{cases}$$

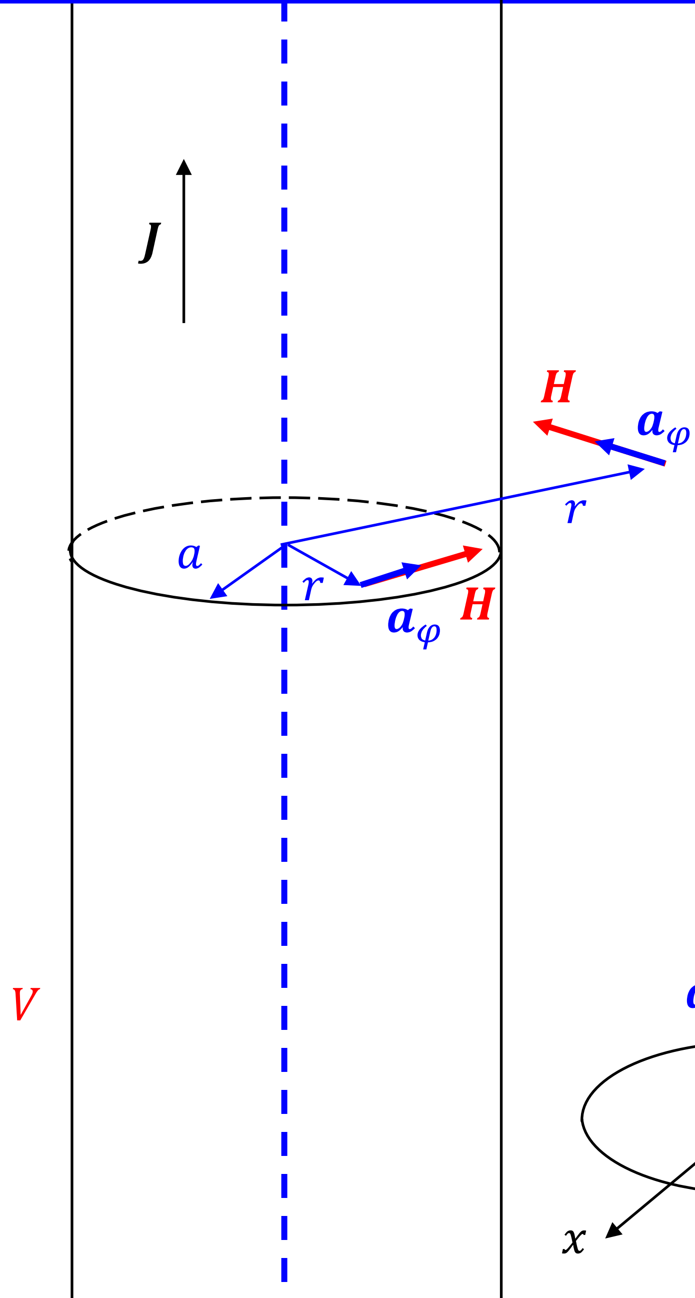
Show that

$$\nabla \cdot \boldsymbol{B} = \boldsymbol{0}.$$

The relative permeability $\mu_r = 1$.



Exercise 9.5.2 (Answer)



$(r \le a)$

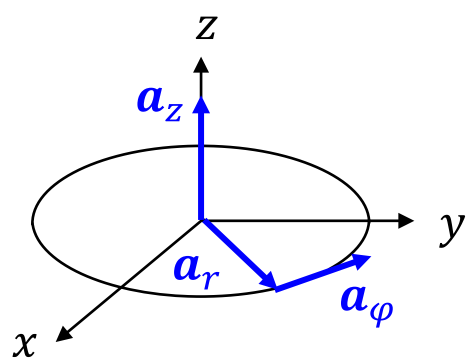
$$\nabla\cdot\boldsymbol{B} = \frac{\mu J}{2}\nabla\cdot\left(r\boldsymbol{a}_\varphi\right) = \frac{\mu J}{2}\nabla\cdot\left(-y\boldsymbol{a}_x + x\boldsymbol{a}_y\right)$$

$$\nabla\cdot\left(r\boldsymbol{a}_\varphi\right) = \frac{\partial}{\partial x}(-y) + \frac{\partial}{\partial y}x = 0$$

$(a \le r)$

$$\nabla\cdot\boldsymbol{B} = \frac{\mu a^2 J}{2}\nabla\cdot\left(\frac{1}{r}\boldsymbol{a}_\varphi\right)$$

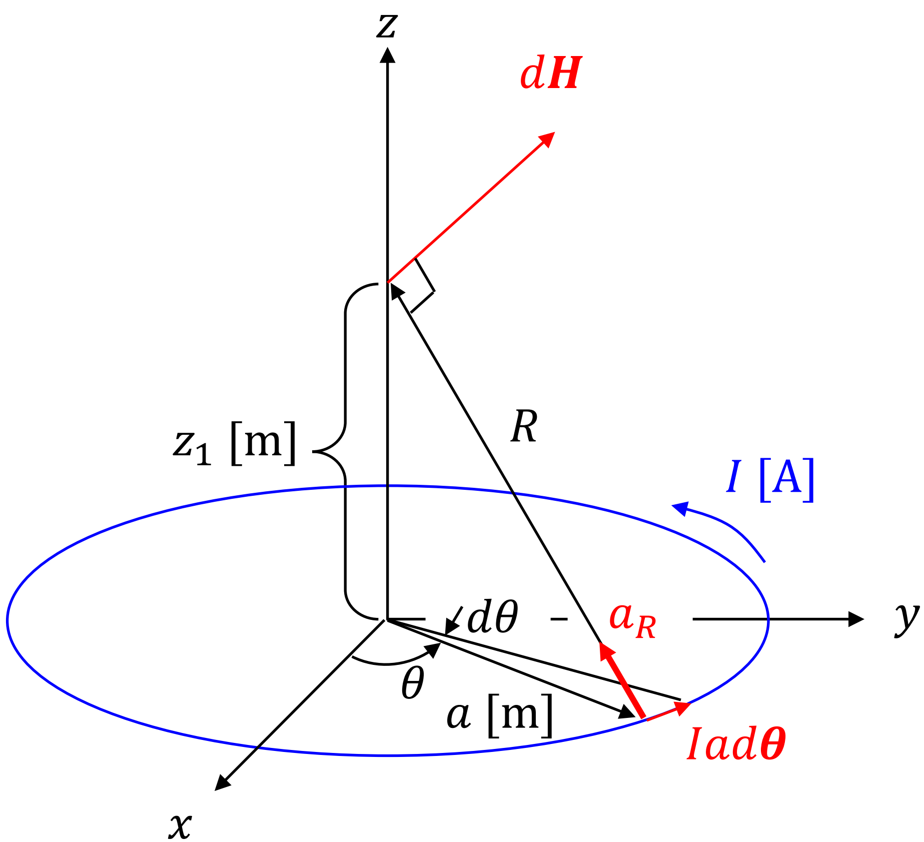
$$\begin{aligned}\nabla\cdot\left(\frac{1}{r}\boldsymbol{a}_\varphi\right) &= \frac{\partial}{\partial x}\left(-\frac{y}{r^2}\right) + \frac{\partial}{\partial y}\left(\frac{x}{r^2}\right)\\ &= \frac{2yx}{r^4} - \frac{2xy}{r^4}\\ &= 0\end{aligned}$$



Exercise 10.1.4 (Homework)

Exercise 10 .1.4. This is your homework.

Exercise 10.1.4 (Homework)

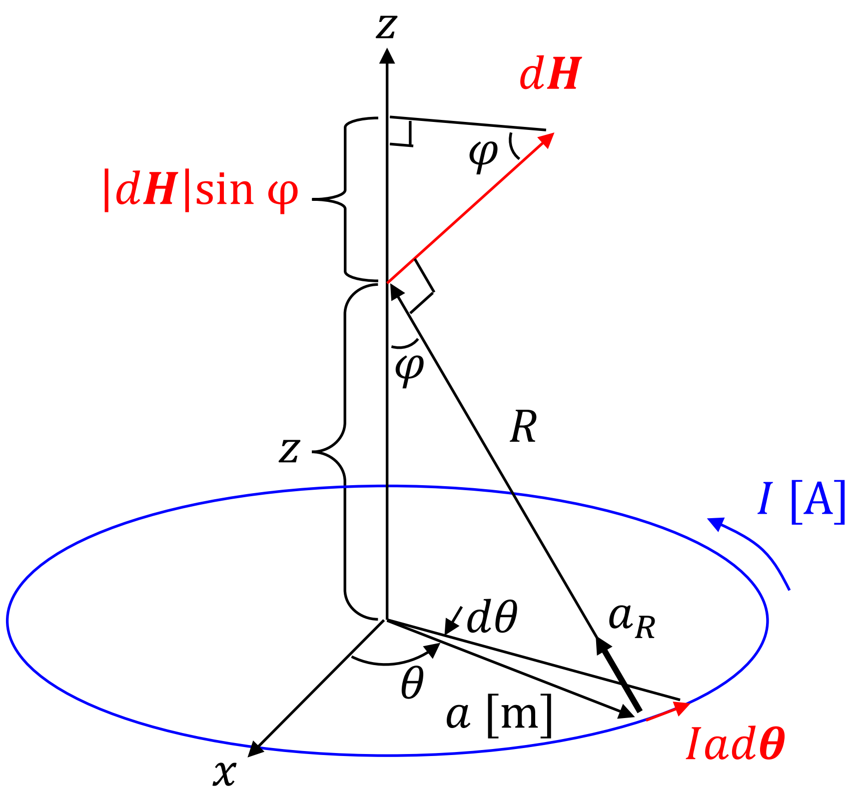


A current I [A] flows on a circular line conductor of radius a [m] and thickness 0 [m]. The conductor lies in the $x - y$ plane.

Find the magnetic field strength at $z = z_1$ [m].

Hint
You can start with a differential current $Iad\theta$. θ is the angle from the x −axis.

Exercise 10.1.4 (Answer)



$$|d\boldsymbol{H}|\sin\varphi = \frac{|Ia\,d\boldsymbol{\theta}\times\boldsymbol{a}_R|}{4\pi R^2}\sin\varphi \qquad [\mathrm{A/m}]$$

$$= \frac{Ia\,d\theta}{4\pi\left(\sqrt{z^2+a^2}\right)^2}\frac{a}{\sqrt{z^2+a^2}}$$

$$= \frac{Ia^2\,d\theta}{4\pi(z^2+a^2)^{3/2}}$$

$$\boldsymbol{H} = \int_0^{2\pi}|d\boldsymbol{H}|\sin\varphi\,\boldsymbol{a}_z\,d\theta$$

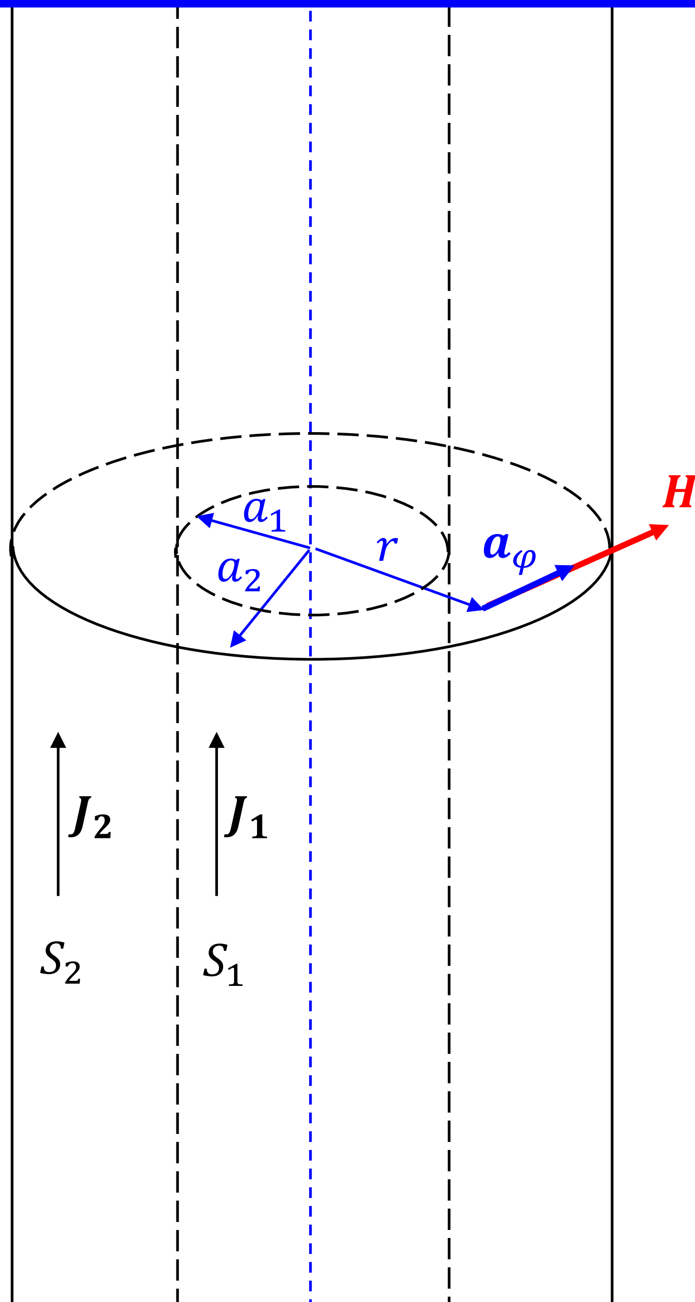
$$= \frac{Ia^2\boldsymbol{a}_z}{4\pi(z^2+a^2)^{3/2}}[\theta]_0^{2\pi}$$

$$= \frac{Ia^2\boldsymbol{a}_z}{2(z^2+a^2)^{3/2}}$$

Exercise 10.2.4 (Homework)

Exercise 10.2.4 (Homework)

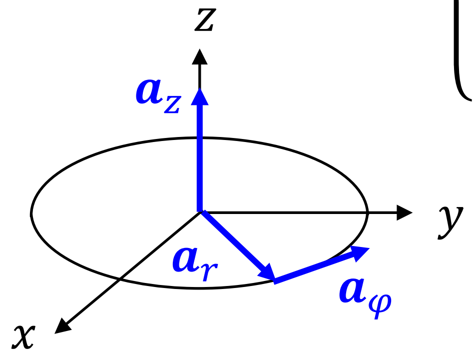
Exercise 10.2.4 (Homework)



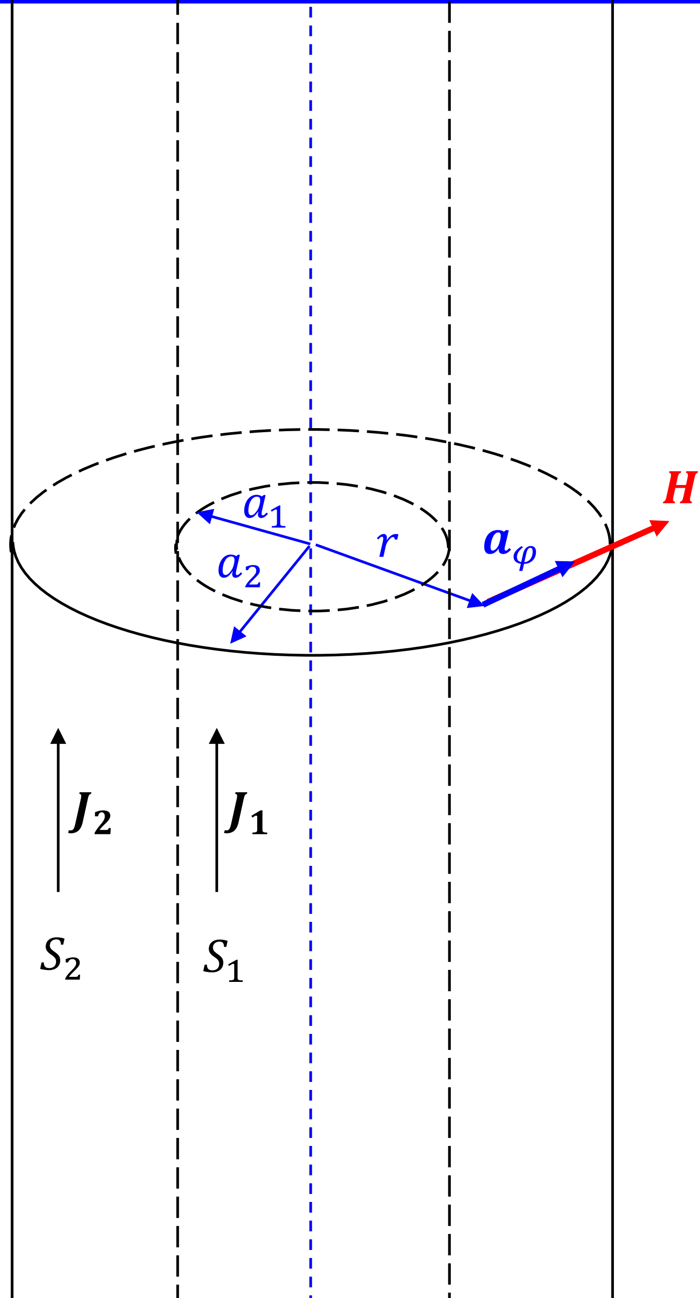
There are two infinite concentric pipe-shaped conductors S_1 and S_2. The radii of S_1 and S_2 are a_1 [m] and a_2 [m], respectively. The thickness of each conductor is zero. A current density $\boldsymbol{J}_1 = J\boldsymbol{a}_z$ [A/m] flows on the surface of S_1, and $\boldsymbol{J}_2 = -J\boldsymbol{a}_z$ [A/m] on the surface of S_2.

Show that

$$\boldsymbol{H} = \begin{cases} 0 & (r \le a_1) \\ \dfrac{a_1 J}{r}\boldsymbol{a}_\varphi & (a_1 \le r \le a_2) \\ 0 & (a_2 \le r) \end{cases}$$



Exercise 10.2.4 (Answer)



$(r \le a_1)$

$$\oint_{CP} \boldsymbol{H} \cdot d\boldsymbol{l} = \int_S 0 \cdot d\boldsymbol{S}$$

$$2\pi r H = 0$$

$$\boldsymbol{H} = 0$$

$(a_1 \le r \le a_2)$

$$2\pi r H = 2\pi a_1 J$$

$$\boldsymbol{H} = \frac{a_1 J}{r} \boldsymbol{a}_\varphi$$

$(a_2 \le r)$

$$2\pi r H = 0$$

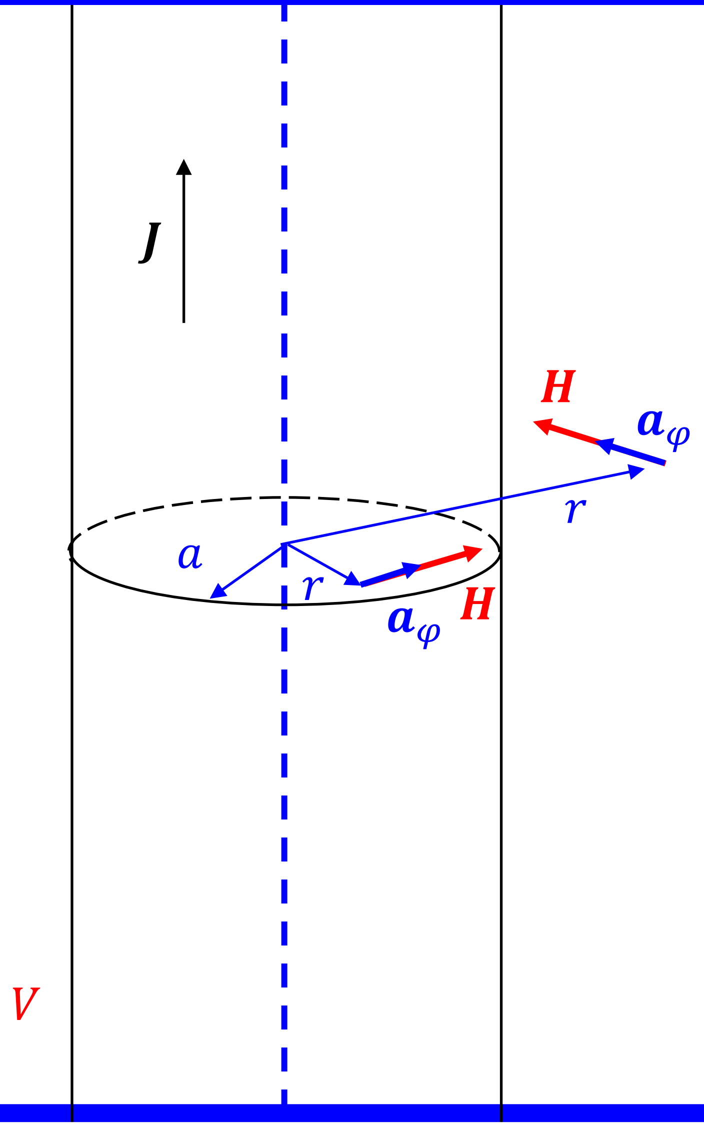
$$\boldsymbol{H} = 0$$

Exercise 11.1.3 (Homework)

Exercise 11.1.3 (Homework)

Exercise 11.1 .3. This is your homework.

Exercise 11.1.3 (Homework)



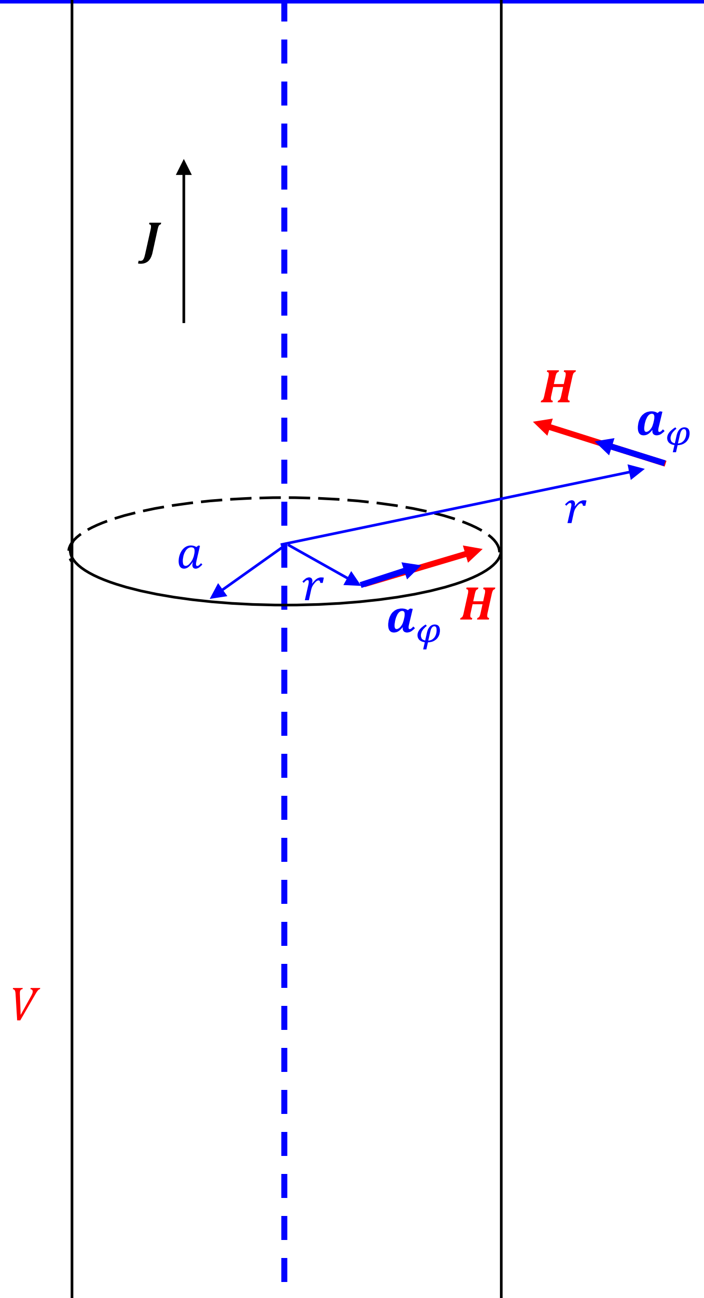
A current of density $\boldsymbol{J} = J\boldsymbol{a}_z$ $[\mathrm{A/m^2}]$ flows throughout an infinite cylinder V of radius a [m]. The answer to Exercise 10.2.2 was

$$\boldsymbol{H} = \begin{cases} \dfrac{rJ}{2}\boldsymbol{a}_\varphi & (0 < r \le a) \\ \dfrac{a^2 J}{2r}\boldsymbol{a}_\varphi & (a \le r). \end{cases}$$

Find $\nabla \times \boldsymbol{H}$ in both regions of $0 < r \le a$ and $a \le r$.

Pause the video, and answer the problem. Find the curl of H both inside and outside of the infinite cylinder V.

Exercise 11.1.3 (Answer)



$0 < r \leq a$

$$
\begin{aligned}
\boldsymbol{H} &= \frac{rJ}{2}\boldsymbol{a}_\varphi \\
&= \frac{rJ}{2}\left(-\sin\varphi\, \boldsymbol{a}_x + \cos\varphi\, \boldsymbol{a}_y\right) \\
&= \frac{J}{2}\left(-y\boldsymbol{a}_x + x\boldsymbol{a}_y\right)
\end{aligned}
$$

$\nabla \times \boldsymbol{H}$

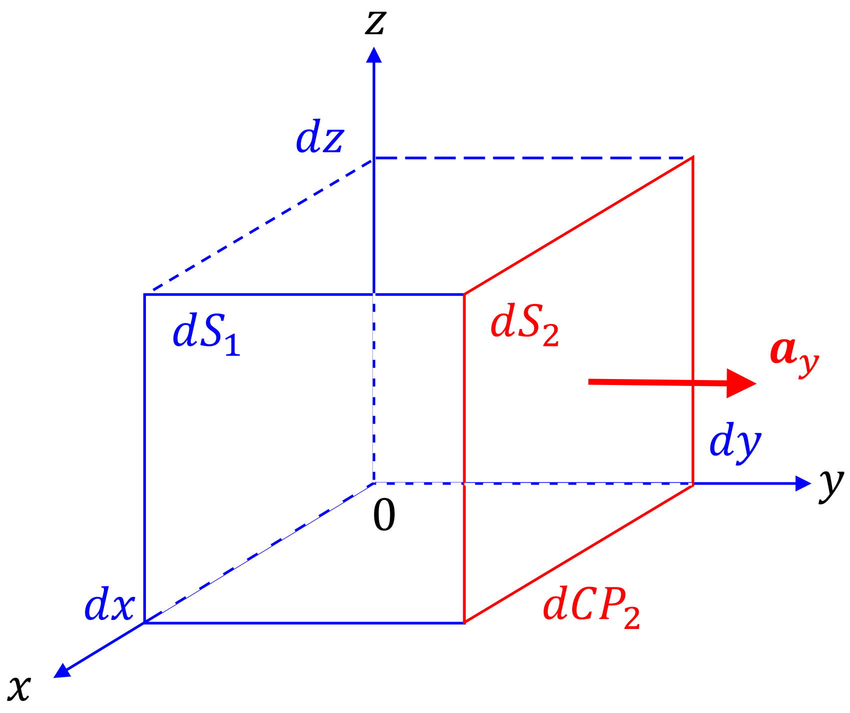
$$
\begin{aligned}
&= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\boldsymbol{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\boldsymbol{a}_y \\
&\quad + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\boldsymbol{a}_z \\
&= \frac{J}{2}\left\{0\boldsymbol{a}_x + 0\boldsymbol{a}_y + (1+1)\boldsymbol{a}_z\right\} \\
&= J\boldsymbol{a}_z
\end{aligned}
$$

$a \leq r$

$\nabla \times \boldsymbol{H} = 0$ (refer to Exercise 11.1.1)

Exercise 11.2 (Homework)

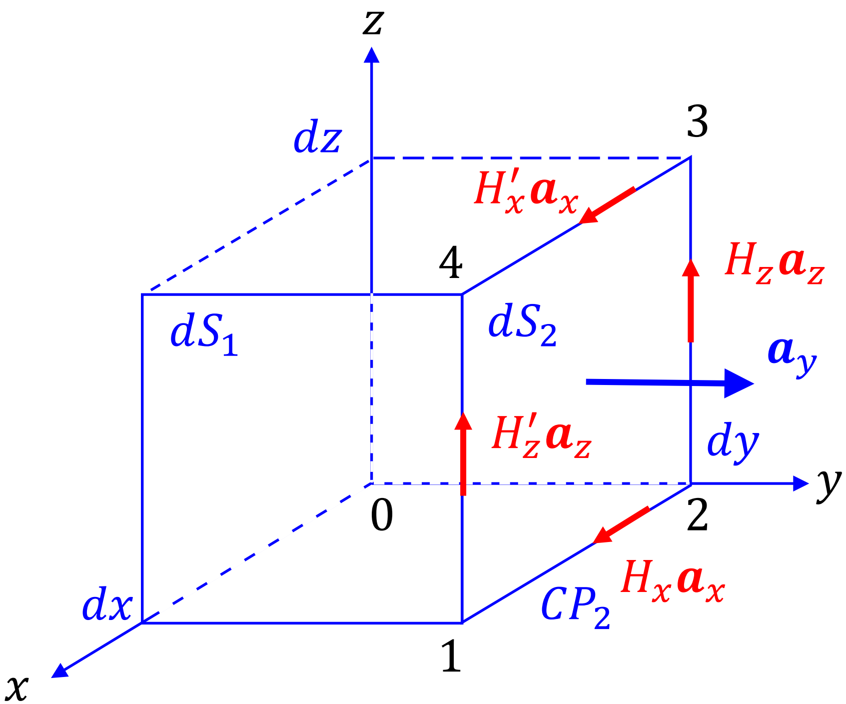
Exercise 11.2 (Homework)



Show that

$$(curl\ \boldsymbol{H}) \cdot \boldsymbol{a}_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}.$$

Exercise 11.2 (Answer)



$$(curl\ \boldsymbol{H}) \cdot \boldsymbol{a}_y \equiv \frac{\oint_{dCP_2} \boldsymbol{H} \cdot d\boldsymbol{l}}{dzdx}$$

$$\int_1^2 \boldsymbol{H} \cdot d\boldsymbol{l} = -H_x dx$$

$$\int_3^4 \boldsymbol{H} \cdot d\boldsymbol{l} = \left(H_x + \frac{\partial H_x}{\partial z} dz\right) dx$$

$$\int_2^3 \boldsymbol{H} \cdot d\boldsymbol{l} = H_z dz$$

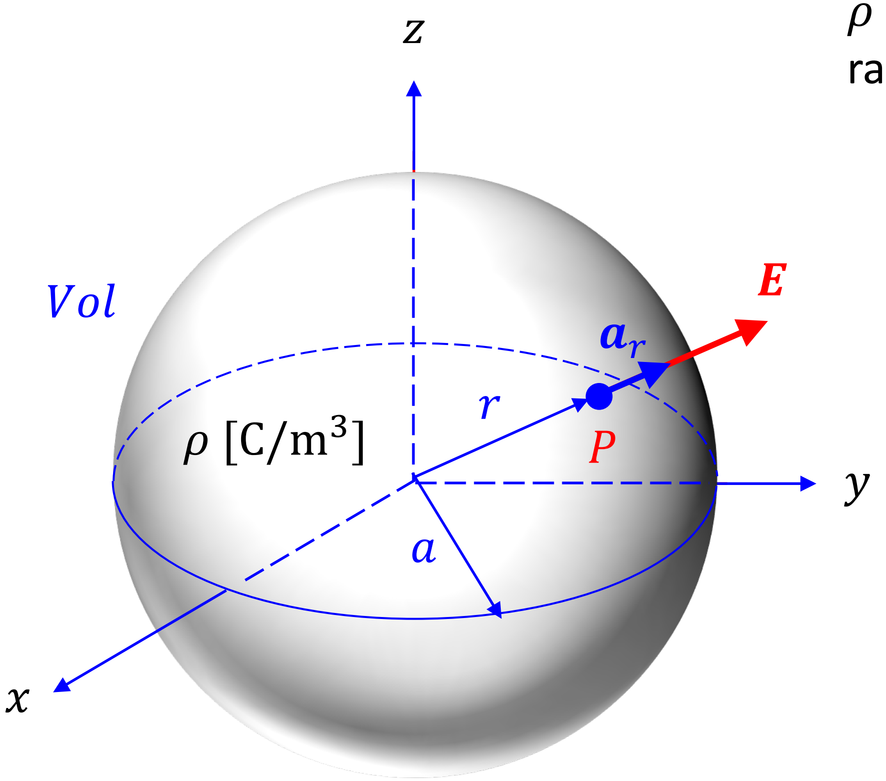
$$\int_4^1 \boldsymbol{H} \cdot d\boldsymbol{l} = -\left(H_z + \frac{\partial H_z}{\partial x} dx\right) dz$$

$$(curl\ \boldsymbol{H}) \cdot \boldsymbol{a}_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}$$

Exercise 11.3 (Homework)

Exercise 11.3 (Homework)

Exercise 11.3 (Homework)



Charge is distributed with a constant density $\rho\ [\mathrm{C/m^3}]$ throughout a spherical volume Vol of radius a [m]. The relative permittivity $\varepsilon_r \approx 1$.

According to Exercise 2.3.1, the electric field intensity $\boldsymbol{E}$ was

$$\boldsymbol{E} = \begin{cases} \dfrac{\rho r}{3\varepsilon}\boldsymbol{a}_r & (r \leq a) \\ \dfrac{\rho a^3}{3\varepsilon r^2}\boldsymbol{a}_r & (a \leq r). \end{cases}$$

Show that $\nabla \times \boldsymbol{E} = \boldsymbol{0}$ both inside and outside of Vol.

Exercise 11.3 (Answer)

$$curl\ \boldsymbol{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\boldsymbol{a}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\boldsymbol{a}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\boldsymbol{a}_z$$

$(r\ \leq a)$

$$\boldsymbol{E} = \frac{\rho r}{3\varepsilon}\boldsymbol{a}_r = \frac{\rho}{3\varepsilon}\left(x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z\right) = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a}_z$$

$$curl\ \boldsymbol{E} = 0$$

$(a \leq r)$

$$\boldsymbol{E} = \frac{\rho a^3}{3\varepsilon r^2}\boldsymbol{a}_r = \frac{\rho a^3}{3\varepsilon r^3}\left(x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z\right) = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a}_z$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = \frac{\rho a^3}{3\varepsilon}\left(\frac{\partial}{\partial y}\frac{z}{r^3} - \frac{\partial}{\partial z}\frac{y}{r^3}\right) = \frac{\rho a^3}{3\varepsilon}\left(\frac{-3zy}{r^5} - \frac{-3yz}{r^5}\right) = 0$$

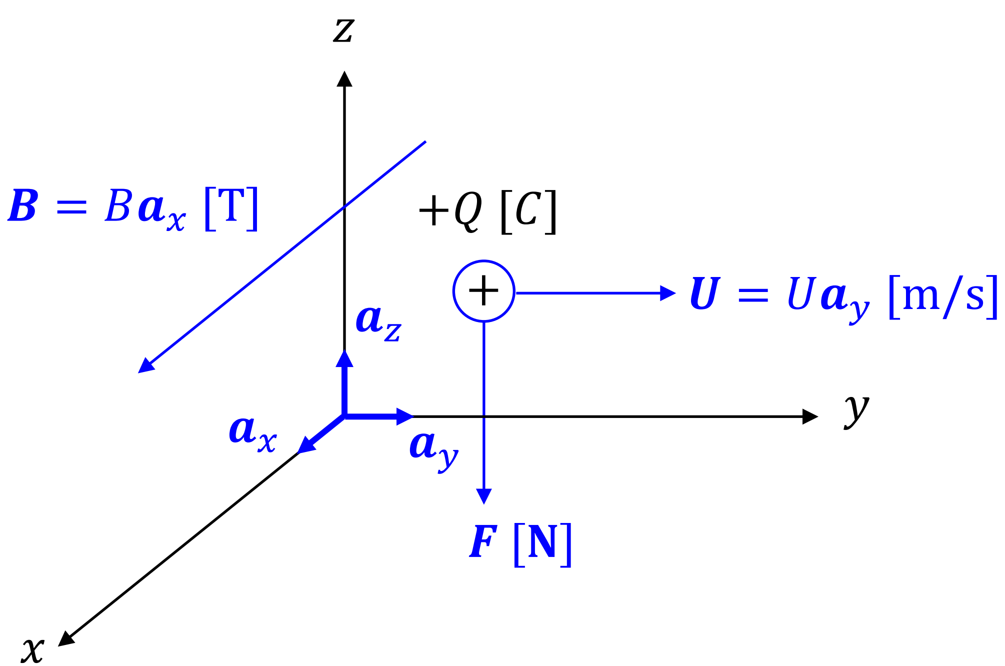
$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$curl\ \boldsymbol{E} = 0$$

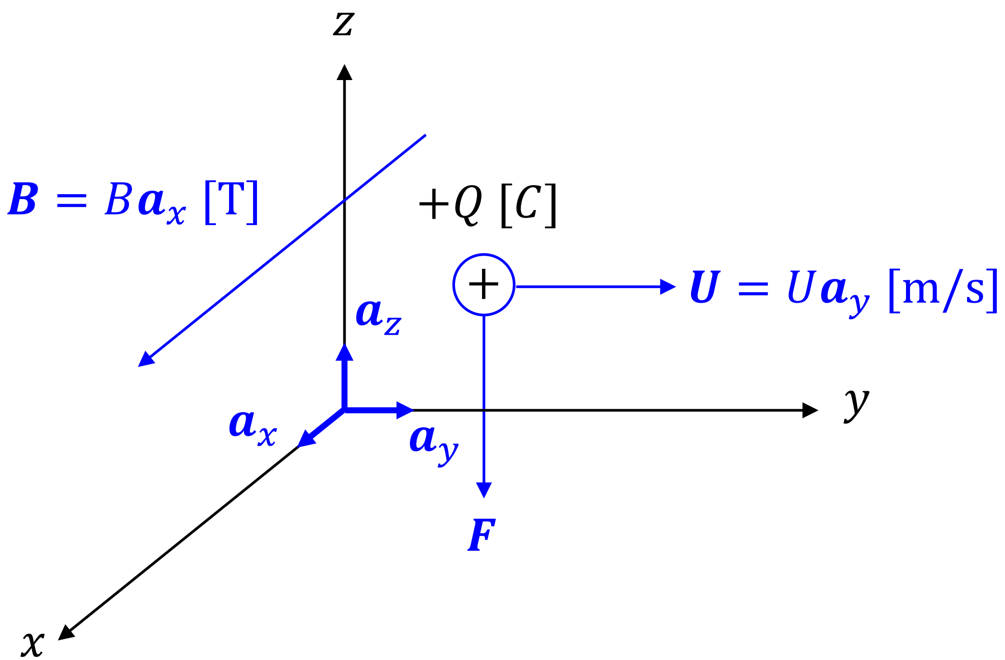
Exercise 12.1.2 (Homework)

Exercise 12.1.2 (Homework)



Show that the radius r [m] of the circle is 1.1 [m], and the time t [s] needed for one revolution is 69 [μs], for the ionized carbon atom in Exercise 12.1.1

Exercise 12.1.2 (Answer)



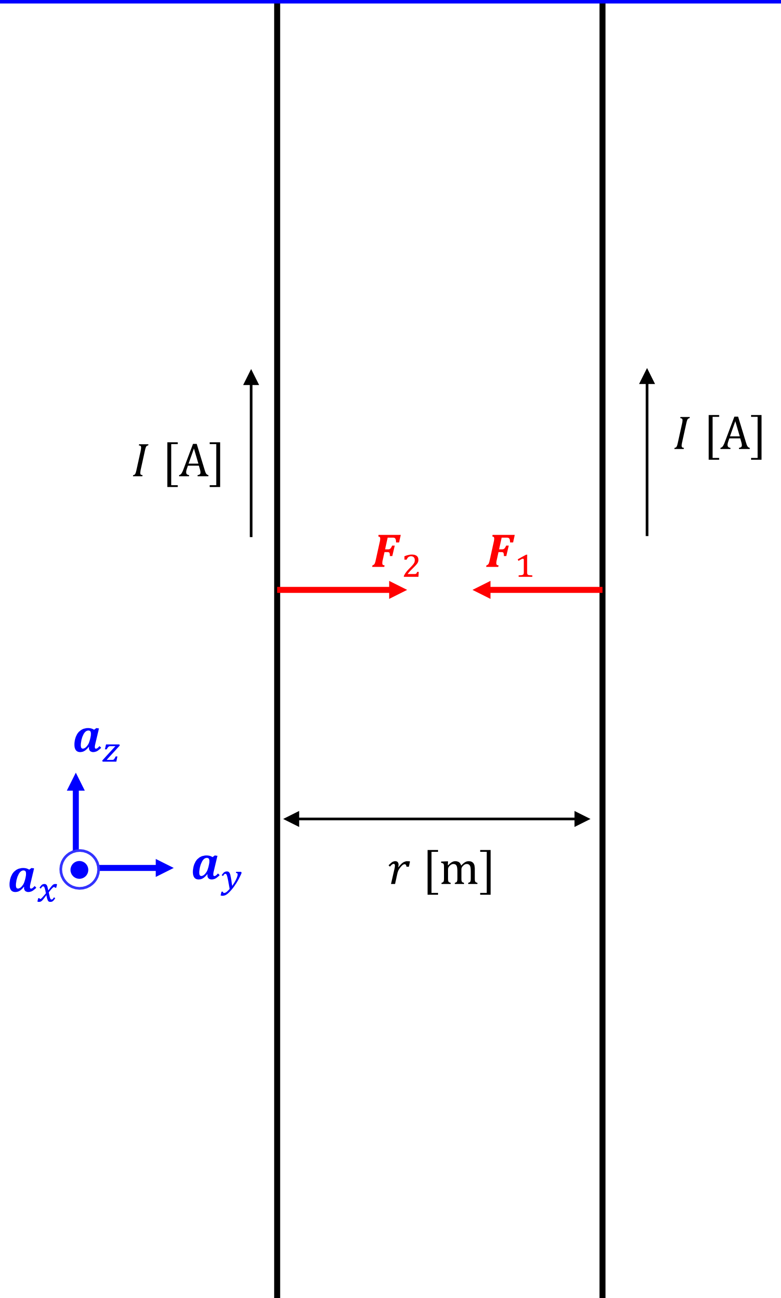
$$r = \frac{mU}{QB}$$
$$= \frac{1.7 \times 10^{-27} \times 100 \times 10^{3}}{1.6 \times 10^{-19} \times 10^{-3}}$$
$$= 1.1\ [\mathrm{m}]$$

$$t = \frac{2\pi r}{U}$$
$$= \frac{2\pi \times 1.1}{100 \times 10^{3}}$$
$$= 69\ [\mu\mathrm{s}]$$

Exercise 12.3.2 (Homework)

Exercise 12.3.2 (Homework)

Exercise 12.3.2 (Homework)



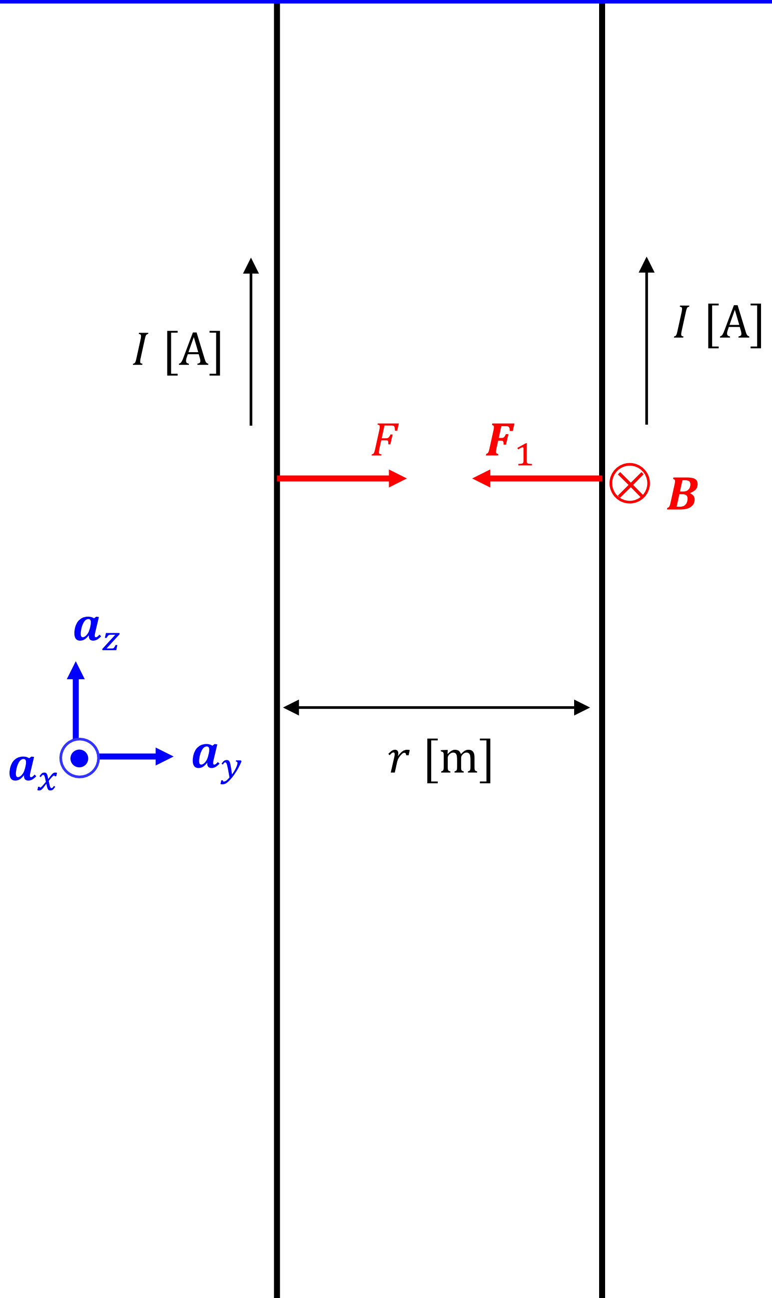
There are two infinite line conductors placed parallel to the $z-$ axis. The distance between the conductors is r [m]. The thicknesses of conductors are assumed to be zero. In each conductor, a current of I [A] flows to the $z-$ direction. The relative permeability $\mu_r = 1$.

Show that the force $\boldsymbol{F}_1$ on the right hand side conductor per unit length is given by

$$\boldsymbol{F}_1 = -\frac{\mu_0 I^2}{2\pi r}\boldsymbol{a}_y$$

Hint: Exercise 10.1.2

Exercise 12.3.2 (Answer)



According to Exercise 10.1.2, the magnetic field strength H at the right-hand side conductor is

$$\boldsymbol{H} = -\frac{I}{2\pi r}\boldsymbol{a}_x.$$

Then

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} = -\frac{\mu_0 I}{2\pi r}\boldsymbol{a}_x.$$

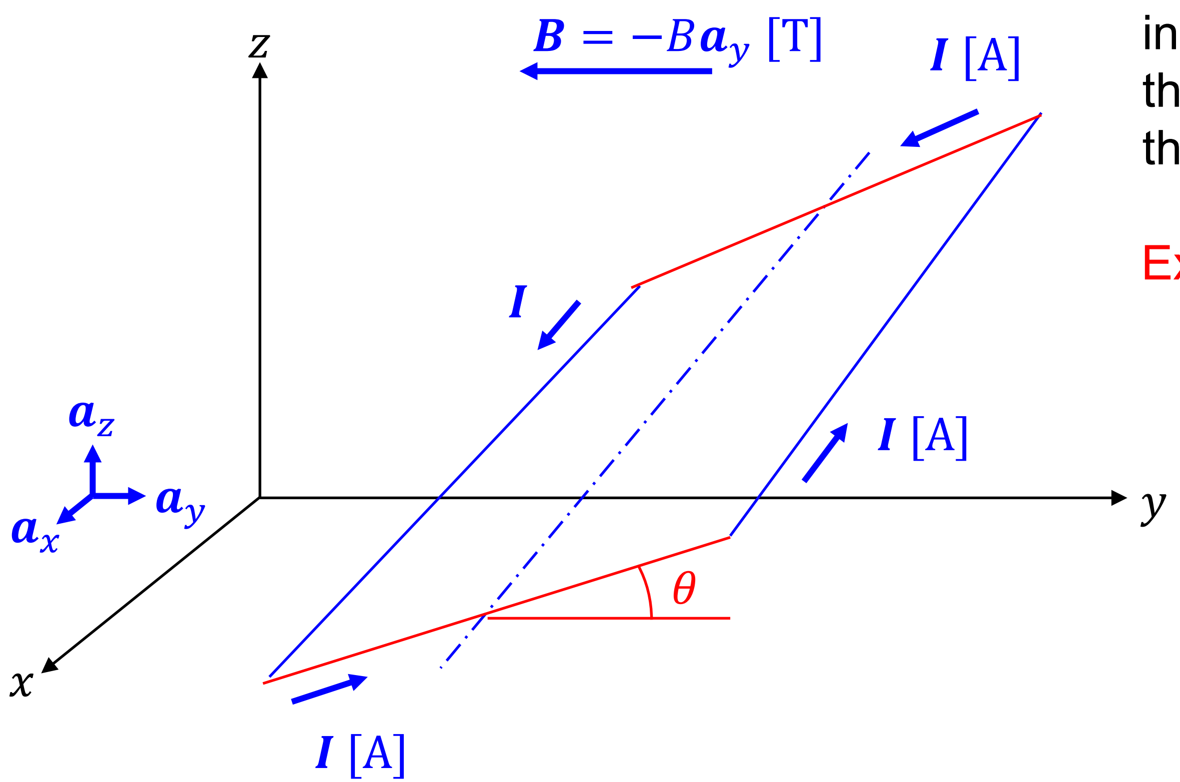
Thus, the force per unit length is

$$F_{\mathbf{1}} = \frac{I\boldsymbol{L} \times \boldsymbol{B}}{L} = -\frac{\mu_0 I^2}{2\pi r}\boldsymbol{a}_y.$$

Exercise 12.4.2 (Homework)

Exercise 12.4.2 (Homework)

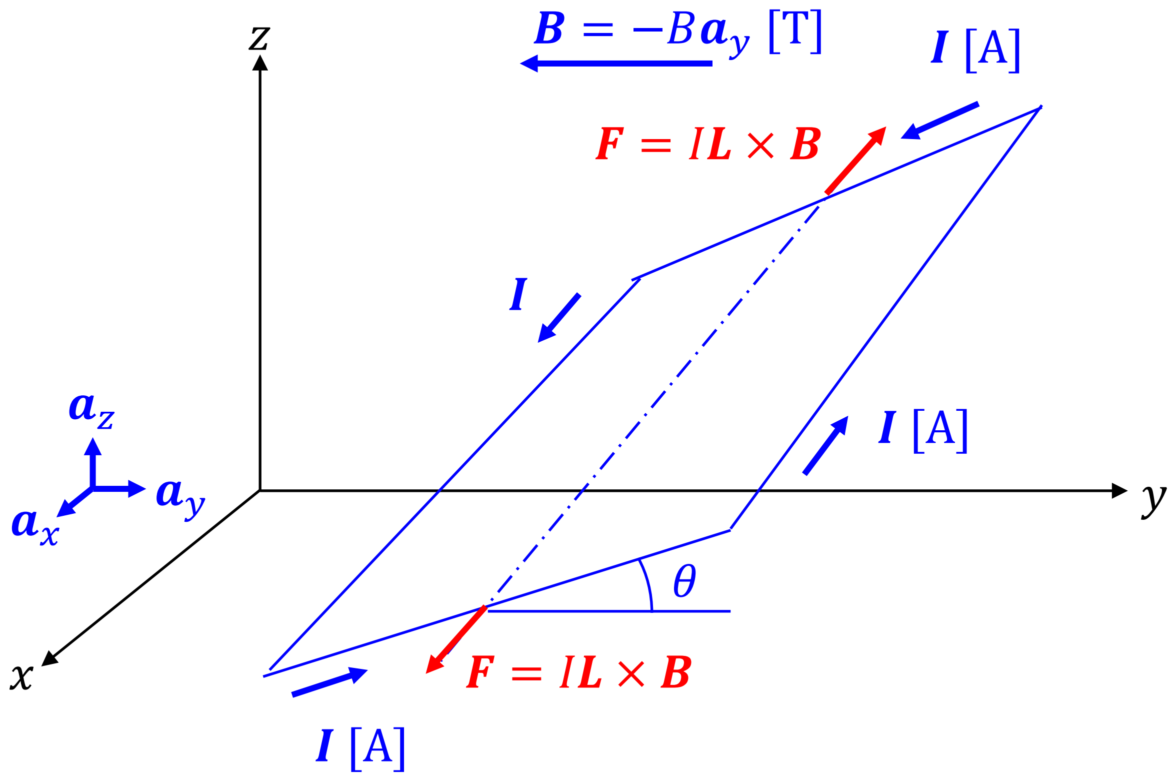
Exercise 12.4.2 (Homework)



The torque of this coil does not include the forces generated on the conductors shown in red in this figure.

Explain why.

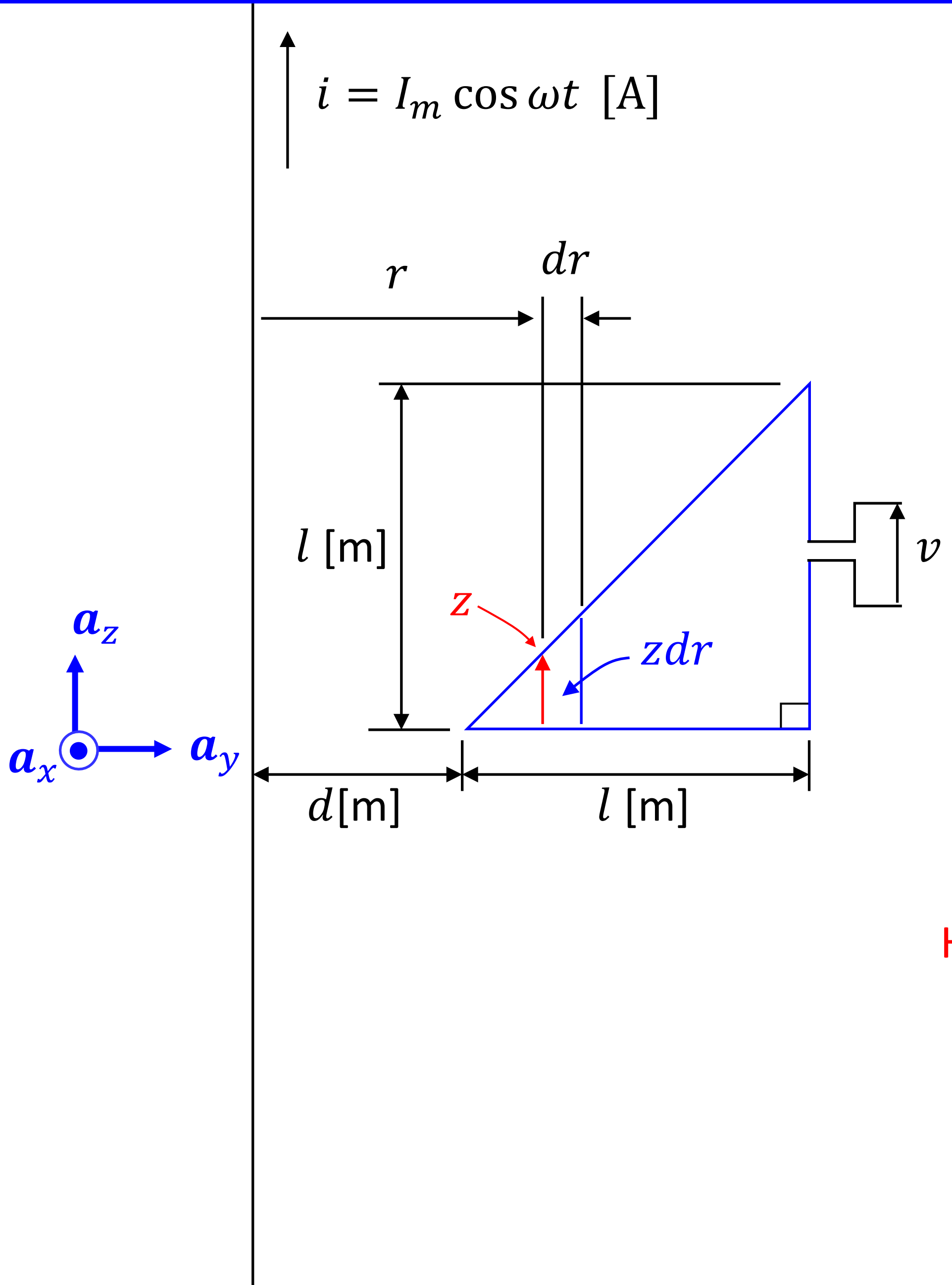
Exercise 12.4.2 (Answer)



Exercise 13.2.3 (Homework)

Exercise 13.2.3 (Homework)

Exercise 13.2.3 (Homework)



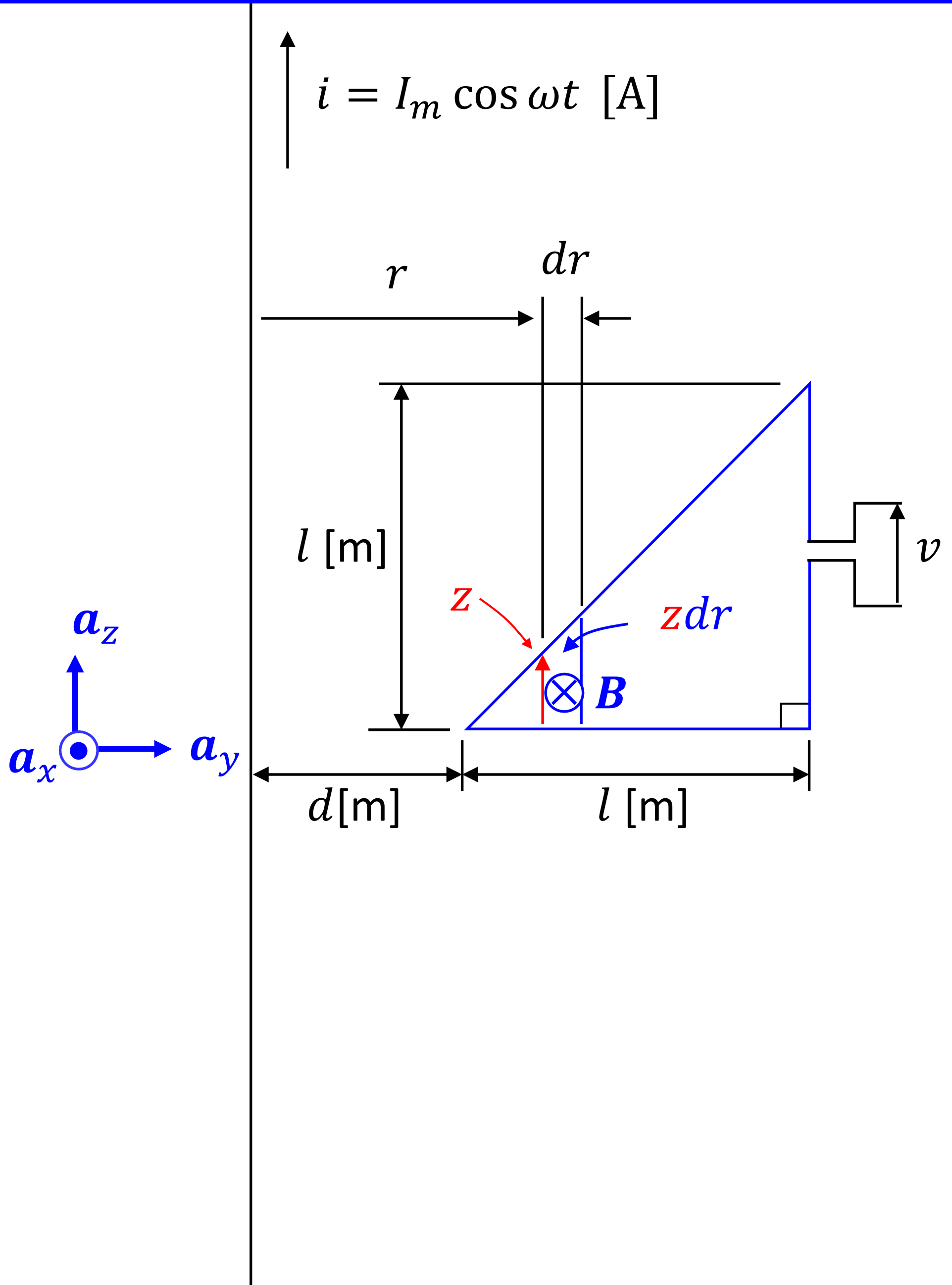
A current i is flowing on an infinite straight line placed parallel to the $z-$ axis. i is given by

$$i = I_m \cos \omega t \quad [\mathrm{A}].$$

Find the voltage v induced on a right triangle conductor. The triangle and the infinite line are on the same $y - z$ plane. The distance from the infinite line to the left vertex of the triangle is d [m]. The base and height of the triangle are l [m]. The relative permeability $r_s \approx 1$.

Hint: Express the height z as a function of r. The differential area is zdr.

Exercise 13.2.3 (Answer)



$$\boldsymbol{B} = -\frac{\mu_0 i}{2\pi r}\boldsymbol{a}_x [\mathrm{Wb/m^2}]$$

$$z = r - d$$

$$\begin{aligned} d\Phi &= 2Bzdr \\ &= \frac{\mu_0 i}{2\pi r}(r-d)dr \ [\mathrm{Wb}] \end{aligned}$$

$$\begin{aligned} \Phi &= \int_d^{d+l} \frac{\mu_0 i}{2\pi r}(r-d)dr \\ &= \frac{\mu_0 i}{2\pi}[r - d\ln r]_d^{d+l} \\ &= \frac{\mu_0 i}{2\pi}\left(l - d\ln\frac{d+l}{d}\right) \end{aligned}$$

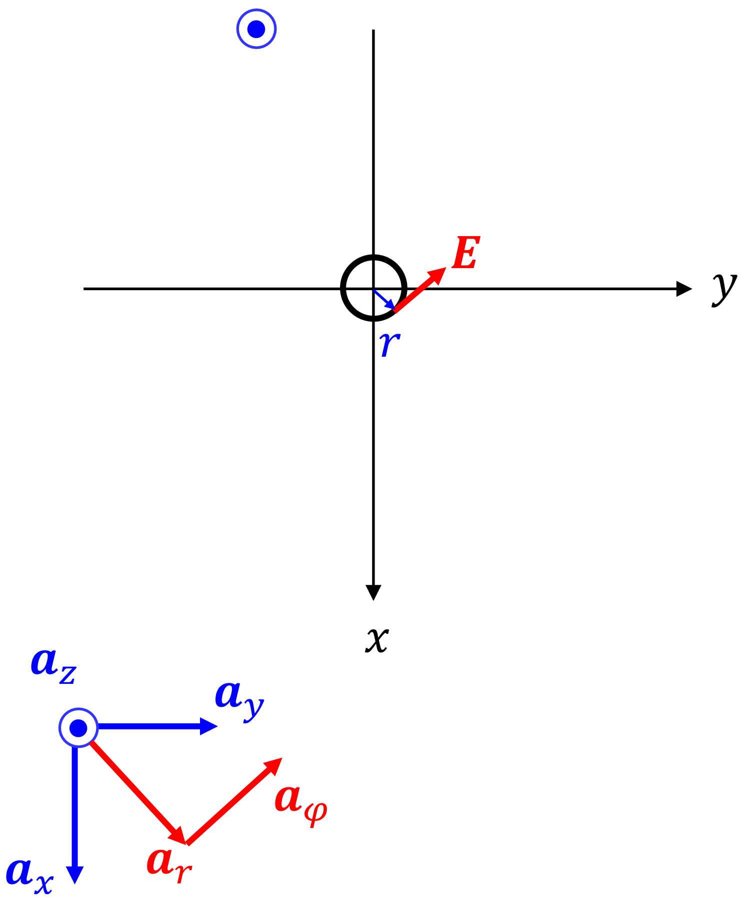
$$\begin{aligned} v &= -\frac{d\Phi}{dt} \\ &= \frac{\mu_0}{2\pi}\left(l - d\ln\frac{d+l}{d}\right)\omega I_m \sin\omega t \ \ [\mathrm{V}] \end{aligned}$$

Exercise 13.3.2 (Homework)

Exercise 13.3.2 (Homework)

Exercise 13.3.2 (Homework)

$\boldsymbol{B} = B_m \cos \omega t \, \boldsymbol{a}_z [\mathrm{Wb/m^2}]$



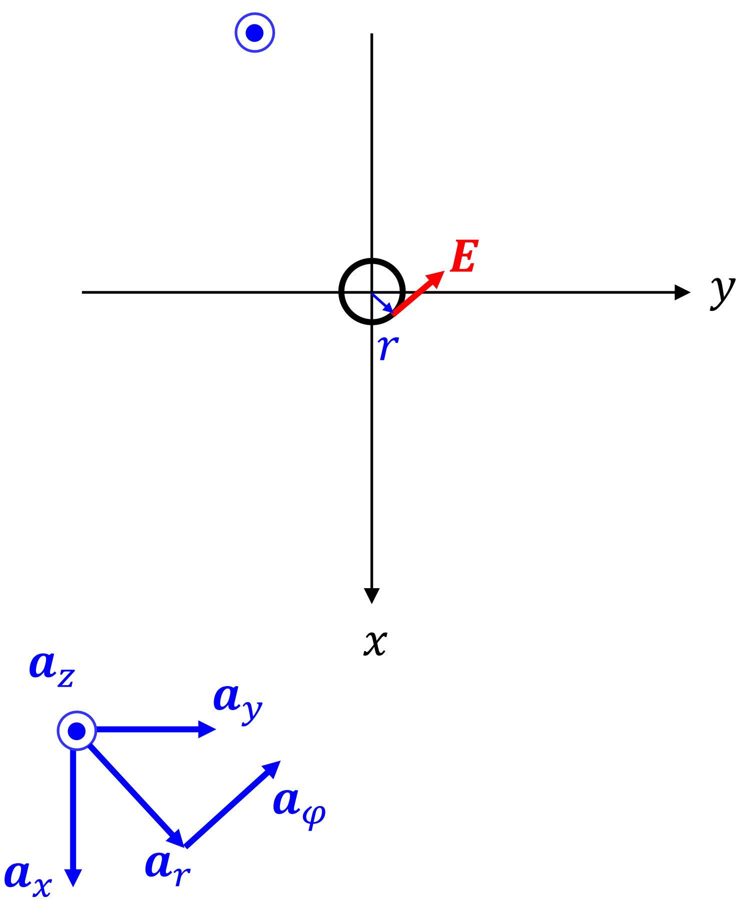
Using the result $\boldsymbol{E}$ of Exercise 13.3.1, show that

$$\nabla \times \boldsymbol{E} = -\frac{d\boldsymbol{B}}{dt}$$

Hint: Exercise 11.1.3

Exercise 13.3.2 (Answer)

$$\boldsymbol{B} = B_m \cos\omega t\, \boldsymbol{a}_z [\mathrm{Wb/m^2}]$$



According to Exercise 13.3.1

$$\boldsymbol{E} = \frac{r}{2}\omega B_m \sin\omega t\, \boldsymbol{a}_\varphi.$$

We can apply the following result of Homework 11.1.3:

$$\boldsymbol{H} = \frac{rJ}{2}\boldsymbol{a}_\varphi$$

$$\nabla \times \boldsymbol{H} = J\boldsymbol{a}_z.$$

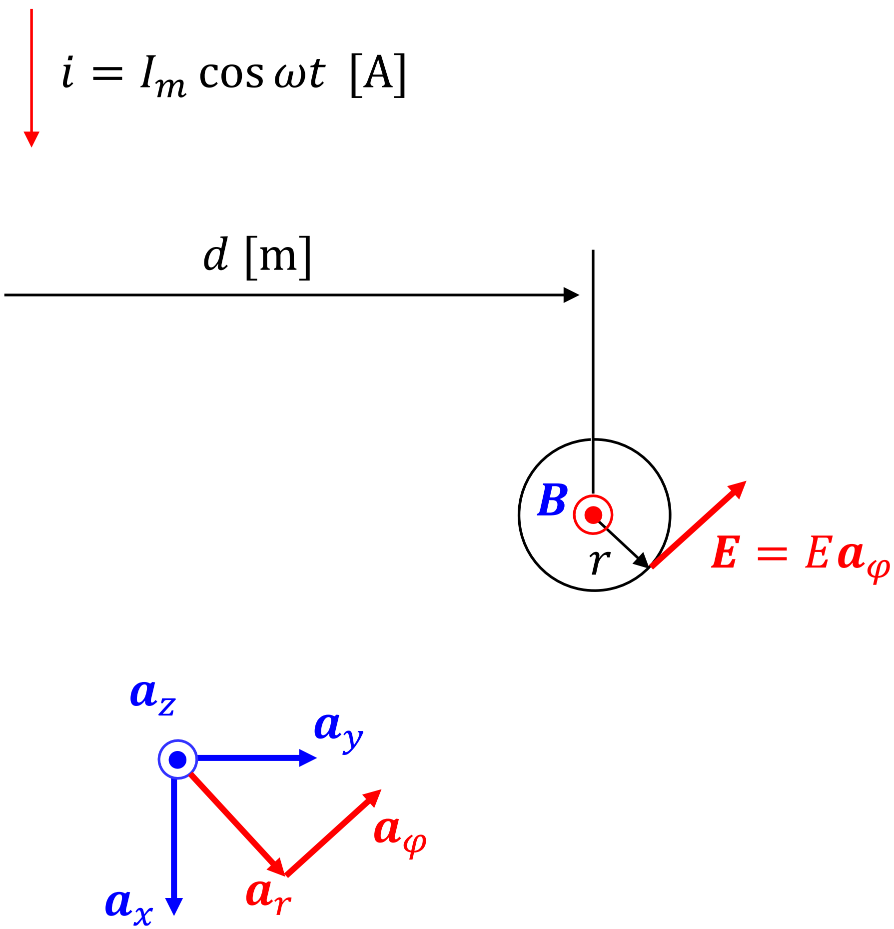
Thus

$$\nabla \times \boldsymbol{E} = \omega B_m \sin\omega t\, \boldsymbol{a}_z$$
$$= -\frac{d\boldsymbol{B}}{dt}$$

Exercise 13.3.3 (Homework)

Exercise 13.3.3 (Homework)

Exercise 13.3.3 (Homework)



A current i expressed as

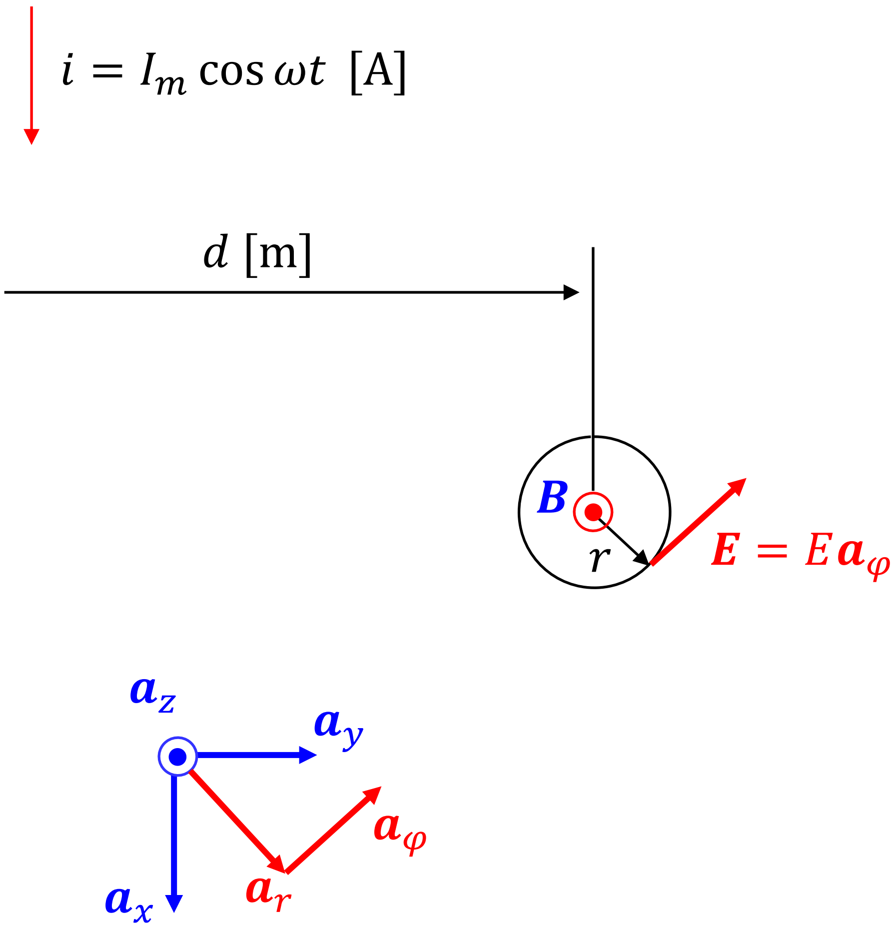
$$i = I_m \cos \omega t \ \text{[A]}$$

flows through an infinite line conductor to the direction of $x-$ axis. A circle conductor of radius r [m] is set parallel to the $x-y$ plane with its center being d [m] away from the infinite line.

Find the electric field intensity $\boldsymbol{E}$ on the circle conductor. You can assume that the radius r is short so that $\boldsymbol{B}$ is uniform within the circle.

Find $\nabla \times \boldsymbol{E}$.

Exercise 13.3.3 (Answer)



According to Exercise 13.2.2

$$\boldsymbol{B} = \frac{\mu_0 I_m \cos \omega t}{2\pi d} \boldsymbol{a}_z [\mathrm{Wb/m^2}].$$

Faraday's Law

$$\oint_{CP} \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt} \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

$$2\pi r E = -\frac{d}{dt} B\pi r^2$$

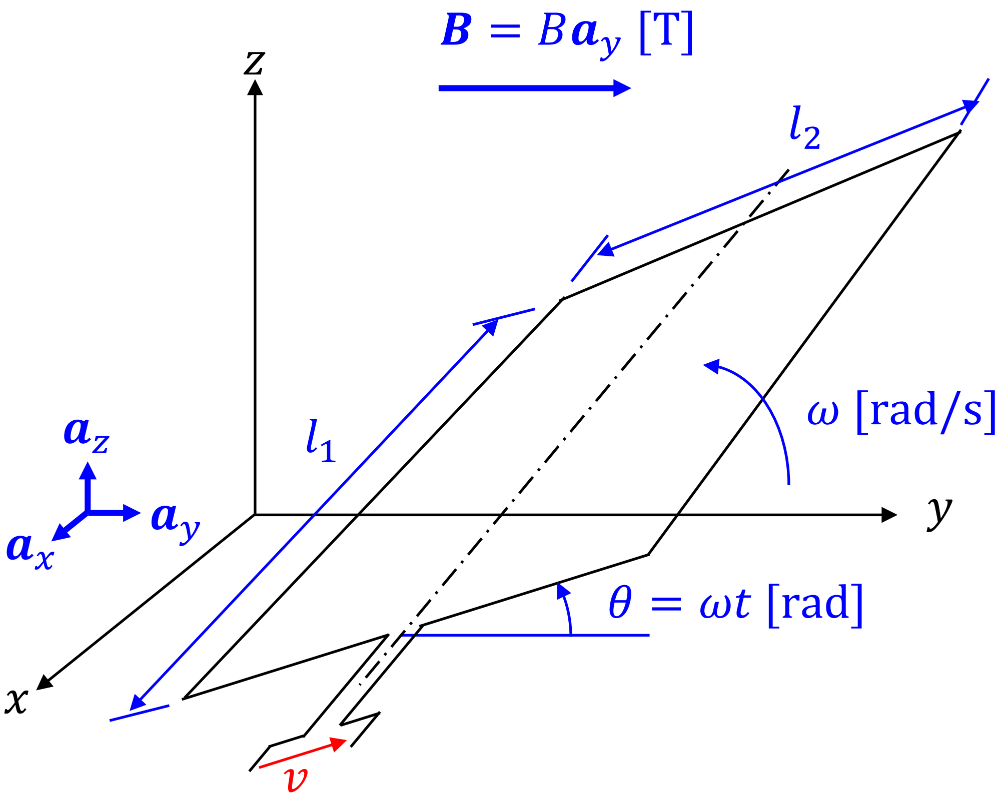
$$\boldsymbol{E} = \frac{r}{2} \frac{\mu_0}{2\pi d} \omega I_m \sin \omega t\, \boldsymbol{a}_\varphi$$

Thus

$$\nabla \times \boldsymbol{E} = \frac{\mu_0}{2\pi d} \omega I_m \sin \omega t\, \boldsymbol{a}_z = -\frac{d\boldsymbol{B}}{dt}$$

Exercise 13.4.3 (Homework)

Exercise 13.4.3 (Homework)



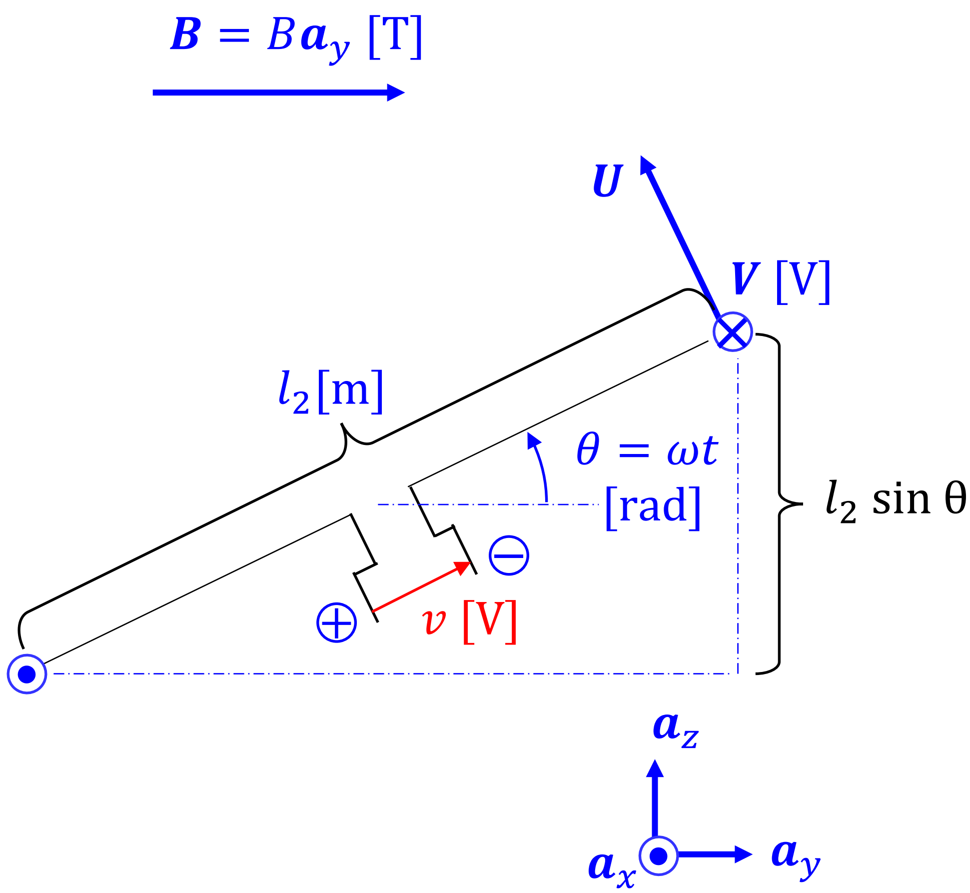
This problem has the same setup as in Exercise 13.4.2.

Find the induced voltage v [V] by applying Faraday's law:

$$v = -\frac{d\Phi}{dt}$$

where Φ is the magnetic flux through the rectangular conductor.

Exercise 13.4.2 (Answer)



$$\Phi = Bl_1 l_2 \sin\theta$$
$$= Bl_1 l_2 \sin\omega t \text{ [Wb]}$$

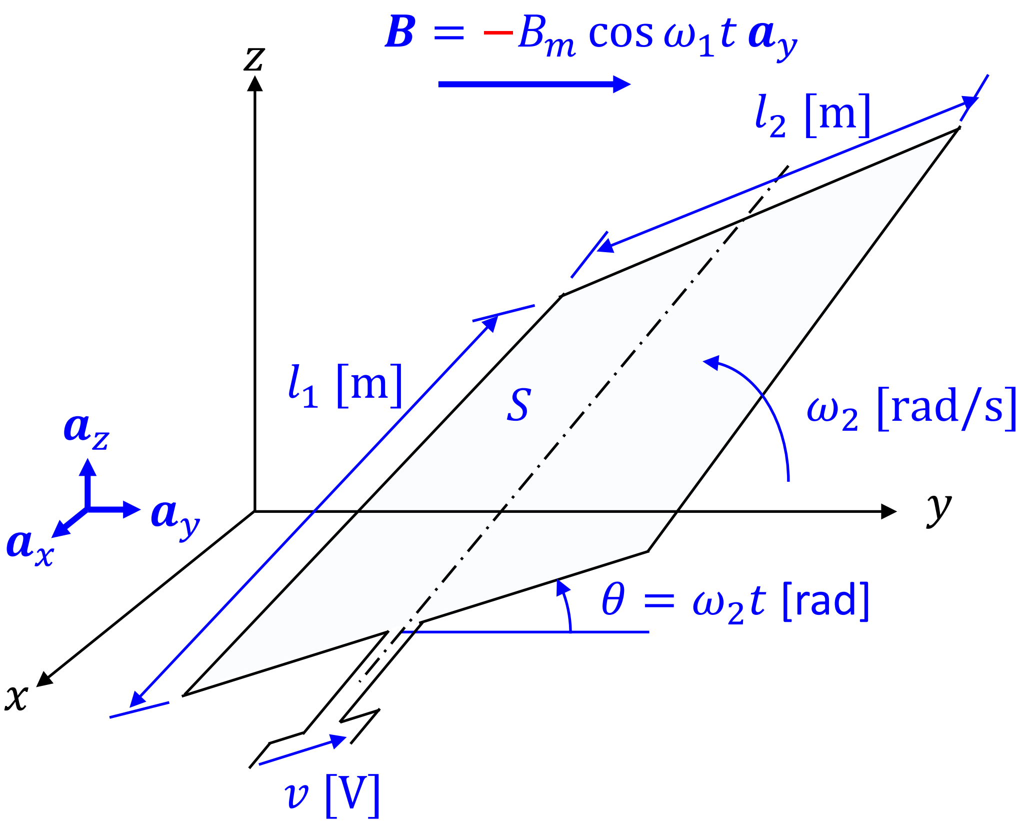
$$v = -\frac{d\Phi}{dt}$$
$$= -Bl_1 l_2 \omega \cos\omega t$$
$$= -\Phi_m \omega \cos\omega t \text{ [V]}$$

$$\Phi_m = Bl_1 l_2 \text{ [Wb]}$$

Exercise 13.5.2 (Homework)

Exercise 13.5.2 (Homework)

Exercise 13.5.2 (Homework)



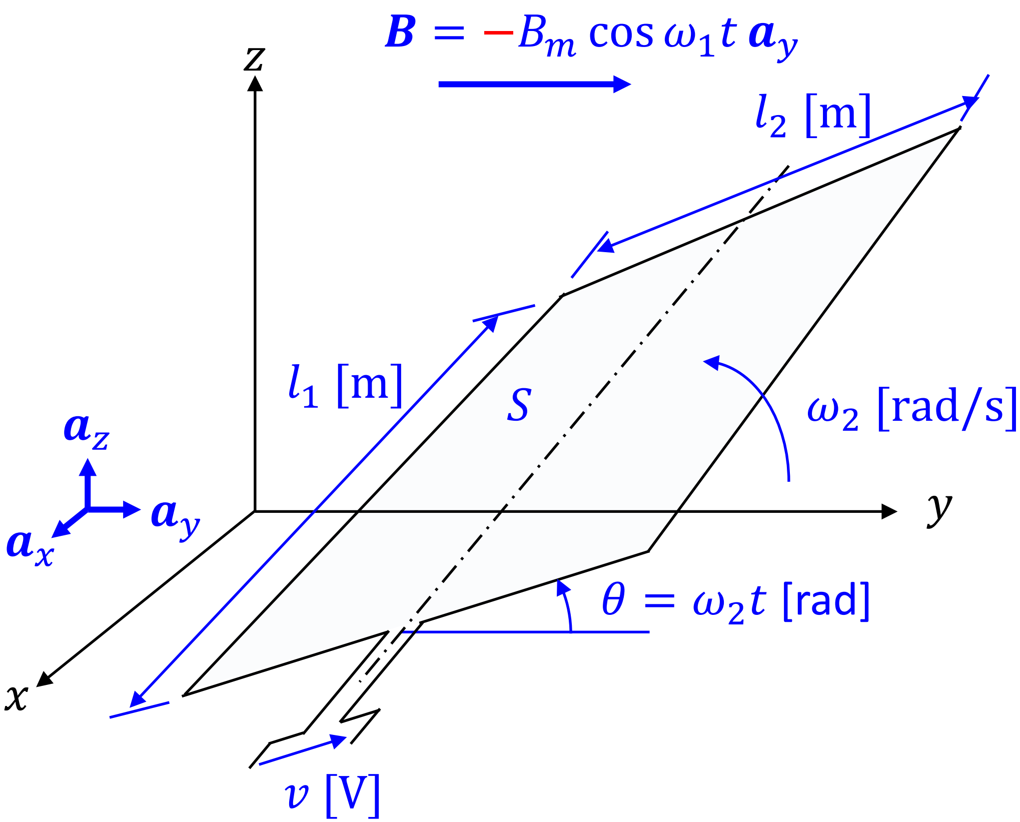
In Exercise 13.5.1, Find v if

$$\boldsymbol{B} = -B_m \cos \omega_1 t \, \boldsymbol{a}_y$$

And

$$\omega_1 = \omega_2 = \omega.$$

Exercise 13.5.2 (Answer)



$$v_1 = -\int_S \frac{d\boldsymbol{B}}{dt} \cdot d\boldsymbol{S}$$
$$= -\frac{d\boldsymbol{B}}{dt} \times l_1 l_2 \sin\omega_2 t$$
$$= -\Phi_m \omega_1 \sin\omega_1 t \sin\omega_2 t$$

$$\Phi_m = B_m l_1 l_2$$

$$v_2 = -\oint_{CP} \boldsymbol{B} \cdot (\boldsymbol{U} \times d\boldsymbol{l})$$
$$= \Phi_m \omega_2 \cos\omega_1 t \cos\omega_2 t$$

$$v = \Phi_m(-\omega_1 \sin\omega_1 t \sin\omega_2 t + \omega_2 \cos\omega_1 t \cos\omega_2 t)$$

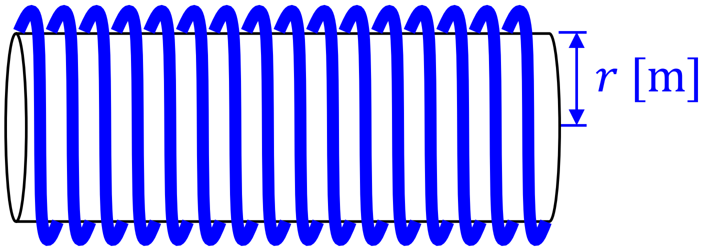
if $\omega_1 = \omega_2 = \omega$,

$$v = \omega\Phi_m(\cos^2\omega t - \sin^2\omega t)$$
$$= \omega\Phi_m \cos 2\omega t$$

Exercise 14.2.2 (Homework)

Exercise 14.2.2 (Homework)

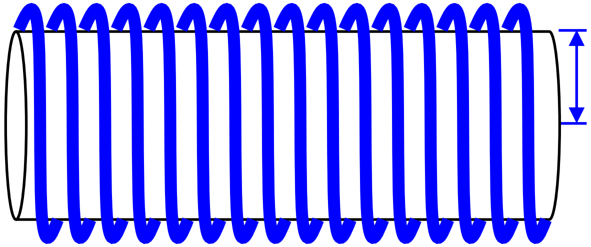
Exercise 14.2.2 (Homework)



Find the inductance of the solenoid coil per unit length in the left figure, where $r = 1$ [cm]. The number of turns within a unit length $n = 100$. Assume $\mu_r = 1$.

Find also the internal inductance of the solenoid coil per unit length.

Exercise 14.2.2 (Answer)

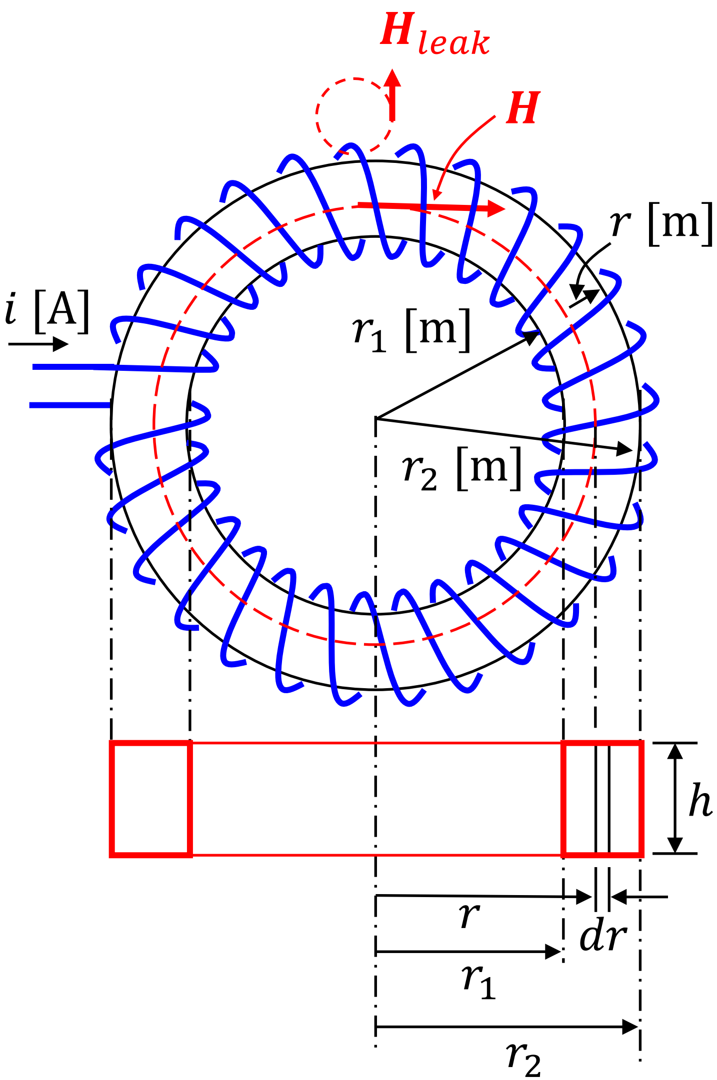


$$L = \mu n^2 \pi r^2$$
$$= 4\pi \times 10^{-7} \times 100^2 \times \pi \times (10^{-2})^2$$
$$= 3.9\ [\mu\mathrm{H/m}]$$

$$L_{internal} = \frac{\mu}{8\pi} \times 2\pi r n$$
$$= \frac{4\pi \times 10^{-7}}{8\pi} \times 2\pi \times 10^{-2} \times 100$$
$$= 0.31\ [\mu\mathrm{H/m}]$$

Exercise 14.2.3 (Homework)

Exercise 14.2.3 (Homework)

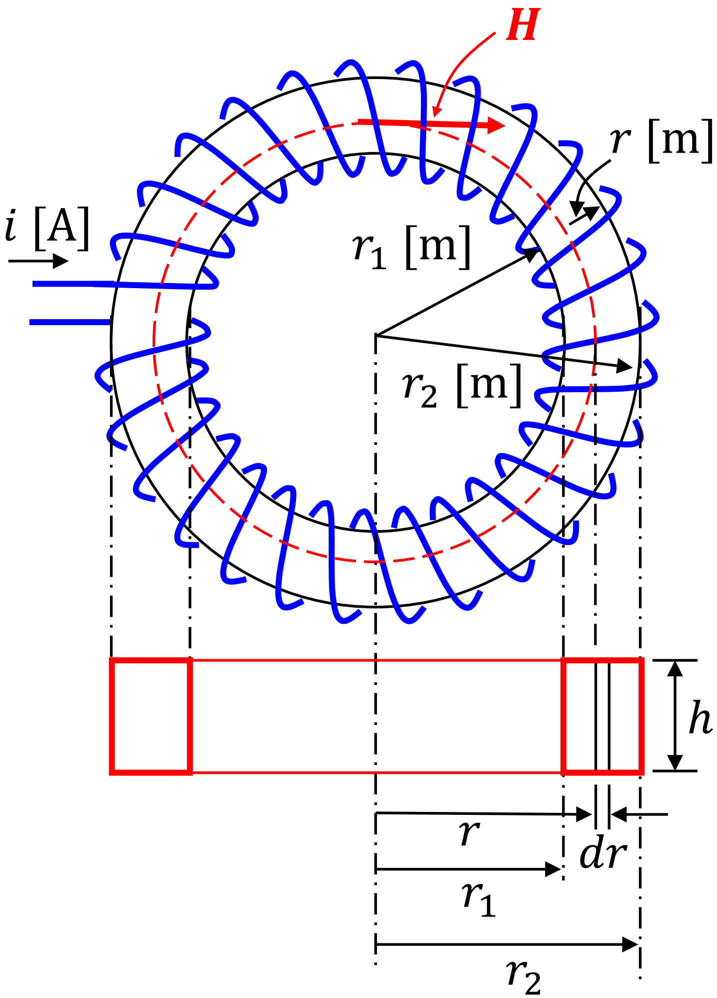


A coil is wound around a tube ring. The cross-sectional view of the ring is rectangular, as shown in the figure below on the left. The inner and outer radii of the ring are r_1[m] and r_2 [m], respectively. The height of the rectangular tube is h [m]. The number of turns of the coil is N. Assume that a leakage flux $\boldsymbol{H}_{leak}$ is negligible, and most of the magnetic flux lines are contained within the tube. Note that $\boldsymbol{H}$ is not uniform within the cross-sectional area of the rectangular tube. A current of i [A] flows in the coil.

Find $\boldsymbol{H}$, the flux linkage of one turn Φ, and the self-inductance of the ring coil. You can neglect the internal inductance of the conductor.

Hint: Apply Ampere's law to the closed path of radius r, and find a differential flux linkage $d\Phi$ through the area of hdr.

Exercise 14.2.3 (Answer)



$$\boldsymbol{H} = -\frac{Ni}{2\pi r}\boldsymbol{a}_\varphi \quad [\mathrm{A/m}]$$

$$\boldsymbol{B} = -\frac{\mu Ni}{2\pi r}\boldsymbol{a}_\varphi \quad [\mathrm{T}]$$

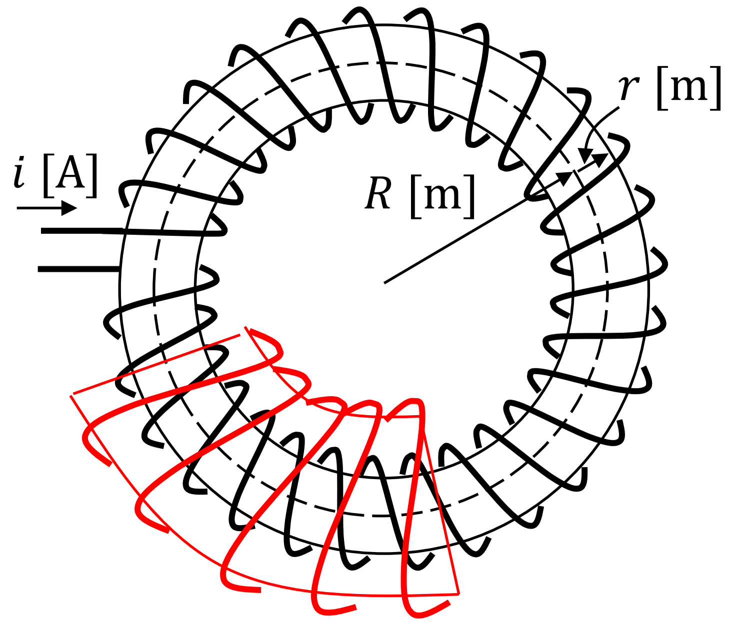
$$d\Phi = \frac{\mu Ni}{2\pi r}hdr$$

$$\Phi = \int_{r_1}^{r_2}\frac{\mu Ni}{2\pi r}hdr = \frac{\mu Nih}{2\pi}\ln\frac{r_2}{r_1} \quad [\mathrm{Wb}]$$

$$L = \frac{N\Phi}{i} = \frac{\mu N^2 h}{2\pi}\ln\frac{r_2}{r_1} \quad [\mathrm{H}]$$

Exercise 14.3.3 (Homework)

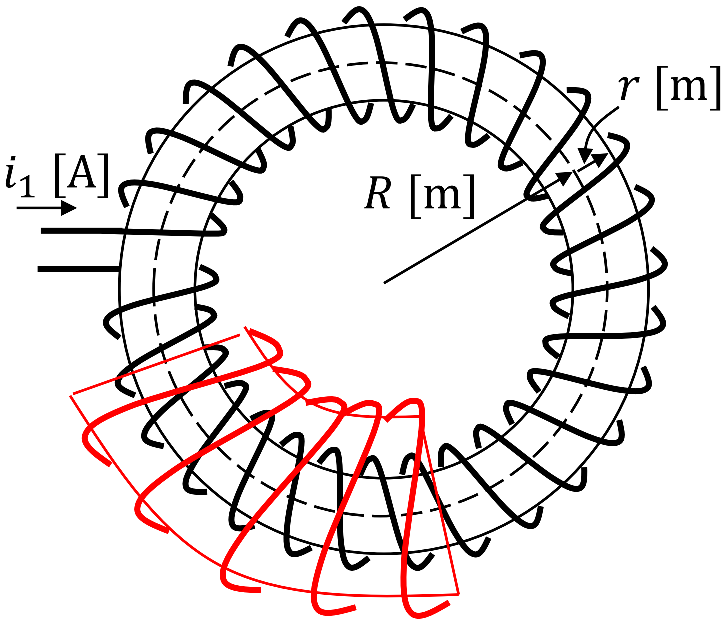
Exercise 14.3.3 (Homework)



The black toroidal coil is the same as in Exercise 14.2.1. The red coil is wound around the toroidal coil as shown in the figure. The number of turns of the black toroidal coil is N_1, and that of the red coil is N_2.

Find the mutual inductance.

Exercise 14.3.3 (Answer)



The flux linkage Φ_1 across the cross sectional area of the black coil is

$$\Phi_1 = \frac{\mu N_1 i_1}{2\pi R}\pi r^2$$
$$= \frac{\mu N_1 i_1 r^2}{2R} \quad [\mathrm{Wb}].$$

The flux linkage of the red coil is

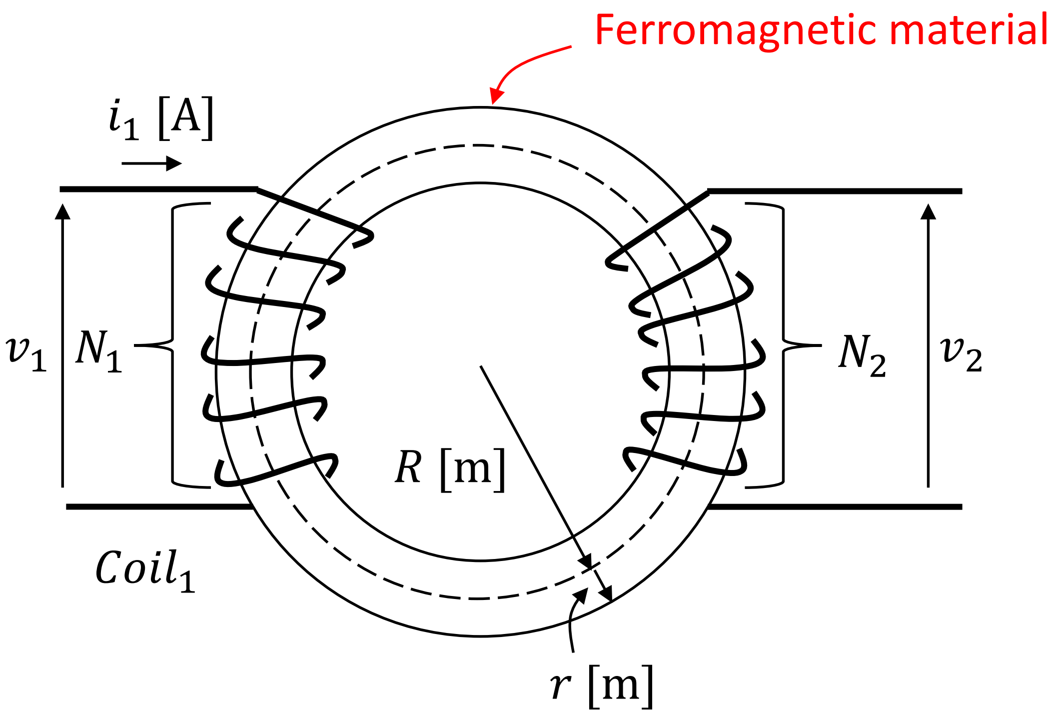
$$N_2\Phi_{21} = N_2\Phi_1$$

Thus

$$M = \frac{N_2\Phi_{21}}{i_1} = \frac{\mu N_1 N_2 r^2}{2R}\,[\mathrm{H}]$$

Exercise 14.3.4 (Homework)

Exercise 14.3.4 (Homework)



The toroidal coils in this figure are the same as in Section 14.3. The relative permeability of the ferromagnetic core μ_r is very large, so the leakage flux outside the core is negligible.

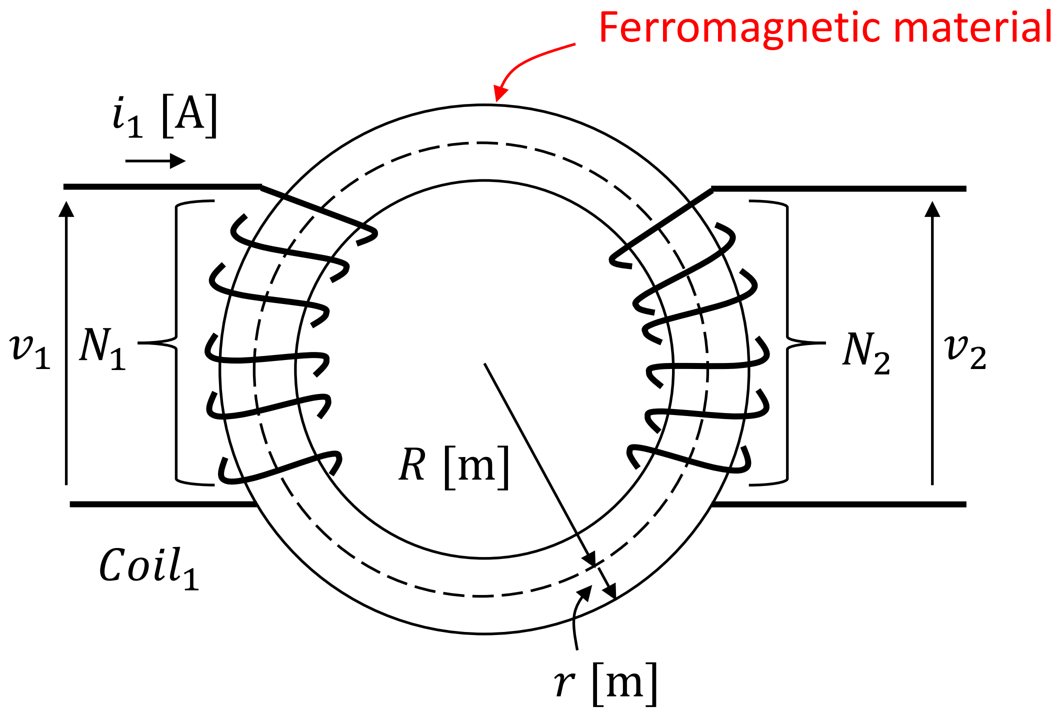
Find the ratio of v_2/v_1.

Hint:

$$v_1 = L_1 \frac{di_1}{dt}$$

$$v_2 = M \frac{di_1}{dt}$$

Exercise 14.3.4 (Answer)



$$v_1 = L_1 \frac{di_1}{dt}$$

$$v_2 = M \frac{di_1}{dt}$$

According to Exercise 14.2.1,

$$L_1 = \frac{\mu N_1^2 r^2}{2R}.$$

In Section 14.3,

$$M = \frac{\mu N_1 N_2 r^2}{2R}.$$

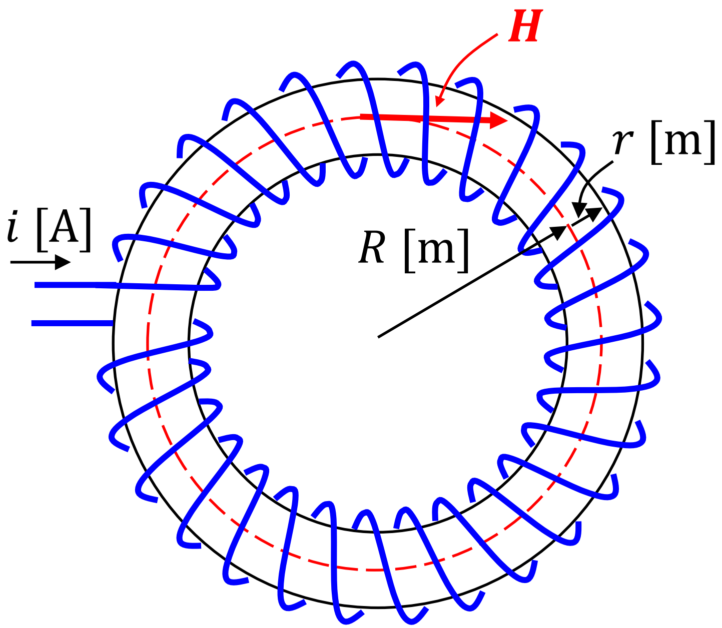
Thus

$$\frac{v_1}{v_2} = \frac{L_1}{M} = \frac{N_1}{N_2}.$$

Exercise 14.4.3 (Homework)

Exercise 14.4 .3. This is your homework.

Exercise 14.4 (Homework)



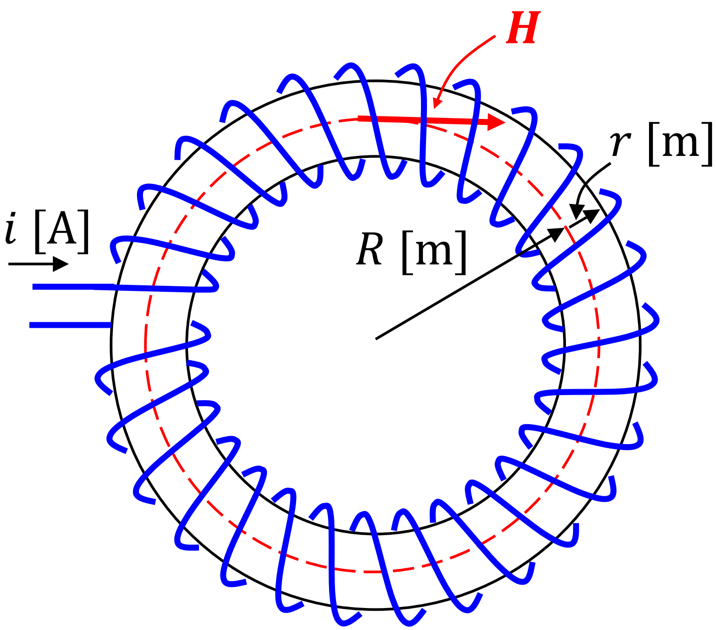
According to Exercise 14.2, the inductance L of this toroidal coil is given by

$$L = \frac{\mu N^2 r^2}{2R},$$

where N represents the number of turns in the coil.

Derive the energy stored in this toroidal coil by using the energy per unit volume, and determine the inductance of this coil so that it satisfies the above equation.

Exercise 14.4 (Answer)



The energy stored in the toroidal coil is

$$E = \int_{Vol} \frac{1}{2} \mu H^2 \, dv$$
$$\approx \frac{1}{2} \mu \left(\frac{Ni}{2\pi R} \right)^2 2\pi R \times \pi r^2$$
$$= \frac{1}{2} \frac{\mu N^2 r^2}{2R} i^2.$$

Since,

$$E = \frac{1}{2} L i^2.$$

Thus,

$$L = \frac{\mu N^2 r^2}{2R}.$$

Exercise 15.1.3 (Homework)

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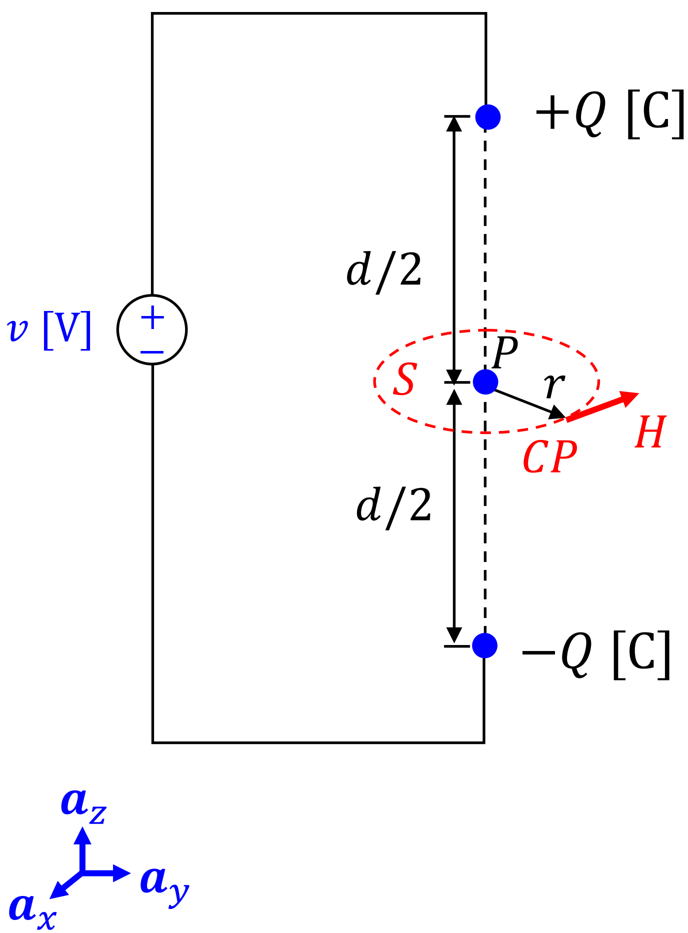


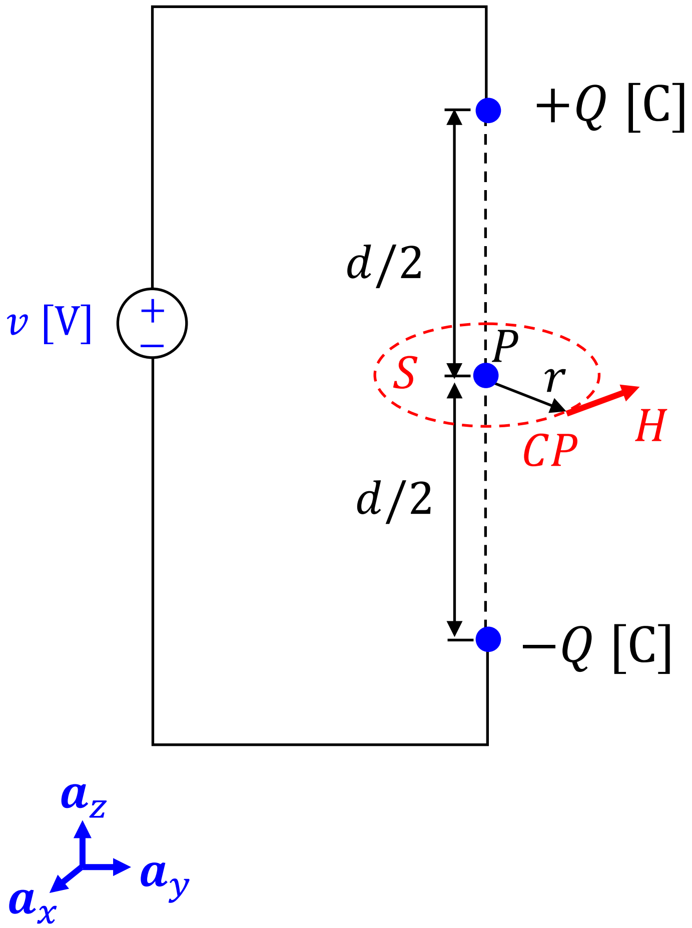
Two point charges are placed on the $z-$ axis with a distance of d [m]. An AC voltage v [V] is applied across the points, and the charges $\pm Q$ [C] vary as follows:

$$Q = Q_m \cos \omega t \,.$$

The dielectric material in the space has a permittivity of ε.

1. Find the electric flux density $\boldsymbol{D}$ at the central point P between the two point charges.
2. Find the magnetic field strength $\boldsymbol{H}$ along a closed path CP of radius r [m]. The center of CP is at P, and the surface S bounded by CP is perpendicular to the $z-$ axis. You can assume that $\boldsymbol{D}$ is constant on S.
3. Find the curl of $\boldsymbol{H}$.

Exercise 15.1.3 (Answer)



1.

$$\boldsymbol{D} = -\frac{2Q}{4\pi\left(\frac{d}{2}\right)^2}\boldsymbol{a}_z = -\frac{2Q}{\pi d^2}\boldsymbol{a}_z$$

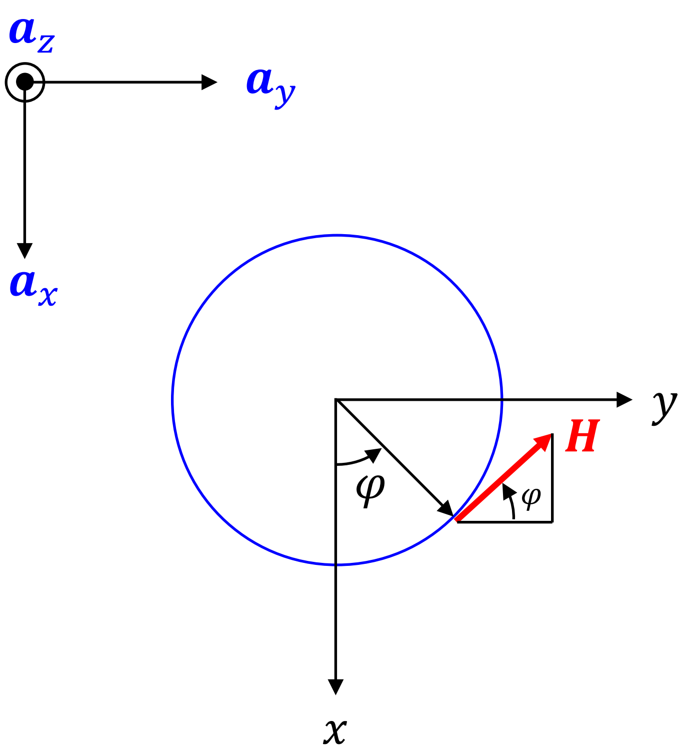
2.

$$\oint_{CP} \boldsymbol{H} \cdot d\boldsymbol{l} = \int_S \frac{d\boldsymbol{D}}{dt} \cdot d\boldsymbol{S}$$

$$2\pi r H = \pi r^2 \frac{dD}{dt}$$

$$\boldsymbol{H} = -\frac{r}{2}\frac{dD}{dt}\boldsymbol{a}_\varphi$$
$$= \frac{r}{\pi d^2}\omega Q_m \sin\omega t\, \boldsymbol{a}_\varphi$$

Exercise 15.1.3 (Answer)



3.

The x, y components of $\boldsymbol{H}$ are

$$H_x = -|\boldsymbol{H}|\sin\varphi = -|\boldsymbol{H}|\frac{y}{r} = \frac{-y}{\pi d^2}\omega Q_m \sin\omega t$$
$$H_y = |\boldsymbol{H}|\cos\varphi = |\boldsymbol{H}|\frac{x}{r} = \frac{x}{\pi d^2}\omega Q_m \sin\omega t\,.$$

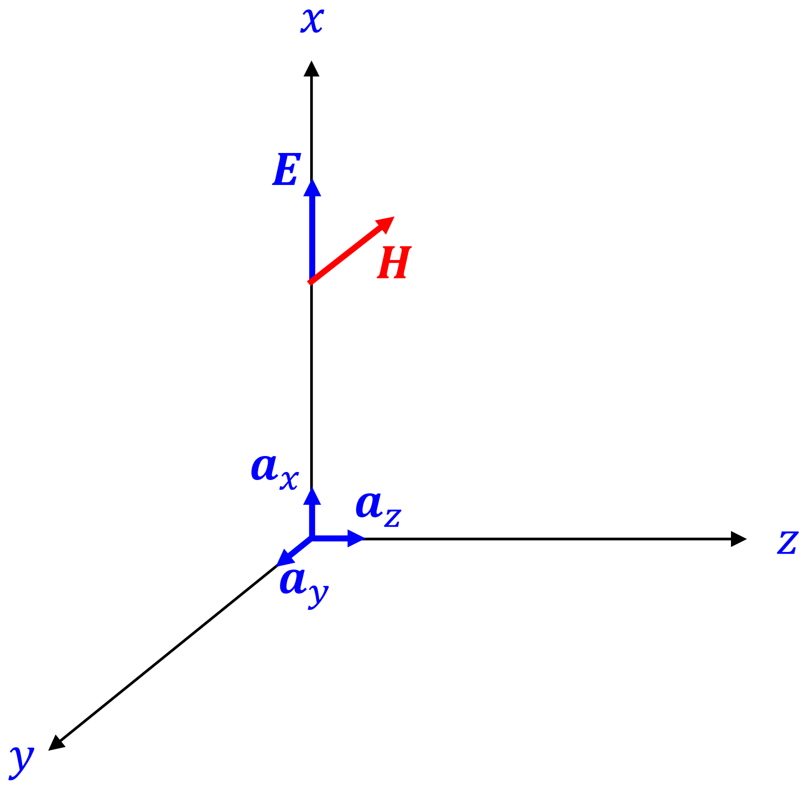
Thus,

$$\nabla\times\boldsymbol{H} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$
$$= \frac{\omega Q_m \sin\omega t}{\pi d^2}(1+1)\boldsymbol{a}_z$$
$$= \frac{2\omega Q_m \sin\omega t}{\pi d^2}\boldsymbol{a}_z$$
$$= -\frac{2}{\pi d^2}\frac{dQ}{dt}\boldsymbol{a}_z$$
$$= \frac{d\boldsymbol{D}}{dt}.$$

Exercise 15.3 (Homework)

Exercise 15.3 (Homework)

Exercise 15.3 (Homework)



Show that the equations below also satisfy both Ampere's law and Faraday's law.

$$\boldsymbol{E} = E_m \cos\omega\left(\frac{z}{c} + t\right)\boldsymbol{a}_x$$

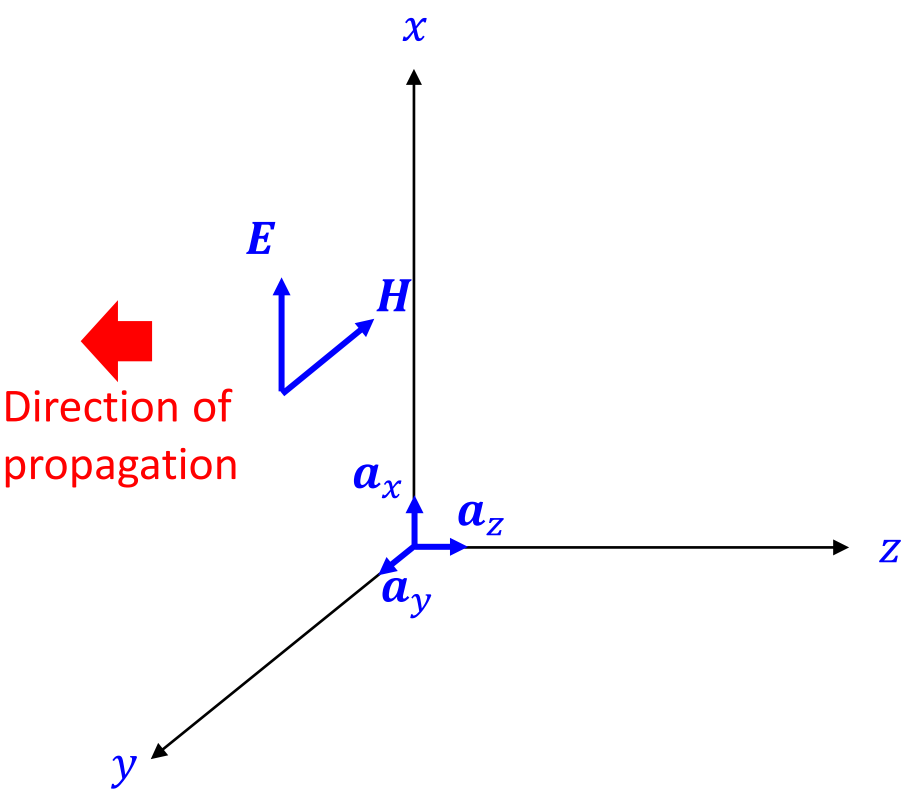
$$\boldsymbol{H} = -H_m \cos\omega\left(\frac{z}{c} + t\right)\boldsymbol{a}_y$$

Find the direction of propagation of E and H.

Exercise 15.3 (Answer)

$$\boldsymbol{E} = E_m \cos\omega\left(\frac{z}{c}+t\right)\boldsymbol{a}_x$$

$$\boldsymbol{H} = -H_m \cos\omega\left(\frac{z}{c}+t\right)\boldsymbol{a}_y$$



$$\varepsilon_0 \frac{d\boldsymbol{E}}{dt} = -\varepsilon_0 \omega E_m \sin\omega\left(\frac{z}{c}+t\right)\boldsymbol{a}_x$$

$$\nabla\times\boldsymbol{H} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix} = -\frac{\partial H_y}{\partial z}\boldsymbol{a}_x$$

$$= -\frac{\omega}{c} H_m \sin\omega\left(\frac{z}{c}+t\right)\boldsymbol{a}_x$$

$$-\mu_0 \frac{d\boldsymbol{H}}{dt} = -\mu_0 \omega H_m \sin\omega\left(\frac{z}{c}+t\right)\boldsymbol{a}_y$$

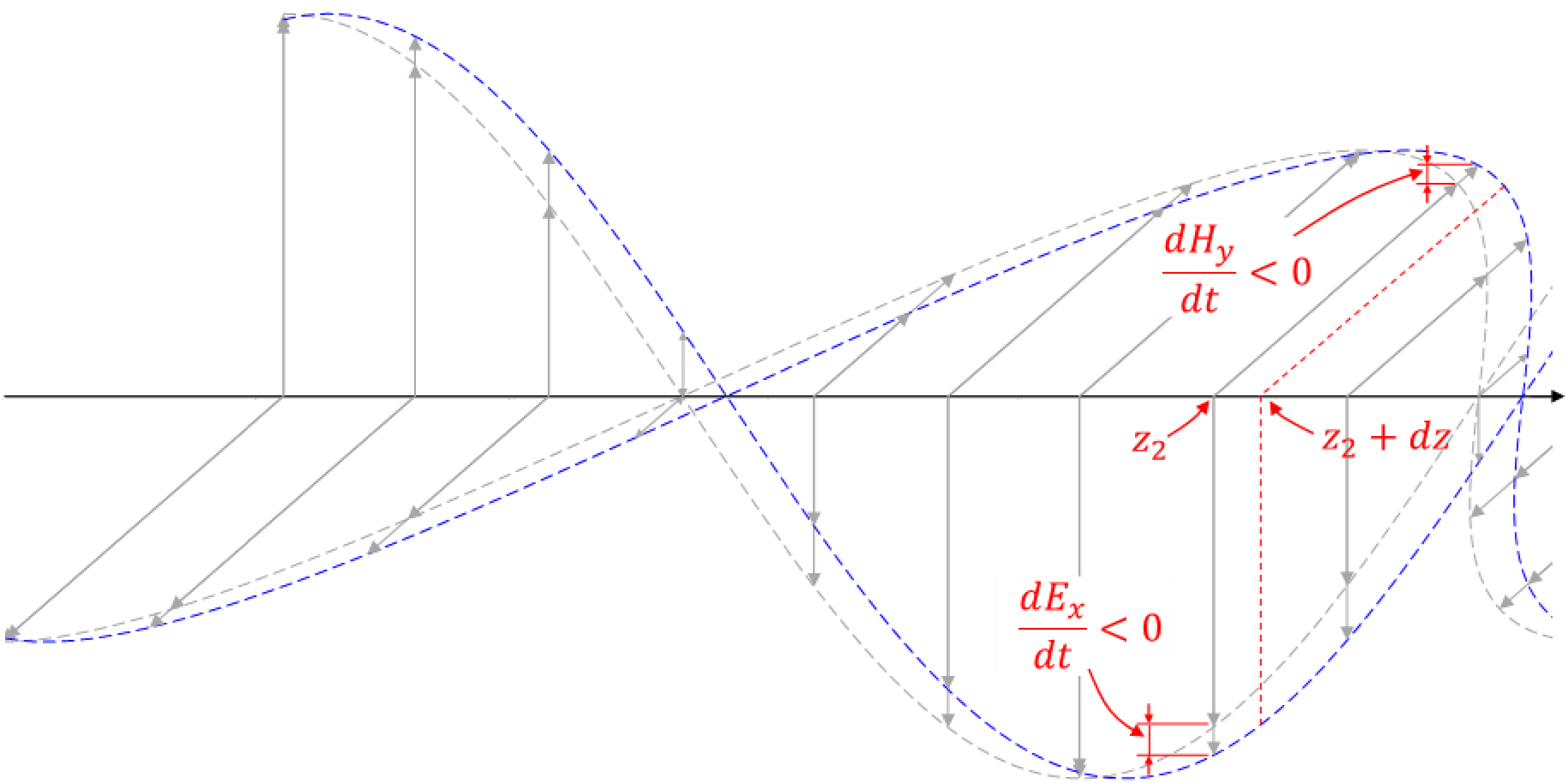
$$\nabla\times\boldsymbol{E} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \frac{\partial E_x}{\partial z}\boldsymbol{a}_x$$

$$= -\frac{\omega}{c} E_m \sin\omega\left(\frac{z}{c}+t\right)\boldsymbol{a}_x$$

$$\omega\left(\frac{z}{c}+t\right) = 0 \quad \Rightarrow \quad \frac{dz}{dt} = -c$$

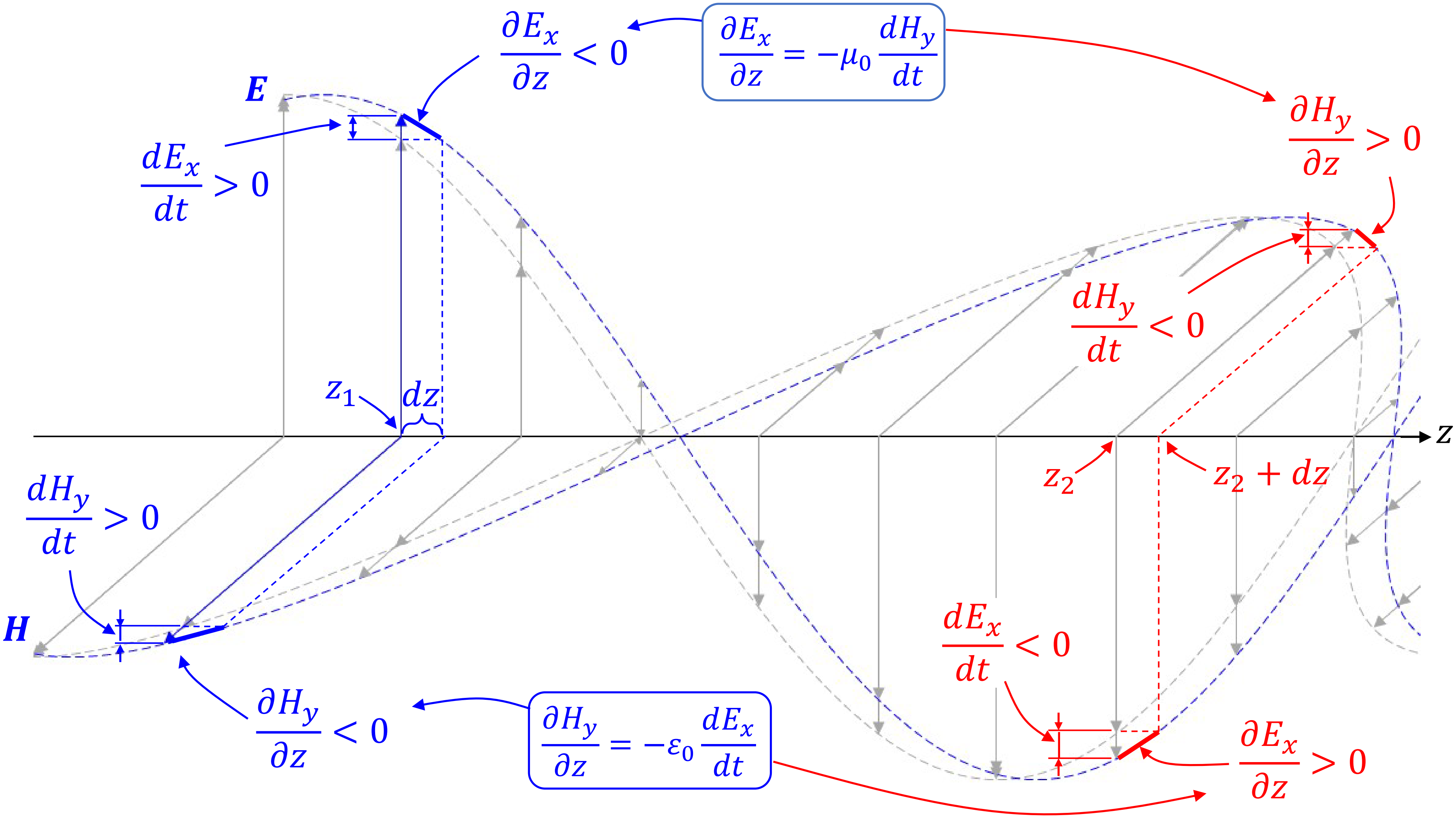
Exercise 15.4 (Homework)

Exercise 15.4 (Homework)



Explain the interaction of E_x and H_y when both E_x and H_y are decreasing over time, as shown at $z = z_2$ in the figure below.

Exercise 15.4 (Answer)



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