Development Microeconomics
Spring 2023
Introduction: Utility Maximization and Choice

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Outline

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Introduction

- In this lecture, we examine the basic model of choice that economists use to explain individuals' behavior.
- The model assumes that individuals, constrained by limited incomes, behave as though they are maximizing utility subject to a budget constraint.
- To maximize utility, individuals choose bundles of commodities for which the rate of trade-off between any two goods (the MRS) is equal to the ratio of the goods' market prices.
- Market prices convey information about opportunity costs to individuals, influencing the choices they make.

Optimization Principle

- To maximize utility, given a fixed amount of income to spend, an individual will buy the goods and services:
  - that exhaust his or her total income
  - for which the psychic rate of trade-off between any goods (the the Marginal Rate of Substitution, MRS) is equal to the rate at which goods can be traded for one another in the marketplace.

A Numerical Illustration

- To see the intuitive reasoning behind this result, consider a numerical illustration:
  - Suppose the individual's MRS is equal to 1, and the prices of goods x and y are \$2 and \$1 per unit, respectively.
  - If the individual is willing to trade 1 unit of x for 1 unit of y and remain equally well off, they can be made better off by reallocating their spending.
  - By reducing x consumption by 1 unit and trading it for 2 units of y in the market, the individual can increase their well-being. The condition for maximum utility must be the equality of the MRS and the price ratio px / py.

The Budget Constraint

- Assume that the individual has I dollars to allocate between good x and good y. If p_x is the price of good x and p_y is the price of good y, then the individual is constrained by $p_x x + p_y y \leq I$.
- This person can afford to choose only combinations of x and y
  - If all of I is spent on good x, it will buy I/p_x units of x.
  - If all is spent on y, it will buy I/p_y units of y.
- The slope of the constraint is easily seen to be $-\frac{p_x}{p_y}$. This slope shows how y can be traded for x in the market. If $p_x = 2$ and $p_y = 1$, then 2 units of y will trade for 1 unit of x.

The Budget Constraint

- The shaded triangle shows affordable combinations of goods x and y.
- The outer boundary, where all funds are spent on either x or y, represents the constraint.
- Its slope is determined by the ratio of their prices: –px / py.

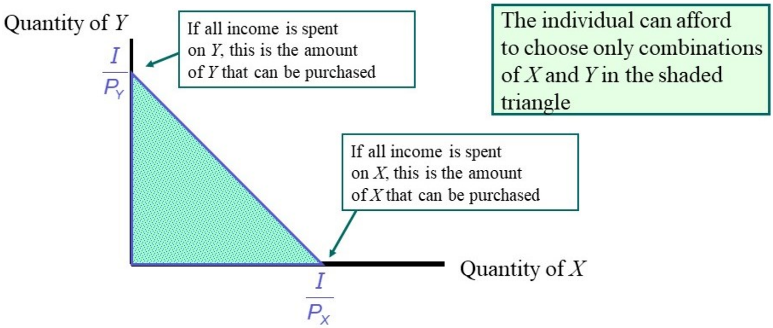

Figure 1: The Individual's Budget Constraint for Two Goods

First-Order Conditions for a Maximum

- We can add the individual's utility map to show the utility-maximization process

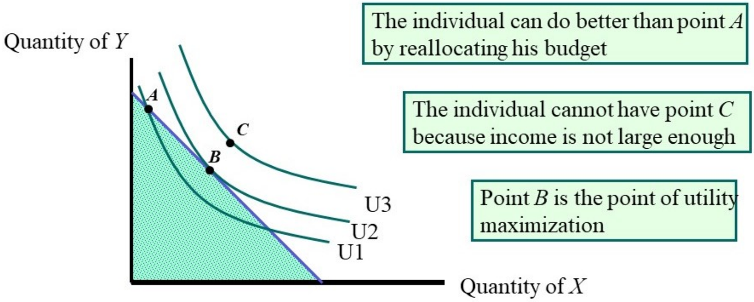

Figure 2: A Graphical Demonstration of Utility Maximization

First-Order Conditions for a Maximum

- Utility is maximized where the indifference curve is tangent to the budget constraint

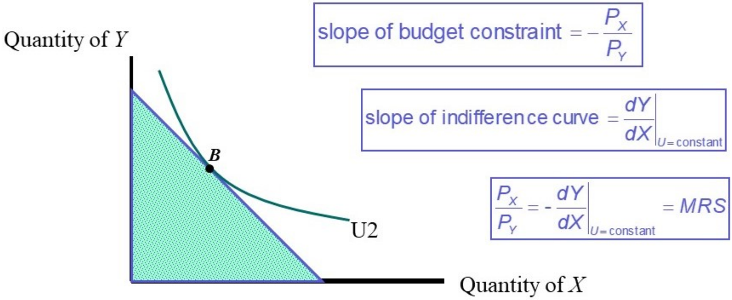

Figure 3: Utility maximization

Second-Order Conditions for a Maximum

- The tangency rule is only necessary but not sufficient unless we assume that *MRS* is diminishing
  - if *MRS* is diminishing, then indifference curves are strictly convex
- If *MRS* is not diminishing, then we must check second-order conditions to ensure that we are at a maximum

Second-Order Conditions for a Maximum

- The tangency rule is only necessary but not sufficient unless we assume that *MRS* is diminishing

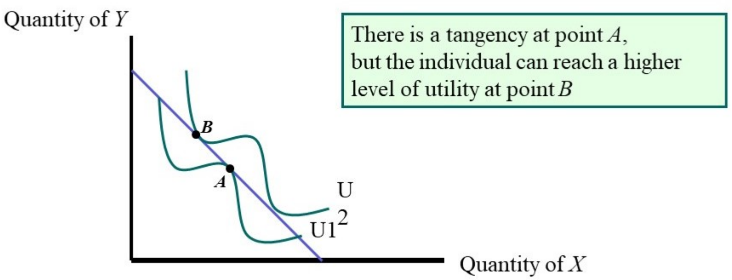

Figure 4: Indifference Curve Map for Which the Tangency Condition Does Not Ensure a Maximum

Corner Solutions

- In some situations, individuals' preferences may be such that they can maximize utility by choosing to consume only one of the goods

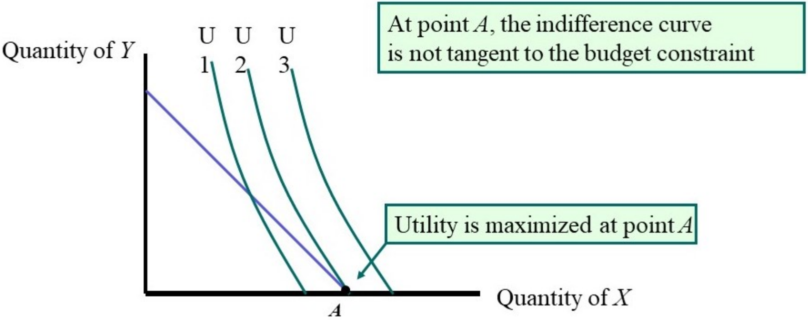

Figure 5: Corner Solution for Utility Maximization

The n-Good Case

With n goods, the individual's objective is to maximize utility from these n goods:

$$\text{utility} = U(x_1, x_2, ..., x_n) \quad (1)$$

subject to the budget constraint:

$$I = p_1x_1 + p_2x_2 + ... + p_nx_n \quad (2)$$

or

$$I - p_1x_1 - p_2x_2 - ... - p_nx_n = 0 \quad (3)$$

For maximizing a function subject to a constraint, we set up the Lagrangian expression:

$$\mathcal{L} = U(x_1, x_2, ..., x_n) + \lambda(I - p_1x_1 - p_2x_2 - ... - p_nx_n) \quad (4)$$

The n-Good Case

- First-order Conditions and Second-order Conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1} &= \frac{\partial U}{\partial x_1} - \lambda p_1 = 0, \\
\frac{\partial \mathcal{L}}{\partial x_2} &= \frac{\partial U}{\partial x_2} - \lambda p_2 = 0, \\
&\vdots \\
\frac{\partial \mathcal{L}}{\partial x_n} &= \frac{\partial U}{\partial x_n} - \lambda p_n = 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= I - p_1 x_1 - p_2 x_2 - \ldots - p_n x_n = 0.
\end{aligned} \tag{5}
$$

- These $n+1$ equations can, in principle, be solved for the optimal $x_1, x_2, \ldots, x_n$ and for λ.
  - These Equations are necessary but insufficient for a maximum.
  - Second-order conditions, complex and matrix-based, are required.
  - Under strict quasi-concavity (diminishing MRS), the Equation confirms a maximum.

Implications of First-Order Conditions

- The first-order conditions represented by Equation 5 can be rewritten in various instructive ways. For instance, considering any two goods, x_i and x_j, we have:

$$\frac{\partial U}{\partial x_i} \frac{\partial U}{\partial x_j} = \frac{p_i}{p_j} \quad (6)$$

- This implies that at the optimal allocation of income:

$$\text{MRS}(x_i \text{ for } x_j) = \frac{p_i}{p_j} \quad (7)$$

- This aligns precisely with the earlier graphical derivation ; to maximize utility, the individual should equate the psychic rate of trade-off to the market trade-off rate.

Interpreting the Lagrange Multiplier

- Solving Equations 5 for λ, we find:

$$\lambda = \frac{\frac{\partial U}{\partial x_i}}{p_i} \quad (8)$$

  - These equations imply that each good purchased should yield the same marginal utility per dollar spent.
  - λ represents the marginal utility of an extra dollar of expenditure.

- Another interpretation of the necessary conditions is:

$$p_i = \frac{\frac{\partial U}{\partial x_i}}{\lambda} \quad (9)$$

  - This compares the extra utility value of one more unit of good i to the common marginal utility of an extra dollar.
  - how much the consumer is willing to pay for the last unit

Corner Solutions

- When corner solutions are involved, the first-order conditions must be modified:

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial U}{\partial x_i} - \lambda p_i \leq 0 \quad (i = 1, \ldots, n) \quad (10)$$

and, if:

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial U}{\partial x_i} - \lambda p_i < 0 \quad (11)$$

then:

$$x_i = 0 \quad (12)$$

- This means that

$$p_i > \frac{\partial U}{\partial x_i} / \lambda \quad (13)$$

  - Hence, any good whose price (p_i) exceeds its marginal value to the consumer will not be purchased ($x_i = 0$).
  - Corollary, individuals will not purchase goods they believe are not worth the money.

Cobb–Douglas Demand Functions

The Cobb–Douglas utility function:

$$U(x, y) = x^a y^b \quad (a + b = 1) \quad (14)$$

Setting up the Lagrangian expression:

$$\mathcal{L} = x^a y^b + \lambda(I - p_x x - p_y y) \quad (15)$$

yields the first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial x} = a x^{a-1} y^b - \lambda p_x = 0,$$

$$\frac{\partial \mathcal{L}}{\partial y} = b x^a y^{b-1} - \lambda p_y = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0 \quad (16)$$

Cobb–Douglas Demand Functions

Taking the ratio of the first two terms shows:

$$\frac{ay}{bx} = \frac{p_x}{p_y} \quad (17)$$

which simplifies to:

$$p_y y = \frac{b}{a} p_x x = \frac{1-a}{a} p_x x \quad (18)$$

Since $\alpha + \beta = 1$:

$$p_y y = \left(\frac{\beta}{\alpha}\right) p_x x = \left[\frac{1-\alpha}{\alpha}\right] p_x x$$

Substituting into the budget constraint:

$$I = p_x x + \left[\frac{1-\alpha}{\alpha}\right] p_x x = \left(\frac{1}{\alpha}\right) p_x x \quad (19)$$

Solving for x and y:

$$x^* = \frac{\alpha I}{p_x} \quad (20), y^* = \frac{\beta I}{p_y} \quad (21)$$

Cobb-Douglas Demand Functions

- Will a change in p_y affect the quantity of x demanded in the demand function?
- Explain your answer mathematically.
- Also develop an intuitive explanation based on the notion that the share of income devoted to good y is given by the parameter of the utility function, beta.

Cobb-Douglas Demand Functions

Will a change in p_y affect the quantity of x demanded in the demand function?

Mathematical Explanation: In the Cobb-Douglas demand function $x^* = \frac{\alpha I}{p_x}$, differentiating with respect to p_y yields:

$$\frac{dx^*}{dp_y} = -\frac{\alpha I}{p_x^2}\frac{dp_x}{dp_y}$$

Since p_x and p_y are typically inversely related, a change in p_y leads to an opposite change in the quantity of x demanded.

Intuitive Explanation: The parameter β of the utility function represents the share of income devoted to good y. An increase in p_y makes consuming y relatively more expensive compared to x. Consumers adjust their consumption to maintain utility, allocating less income to y and more to x, resulting in a decrease in the quantity of x demanded.

Limitations of Cobb-Douglas Utility Function

- An individual with the Cobb–Douglas utility function allocates a proportion of income to buying goods x and y corresponding to parameters α and β.
- However, the simplicity of the Cobb–Douglas function may limit its ability to explain actual consumption behavior.
- The share of income devoted to particular goods often changes in response to changing economic conditions.
- A more general functional form might be more useful in explaining consumption decisions.

Cobb–Douglas Demand Functions (cont'd)

Numerical example: Suppose $p_x = 1$, $p_y = 4$, $I = 8$, $a = b = 0.5$. Then,

$$x^* = 4, \quad y^* = 1 \quad \text{(optimal choices)}$$

$$\text{utility} = x^{0.5}y^{0.5} = 2$$

The Lagrange multiplier:

$$\lambda = \frac{ax^{a-1}y^b}{p_x} = 0.25$$

A \$0.08 increase in income would increase utility by 0.02, as predicted by $\lambda = 0.25$.

Constant elasticity of substitution (CES) Demand

Case 1: $d = 0.5$. In this case, utility is given by:

$$U(x, y) = x^{0.5} + y^{0.5} \quad (22)$$

Setting up the Lagrangian expression:

$$\mathcal{L} = x^{0.5} + y^{0.5} + \lambda(I - p_x x - p_y y) \quad (23)$$

yields the following first-order conditions for a maximum:

$$\frac{\partial \mathcal{L}}{\partial x} = 0.5x^{-0.5} - \lambda p_x = 0,$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0.5y^{-0.5} - \lambda p_y = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0 \quad (24)$$

Division of the first two of these shows that:

$$\left(\frac{y}{x}\right)^{0.5} = \frac{p_x}{p_y} \quad (25)$$

By substituting this into the budget constraint and doing some messy algebraic manipulation, we can derive the demand functions associated with this utility function:

$$x^* = \frac{I}{p_x}\left(\frac{1}{1+\left(\frac{p_x}{p_y}\right)}\right), \quad (26)$$

$$y^* = \frac{I}{p_y}\left(\frac{1}{1+\left(\frac{p_y}{p_x}\right)}\right) \quad (27)$$

Price Responsiveness: In these demand functions, the share of income spent on either X or Y is not a constant; it depends on the ratio of the two prices.
The higher the relative price of X (or Y), the smaller will be the share of income spent on X (or Y).

CES Demand

Case 2: $d = -1$. The utility function is given by:

$$U(x, y) = -x^{-1} - y^{-1} \quad (28)$$

First-order conditions imply that:

$$\frac{y}{x} = \left(\frac{p_x}{p_y}\right)^{0.5} \quad (29)$$

Again, substitution of this condition into the budget constraint, together with some messy algebra, yields the demand functions:

$$x^* = \frac{I}{p_x}\left(\frac{1}{1 + \left(\frac{p_y}{p_x}\right)^{0.5}}\right),$$

$$y^* = \frac{I}{p_y}\left(\frac{1}{1 + \left(\frac{p_x}{p_y}\right)^{0.5}}\right) \quad (30)$$

CES Demand

- That these demand functions are less price responsive can be seen in two ways.
  - First, now the share of income spent on good x responds positively to increases in p_x.
  - As the price of x increases, this individual cuts back only modestly on good x;
  - thus, total spending on that good increases.
- That the demand functions are less price responsive than the Cobb–Douglas is also illustrated by the relatively small implied exponents of each good's own price (-0.5).

Indirect Utility Function

Indirect Utility Function:

- It is often possible to manipulate the first-order conditions for a constrained utility-maximization problem to solve for the optimal values of $x_1, x_2, ..., x_n$.
- These optimal values in general will depend on the prices of all the goods and on the individual's income. That is:

$$\begin{aligned} x_1^* &= x_1(p_1, p_2, ..., p_n, I), \\ x_2^* &= x_2(p_1, p_2, ..., p_n, I), \\ &\vdots \\ x_n^* &= x_n(p_1, p_2, ..., p_n, I). \end{aligned} \quad (31)$$

Indirect Utility Function

We use the optimal values of the x's from Equation 42 to substitute in the original utility function :

$$\text{Maximum utility} = U\left[x_1^*(p_1, ..., p_n, I), x_2^*(p_1, ..., p_n, I), ..., x_n^*(p_1, ..., p_n, I)\right]$$

to yield

$$\text{Maximum utility} = V(p_1, p_2, ..., p_n, I) \quad (33)$$

- the optimal level of utility obtainable will depend indirectly on the prices of the goods being bought and the individual's income.
  - This dependence is reflected by the indirect utility function V.
  - If either prices or income were to change, the level of utility that could be attained would also be affected.

Indirect Utility in the Cobb-Douglas

the Cobb–Douglas utility function with $\alpha = \beta = 0.5$, optimal purchases are:

$$x^* = \frac{I}{2p_x}, \quad y^* = \frac{I}{2p_y} \quad (34)$$

Thus, the indirect utility function in this case is:

$$V(p_x, p_y, I) = U(x^*, y^*) = (x^*)^{0.5}(y^*)^{0.5} = \frac{I}{2p_x^{0.5}p_y^{0.5}} \quad (35)$$

Expenditure Minimization

- Dual constrained minimization problem: allocate income to achieve a given utility level with minimal expenditure.
- Expenditure level E_2 provides the minimal expenditure to achieve desired utility U_2.
- Both utility-maximization and expenditure-minimization yield the same solution.
- Expenditure-minimization is advantageous as expenditures are directly observable.
- Dual minimization problem for utility maximization allocating income in such a way as to achieve a given level of utility with the minimal expenditure this means that the goal and the constraint have been reversed

Expenditure Minimization

Point A is the solution to the dual problem

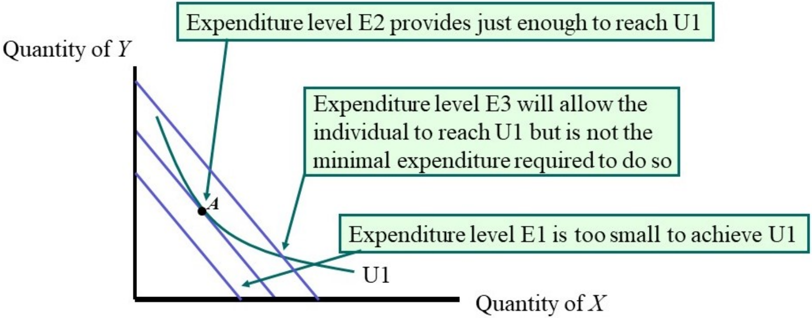

Figure 6: The dual expenditure-minimization problem

Expenditure Minimization: Mathematical Formulation

- The individual's dual expenditure-minimization problem:

$$\text{Minimize Total expenditures} = E = p_1x_1 + p_2x_2 + \ldots + p_nx_n \quad (36)$$

$$\text{Subject to Constraint: Utility} = \Phi(U) = U(x_1, x_2, \ldots, x_n) \quad (37)$$

- The optimal amounts of $x_1, x_2, \ldots, x_n$ depend on prices $(p_1, p_2, \ldots, p_n)$ and the required utility level U.
  - Changes in prices or utility target lead to different optimal commodity bundles.
  - This dependence can be summarized by an expenditure function.

Expenditure Function

- The individual's expenditure function shows the minimal expenditures necessary to achieve a given utility level for a particular set of prices.
- Mathematically, the expenditure function is denoted as $E(p_1, p_2, \ldots, p_n, U) \quad (38)$.
- The expenditure function and the indirect utility function are inversely related
  - Both depending on market prices but involving different constraints (income or utility).
  - This relationship is useful for examining how individuals respond to price changes.

Expenditure Functions

- Two methods for computing expenditure functions:
  1. Direct application of the expenditure-minimization problem using the Lagrangian technique.
  2. Taking advantage of the relationship between expenditure functions and indirect utility functions.
- Indirect utility functions and expenditure functions are inverses of each other, simplifying calculations.

Expenditure Function from the Cobb-Douglas

- Objective: Minimize $E = P_X X + P_Y Y$ subject to $U' = X^{0.5} Y^{0.5}$, where U' is the utility target.
- The Lagrangian expression is:

$$L = P_X X + P_Y Y + \lambda(U' - X^{0.5} Y^{0.5})$$

- First-order conditions:

$$\frac{\partial L}{\partial X} = P_X - 0.5\lambda X^{-0.5} Y^{0.5} = 0$$
$$\frac{\partial L}{\partial Y} = P_Y - 0.5\lambda X^{0.5} Y^{-0.5} = 0$$
$$\frac{\partial L}{\partial \lambda} = U' - X^{0.5} Y^{0.5} = 0$$

Expenditure Function from the Cobb-Douglas (Cont'd)

- These first-order conditions imply that $P_X X = P_Y Y$.
- Substituting into the expenditure function:

$$E = P_X X^* + P_Y Y^* = 2P_X X^*$$

- Solving for the optimal values of X^* and Y^*:

$$X^* = \frac{E}{2P_X}$$
$$Y^* = \frac{E}{2P_Y}$$

- Substituting into the utility function, we can get the indirect utility function

$$U' = \left(\frac{E}{2P_X}\right)^{0.5} \left(\frac{E}{2P_Y}\right)^{0.5} = \frac{E}{\left(2P_X^{0.5} P_Y^{0.5}\right)}$$

- Therefore, the expenditure function becomes:

$$E = 2U' \cdot P_X^{0.5} P_Y^{0.5}$$

Before we forget

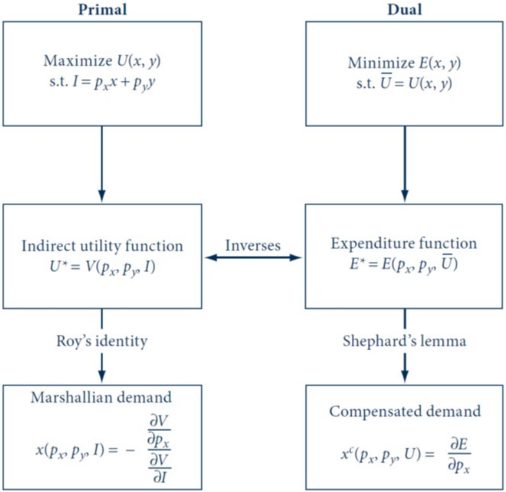

Figure 7: Relationships among Demand Concepts

Outline

Optimization Problem

- **Objective:** Solve for the optimal consumption bundle given a Cobb-Douglas utility function and budget constraint.

Utility Function

$$U(x, y) = x^a \cdot y^b$$

Budget Constraint

$$I = p_x \cdot x + p_y \cdot y$$

Optimal Consumption Bundle

$$x^* = \frac{a \cdot I}{p_x}$$

$$y^* = \frac{b \cdot I}{p_y}$$

Calibration Step by Step

1. **Define Parameters:** Choose values for a, b, p_x, p_y, and I.
2. **Calculate Optimal Bundle:** Use the formulas to calculate the optimal quantities of goods x and y.
3. **Generate Data:** Create data points for plotting indifference curves and the budget constraint.
4. **Plotting:** Use ggplot2 to plot the indifference curves and the budget constraint.

Calibrating Model Parameters

- **Initial Conditions:**
  - The initial prices (p_x and p_y), utility function parameters (a and b), and initial income (I) can be calibrated based on empirical data or theoretical assumptions.
  - Empirical calibration may involve analyzing historical data on prices, incomes, and consumption patterns to estimate reasonable initial values for these parameters.
  - Theoretical calibration may rely on economic theory or existing literature to determine plausible values for the parameters based on economic principles or behavioral assumptions.
- **Utility Function Parameters:**
  - The parameters a and b in the Cobb-Douglas utility function represent the preferences of the consumer for goods x and y.
  - These parameters can be calibrated based on empirical data on consumer behavior, such as expenditure patterns or survey data on preferences.
  - Alternatively, theoretical assumptions or insights from economic theory can inform the choice of parameter values. For example, $a = b = 0.5$ represents equal preference for both goods.

R Code

- Define the parameters: $a = 0.5$, $b = 0.5$, $p_x = 1$, $p_y = 4$, $I = 8$.
- Calculate the optimal consumption bundle using the Cobb-Douglas utility function.
- Generate data for plotting.
- Plot the indifference curves and the budget constraint.

R Code

```r
# Load necessary libraries
library(ggplot2)

# Define the utility function
U <- function(x, y) {
  x^a * y^b  # Cobb-Douglas utility function
}

# Define budget constraint
budget_constraint <- function(px, py, I, x) {
  (I - px * x) / py  # Calculate y from the budget
  constraint
}
```

R Code: Calibration

```r
# Define price, income, and parameters
px <- 1  # Price of good x
py <- 4  # Price of good y
I <- 8   # Income
a <- 0.5 # Parameter a in the Cobb-Douglas utility function
b <- 0.5 # Parameter b in the Cobb-Douglas utility function

# Calculate optimal consumption bundle
result <- optimize_expenditure_cobb_douglas(px, py, I, a, b)
print(round(result, 2))  # Optimal quantities of goods x
and y
```

R Code: Generate Data

```r
# Generate data for plotting indifference curves
x_values <- seq(0.1, 10, by = 0.1)
y_values <- seq(0.1, 10, by = 0.1)
ind_curves <- expand.grid(x = x_values, y = y_values)
ind_curves$U <- U(ind_curves$x, ind_curves$y)

# Generate data for plotting budget constraint
x_budget <- seq(0, I / px, length.out = 100)
y_budget <- budget_constraint(px, py, I, x_budget)
budget_data <- data.frame(x = x_budget, y = y_budget)
```

R Code: Plotting

```r
# Plotting
ggplot() +
  # Indifference curves
  geom_contour(data = ind_curves, aes(x = x, y = y, z = U), color = "blue") +
  # Budget constraint
  geom_line(data = budget_data, aes(x = x, y = y), color = "red") +
  # Optimal consumption bundle
  geom_point(data = data.frame(x = result[1], y = result[2]), aes(x = x, y = y),
   color = "green", size = 3) +
  # Utility value annotation
  annotate("text", x = result[1], y = result[2], label = paste("U =", round(U(result[1],
  # Labels and titles
  labs(x = "Good 1", y = "Good 2",
   title = "Optimal Consumption Bundle and Indifference Curves") +
  theme_minimal()
```

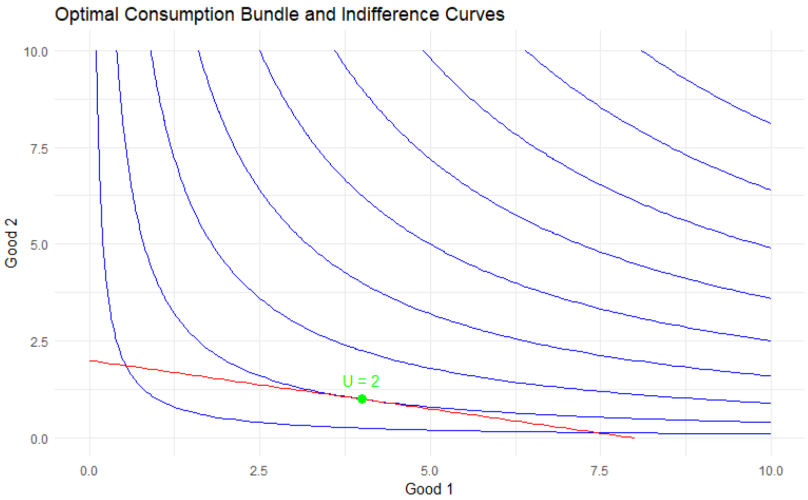

Figure 8: Optimal Consumption Bundle and Indifference Curves

Comparative microsimulation analysis

1. **Sensitivity Analysis:** See sim01.R
   - Simulate changes in parameters a and b.
2. **Price Change:** See sim02.R
   - Simulate changes in the prices of goods p_x and p_y.
3. **Income Changes:** See sim03.R
   - Simulate changes the income I and observe its effects on the budget constraint and the optimal consumption bundle.
4. **Consumer Surplus and Welfare:** See sim04.R
   - Calculate and visualize consumer surplus and overall welfare for different consumption bundles.
5. **Long-Run vs. Short-Run Analysis:** See sim05.R
   - Discuss the difference between short-run and long-run consumption decisions.

SIM1: Sensitivity Analysis of Optimal Consumption Bundle

- Explanation:
  1. Parameter Variation:
     - A sequence of a and b values is defined from 0.1 to 1 with a step size of 0.1.
  2. Looping Through Combinations:
     - Empty vectors store results for each combination of a and b.
     - Optimal consumption bundle is calculated for each combination.
  3. Storing Results:
     - Optimal quantities of x and y are stored in separate vectors.
  4. Visualization:
     - Results are visualized using ggplot2, showing optimal x values across different a and b combinations.
- Interpretation:
  - The heatmap reveals how changes in a and b affect the optimal consumption bundle.
  - Higher values of a or b may indicate stronger preferences for the corresponding good.

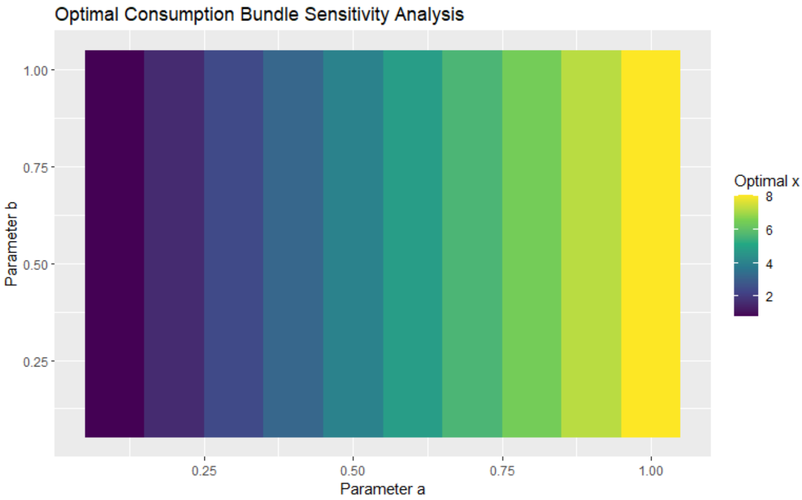

Figure 9: Optimal Consumption Bundle Sensitivity Analysis

SIM2: Optimal Consumption Bundle with Price Changes

- Objective:
  - Simulate changes in the optimal consumption bundle for a Cobb-Douglas utility function in response to variations in the prices of goods.
- Procedure:
  1. Define a range of price values for goods x and y.
  2. Simulate changes in the optimal consumption bundle for each combination of price values.
  3. Plot the optimal consumption bundle on a scatter plot, with different colors representing different price levels of good x.
- Intuition:
  - As the prices of goods x and y change, the optimal consumption bundle shifts accordingly.
  - Higher prices of good x lead to lower quantities consumed, while lower prices lead to higher quantities consumed.
  - The plot helps visualize the relationship between the quantity of good x and its price, providing insights into consumer behavior and market equilibrium.

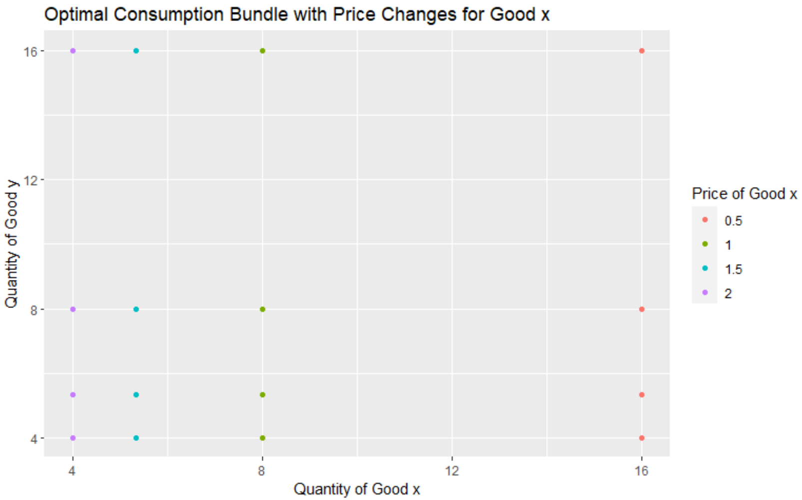

Figure 10: Optimal Consumption Bundle Sensitivity Analysis

SIM3: Optimal Consumption Bundle with Income Changes

- Objective:
  - Simulate changes in the optimal consumption bundle for a Cobb-Douglas utility function in response to variations in income levels.
- Procedure:
  1. Create empty vectors to store results for optimal quantities of goods x and y.
  2. Simulate changes in the optimal consumption bundle for each income level.
  3. Plot the optimal consumption bundle, with income levels on the x-axis and quantities of goods on the y-axis.
- Intuition:
  - As income levels change the optimal consumption bundle shifts accordingly.
  - Higher income levels lead to higher quantities consumed of both goods x and y, while lower income levels lead to lower quantities consumed.
  - We visualize the relationship between income levels and quantities consumed, providing insights into consumer behavior and welfare analysis.

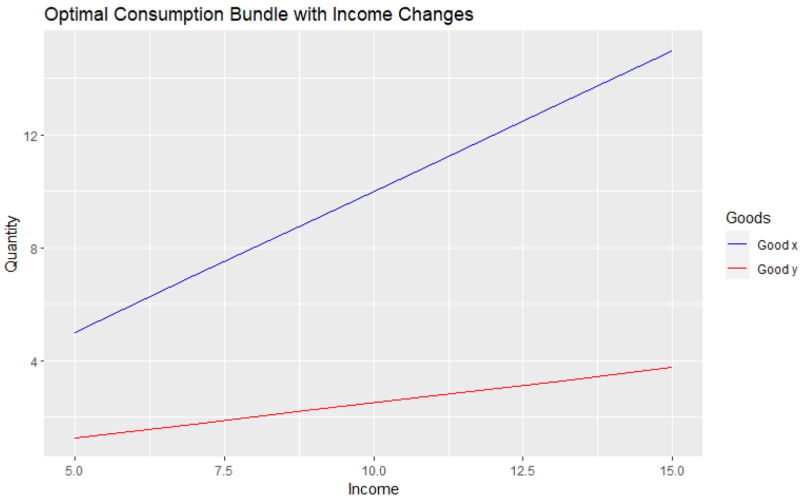

Figure 11: Optimal Consumption Bundle with Income Changess

SIM4: Changes in Consumer Surplus and Overall Welfare

- Objective:
  - Simulate changes in consumer surplus and overall welfare for a Cobb-Douglas utility function in response to variations in prices and income levels.
- Procedure:
  1. Define a range of price values for goods x and y, and income levels.
  2. Simulate changes in consumer surplus and overall welfare for each combination of price values and income levels.
  3. Plot the changes in consumer surplus and overall welfare on bar plots, with scenarios on the x-axis and respective values on the y-axis.
- Intuition:
  - Changes in prices and income levels affect consumer surplus and overall welfare.
  - Higher prices of goods x and y reduce consumer surplus, while higher income levels increase it.
  - Overall welfare is the sum of consumer surplus and total utility, reflecting the satisfaction derived from consumption.

SIM4: Program Overview

- **Functions:**
  - **Consumer Surplus Function (consumer_surplus):** This function calculates the consumer surplus given the prices of goods (px, py), income (I), and the quantities of goods consumed (x, y). Consumer surplus is calculated as the difference between the total amount of money a consumer is willing to pay for a good or service (reflected by their income) and the total amount they actually pay for it (reflected by the total expenditure on the goods).
  - **Overall Welfare Function (overall_welfare):** This function calculates the overall welfare, which is the sum of consumer surplus and total utility. Total utility typically represents the satisfaction or happiness derived from consuming goods and services.

SIM4: Program Overview

- **Simulation of Changes in Prices and Income:**
  - The program simulates changes in prices (*px* and *py*) and income (*I*) by looping through the specified values of *px_values*, *py_values*, and *I_values*.
  - For each combination of price changes and income changes, the program calculates the optimal consumption bundle (x and y) using the `optimize_expenditure_cobb_douglas` function. This function likely finds the quantities of goods that maximize utility given the new prices and income, assuming a Cobb-Douglas utility function.
  - Consumer surplus and overall welfare are then calculated for each scenario using the previously defined functions.
  - Results for each scenario, including the new prices, income, optimal quantities, consumer surplus, and overall welfare, are stored in a list.

SIM4: Program Overview

- **Dataframe Creation and Plotting:**
  - The results are converted into a dataframe (`results_df`) for ease of manipulation and visualization.
  - A unique identifier for each scenario (`scenario`) is created using the combination of income, price of good x, and price of good y.
  - The changes in consumer surplus and overall welfare are plotted against these unique scenarios using bar plots.
- **Visualization:**
  - The bar plots visualize the changes in consumer surplus and overall welfare across different scenarios.

SIM4: Example Breakdown

- **px (Price of Good x):** 0.5
- **py (Price of Good y):** 2.0
- **I (Income):** 5.0
- **x (Quantity of Good x):** 10.0
- **y (Quantity of Good y):** 2.5
- **Consumer Surplus:** -5.0
- **Overall Welfare:** 20.0

Interpretation:

- The consumer's income is 5.0, and they face prices of 0.5 for Good x and 2.0 for Good y.
- With these prices and income, the consumer purchases 10.0 units of Good x and 2.5 units of Good y.
- The consumer surplus is -5.0, indicating that the consumer is willing to pay 5.0 units of currency more for the goods than what they actually pay.
- The overall welfare, which is the sum of consumer surplus and total utility, is 20.0, indicating the level of satisfaction or utility derived from consuming the given quantities of goods.

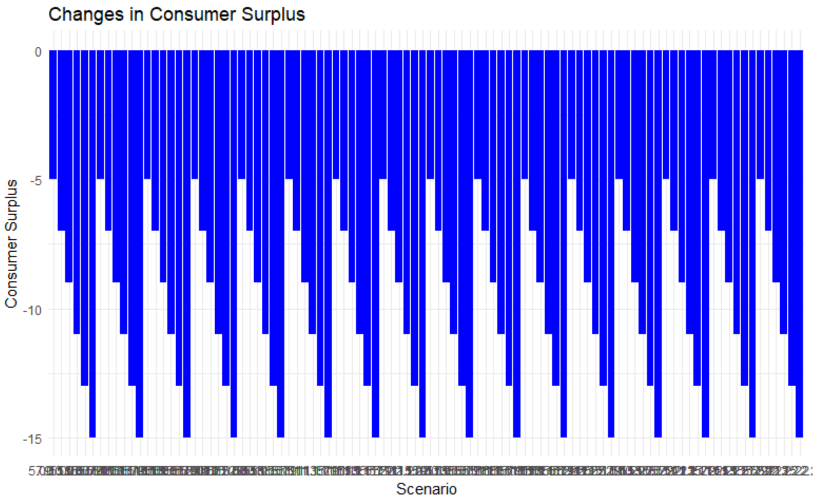

Figure 12: Changes in Consumer Surplus

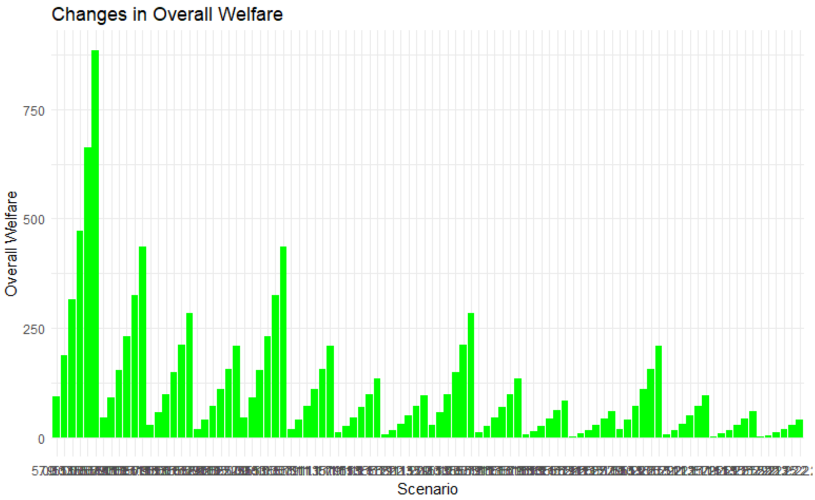

Figure 13: Changes in Overall Welfare

SIM5: Adjustments in Consumption Patterns Over Time

- Objective
  - Simulate adjustments in consumption patterns for a Cobb-Douglas utility function over a period of time.
- Procedure
  1. Define initial conditions including prices of goods (p_x, p_y), income level (I), and utility function parameters (a, b).
  2. Utilize the Cobb-Douglas utility function $U(x, y) = x^a \cdot y^b$ to model consumer preferences, where x and y represent the quantities of goods consumed.
  3. Simulate changes in consumption patterns over time by updating prices and income linearly. For example:

$$p_{x_t} = p_x + \alpha \cdot t \quad \text{and} \quad p_{y_t} = p_y - \beta \cdot t$$

$$I_t = I + \gamma \cdot t$$

     where α, β, and γ are constants representing the rate of change.
  4. Solve optimization problems at each time step to determine the optimal consumption bundle (x_t, y_t).

SIM5: Adjustments in Consumption Patterns Over Time

- Calibration of the Rate of Change
  - In the model, the rates of change (0.1 for prices and -0.05 for p_y per time unit, and 2 for income) are chosen arbitrarily.
  - These rates of change can be adjusted based on the specific context or dynamics being modeled. For example, if analyzing a period of rapid economic growth, higher rates of change for income may be appropriate.
  - Sensitivity analysis can be conducted to assess the model's robustness to variations in the rates of change.
- Intuition
  - Consumption patterns adjust over time in response to changes in prices and income levels.
  - Higher prices of goods x and y lead to lower quantities consumed, while higher income levels lead to higher quantities consumed.
  - The Cobb-Douglas utility function captures the trade-off between goods x and y based on their respective parameters a and b. Higher values of a and b indicate a greater preference for the corresponding goods.

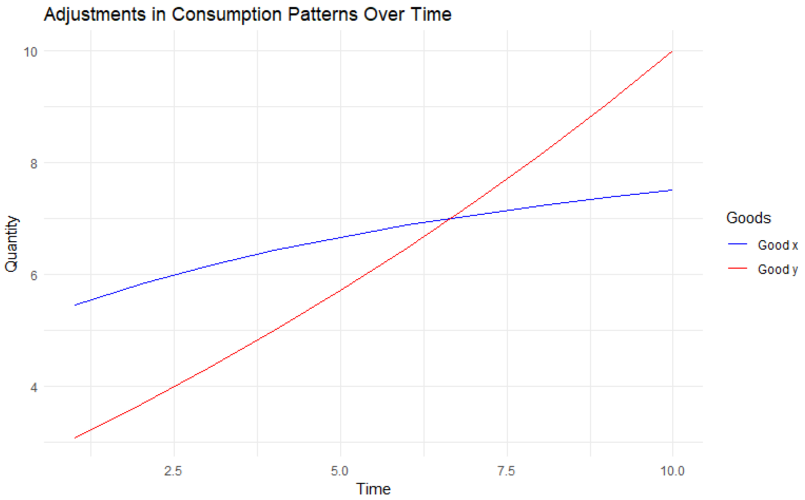

Figure 14: Adjustments in Consumption Patterns Over Time

Outline

Key Questions

1. How do individuals' consumption choices vary in response to changes in market prices and income levels, and what are the implications for welfare analysis?
2. Can behavioral economics concepts and experimental methods enhance our understanding of consumer choice behavior in developing economies?
3. What are the distributional effects of changes in commodity prices on different income groups within a society, and how do these effects influence overall welfare?
4. How do cultural and social factors shape individuals' preferences and decision-making processes regarding consumption patterns in developing countries?
5. What are the implications of corner solutions in utility maximization models for policy design aimed at improving welfare outcomes?
6. How do changes in taxation policies, such as introducing or removing subsidies on essential goods, affect consumer behavior and overall welfare in low-income communities?