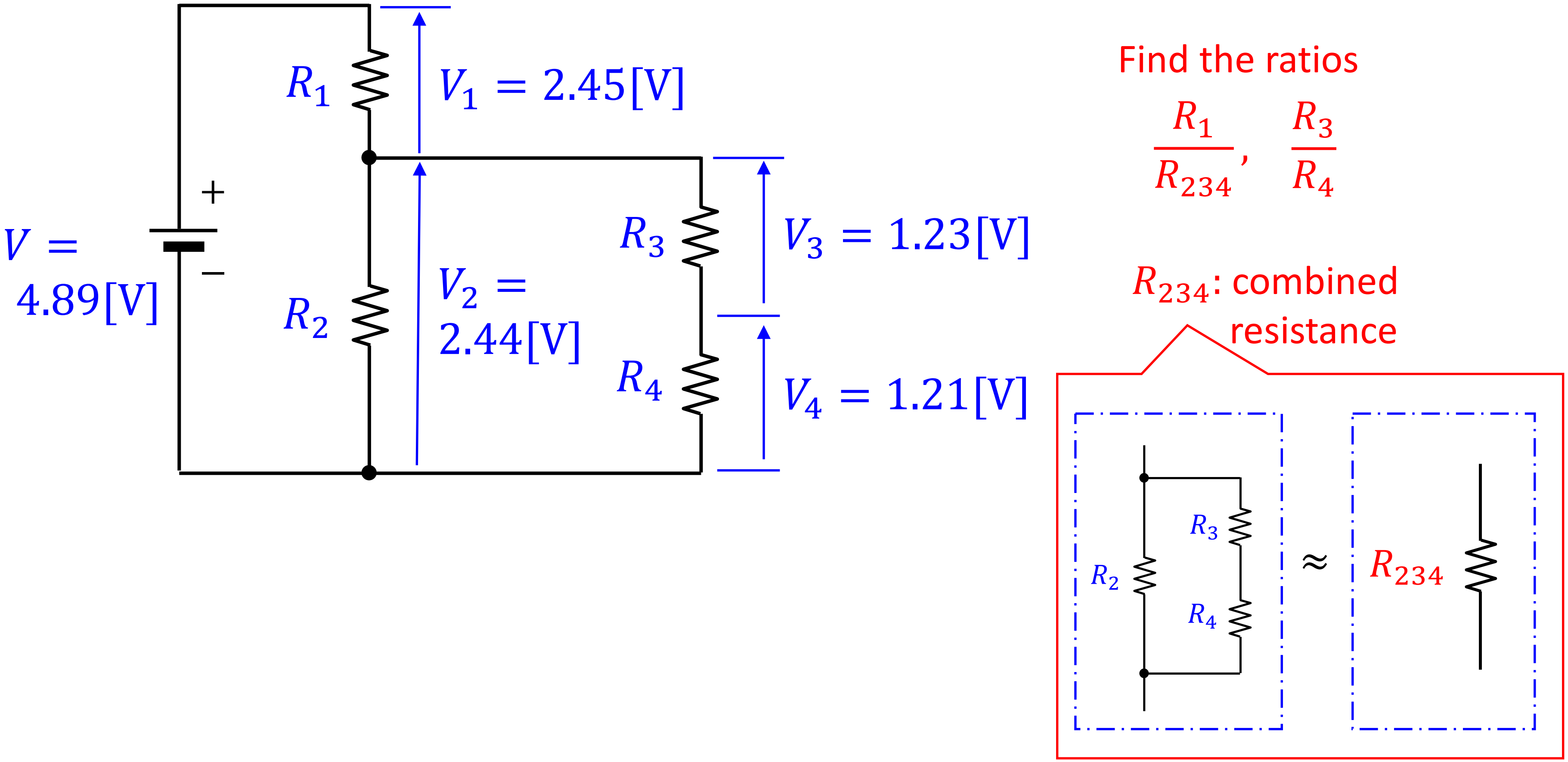
Answers to Homework

Exercise 1.4.2 (Homework)

Exercise 1.4.2 (Homework)

Exercise 1.4.2. This is your homework.

Exercise 1.4.2 (Homework)



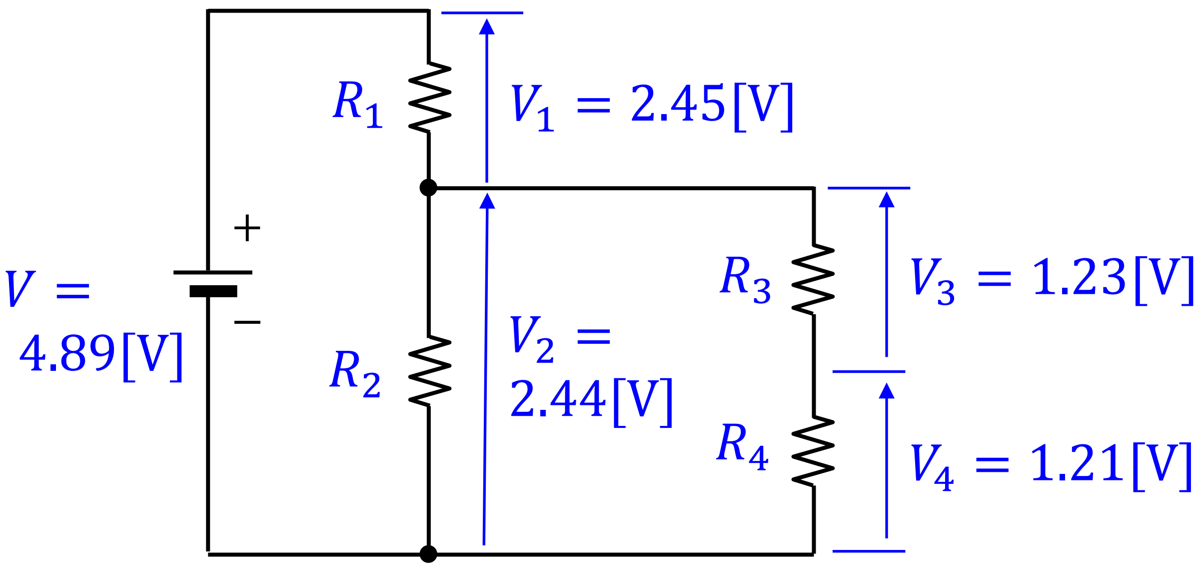
Find the ratios of R_1/R_{234}, and R_3/R_4, where R_{234} is the combined resistance of R_2, R_3, and R_4. The right bottom figure shows an image of the combined resistance.

Exercise 1.4.2 (Answers)

Exercise 1.4.2 (Answers)

The answers to Exercise 1.4.2

Exercise 1.4.2 (Answers)



$$\frac{R_1}{R_{234}} = \frac{V_1}{V_2} = \frac{V_1}{V_3 + V_4} = \frac{2.45}{2.44} = 1.00$$

$$\frac{R_3}{R_4} = \frac{V_3}{V_4} = \frac{1.23}{1.21} = 1.02$$

$R_1/R_{234} = 1.00$, and $R_3/R_4 = 1.02$. The numbers are rounded at the third decimal places.

Exercise 2.2.1 (Homework)

Exercise 2.2.1 (Homework)

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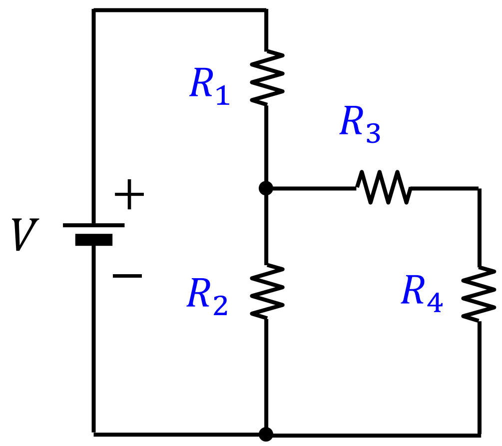
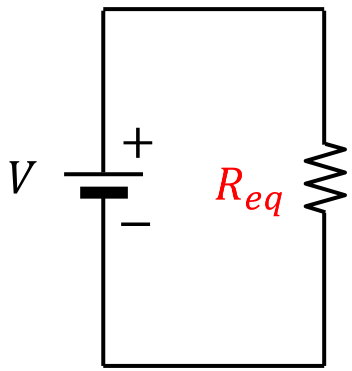




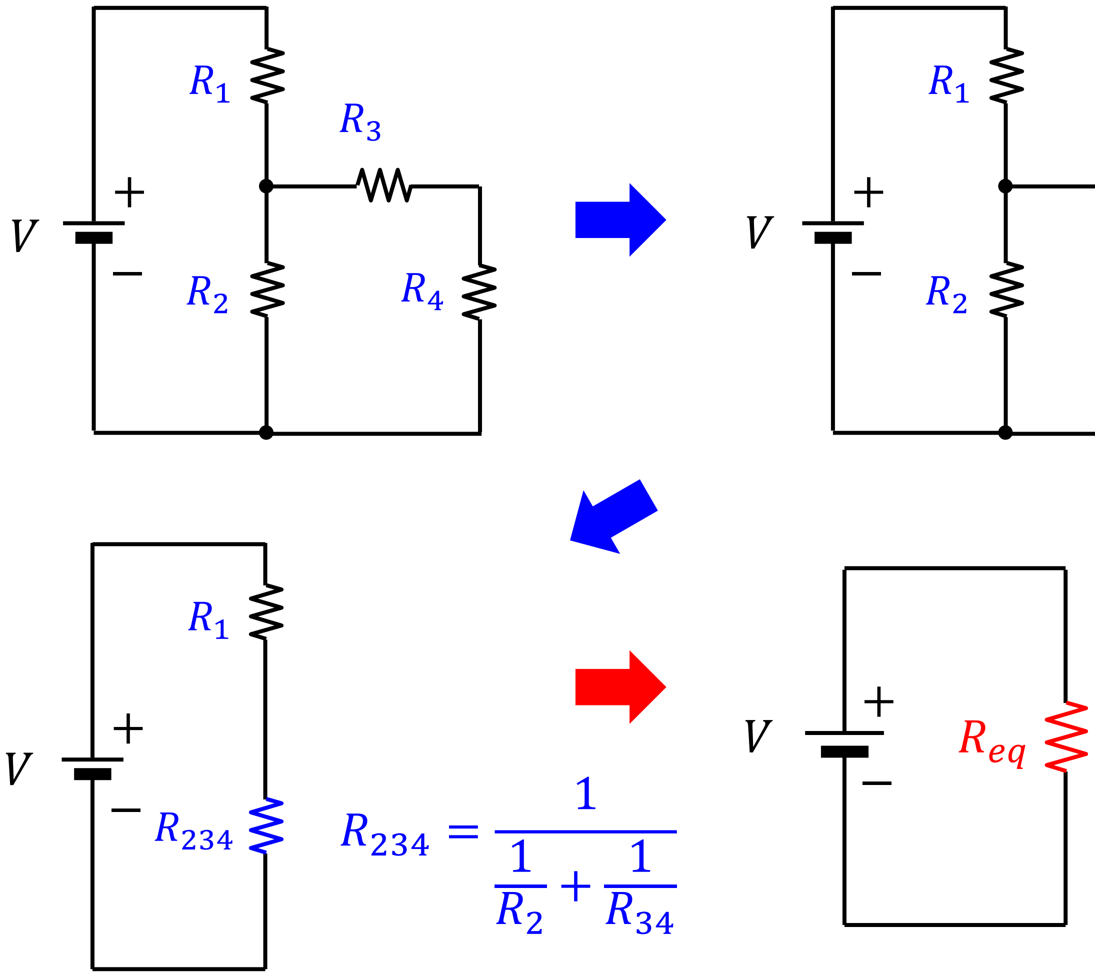
Write R_{eq}.

Write the resistance R_{eq} which is equivalent to the combined resistance of $R_1, \cdots, R_4$ in the top circuit.

Exercise 2.2.1 (Answer)

Exercise 2.2.1 (Answer)

Exercise 2.2.1



$$R_{34} = R_3 + R_4$$

$$R_{234} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_{34}}}$$

$$\begin{aligned} R_{eq} &= R_1 + R_{234} \\ &= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_{34}}} \\ &= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + R_4}} \end{aligned}$$

The equivalent resistance R_{eq} of $R_1, \cdots, R_4$ is that $R_{eq} = R_1 + 1/(1/R_2 + 1/(R_3{+}R_4))$.

Exercise 2.2.2 (Homework)

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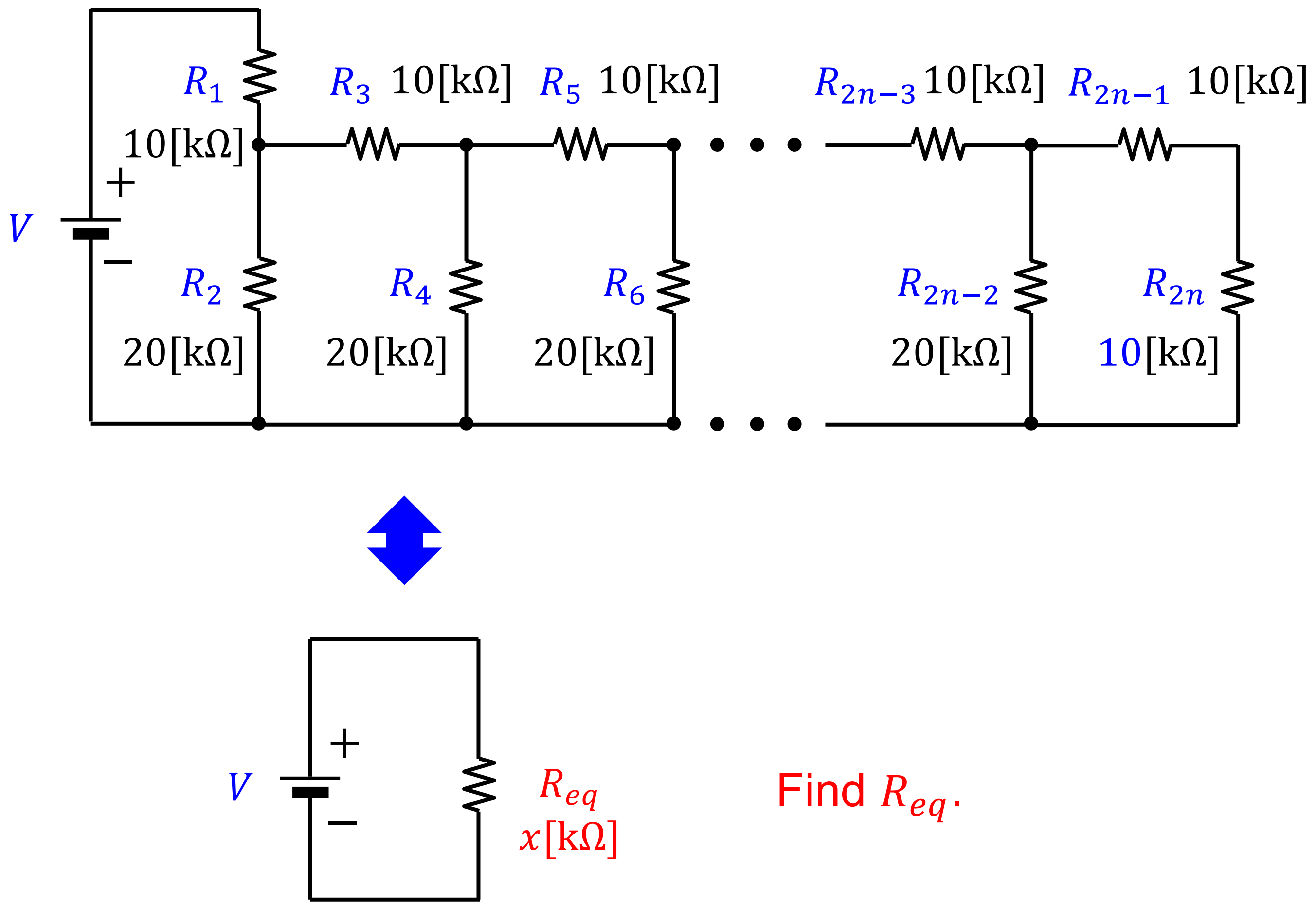


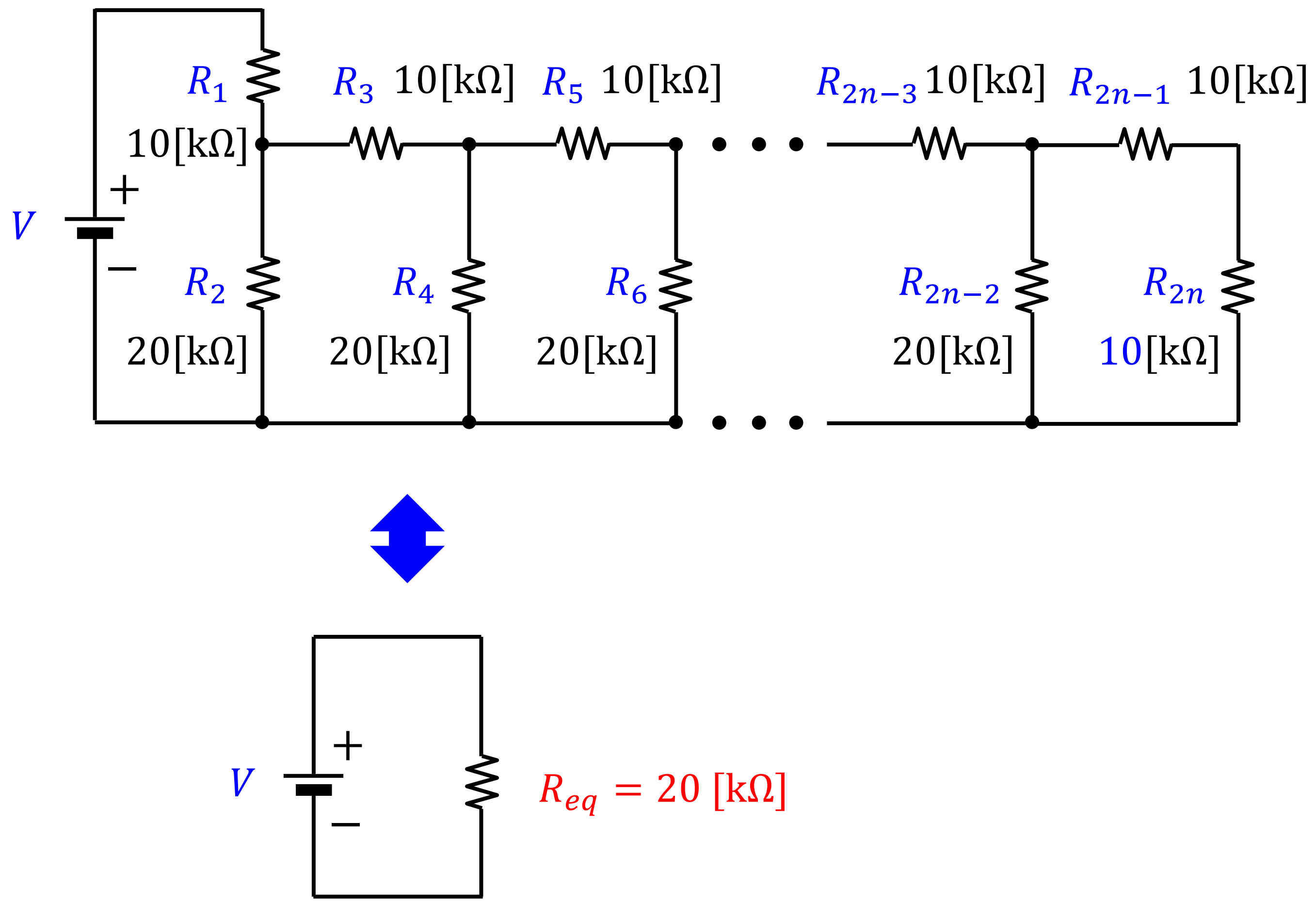
Find R_{eq}.

Find the equivalent resistance R_{eq}.

Exercise 2.2.2 (Answer)

Exercise 2.2.2 (Answer)

Exercise 2.2.2 (Answer)



The result of Exercise 2.2.1 can be applied to this circuit.

Exercise 3.2 (Homework)

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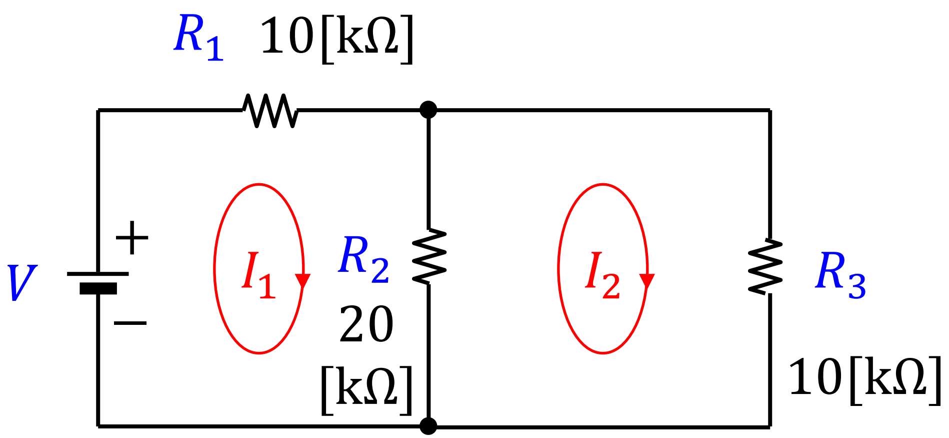
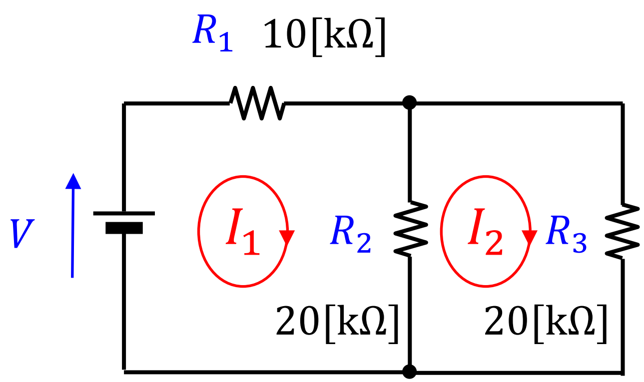


1. Write the currents I_1, I_2.
2. Calculate I_1 and I_2 using the voltage V measured in Experiment 3.2.2.

Calculate I_1, I_2 with the measured V in Experiment 3.2.2.

Exercise 3.2 (Answers)

Exercise 3.2 (Answers)



KVL gives that

$$V = (R_1 + R_2)I_1 - R_2 I_2$$
$$0 = -R_2 I_1 + (R_2 + R_3)I_2$$

Then

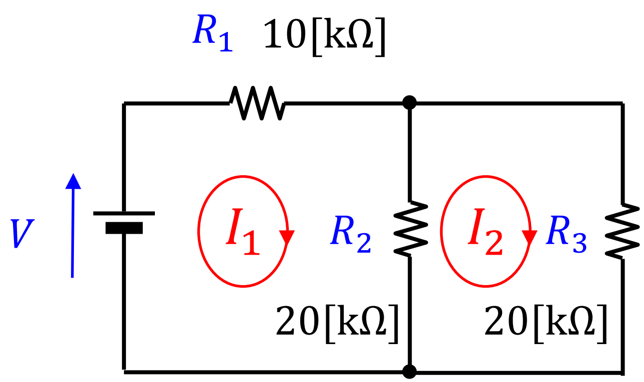
$$\begin{pmatrix} V \\ 0 \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix}^{-1} \begin{pmatrix} V \\ 0 \end{pmatrix}$$

$$I_1 = \frac{R_2 + R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} V$$

$$I_2 = \frac{R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} V$$

Exercise 3.2 (Answers)



Given that $R_1 = 10[\mathrm{k\Omega}]$, $R_2 = R_3 = 20[\mathrm{k\Omega}]$, and the measured $V = 3.88[\mathrm{V}]$, then

$$I_1 = \frac{3.88[\mathrm{V}]}{20[\mathrm{k\Omega}]} = 194 \quad [\mathrm{\mu A}]$$

$$I_2 = \frac{3.88[\mathrm{V}]}{40[\mathrm{k\Omega}]} = 97 \quad [\mu\mathrm{A}]$$

The measured values in Exp. 3.2.2:

$V = 3.88[\mathrm{V}]$
$I_1 = 194\ [\mathrm{\mu A}]$
$I_2 = 97\ [\mathrm{\mu A}]$

Exercise 3.5 (Homework)

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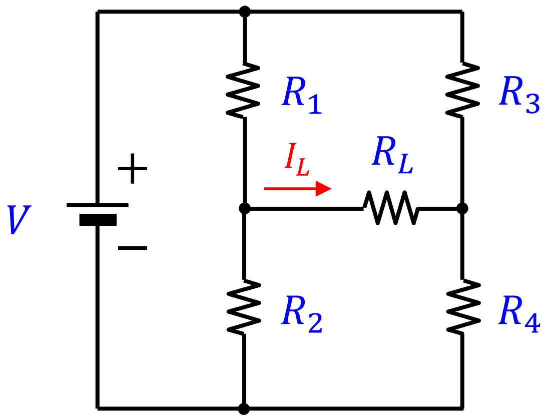

Show that if $R_2R_3 = R_2R_4$, then $I_L = 0$.

Exercise 3.5 (Answer)

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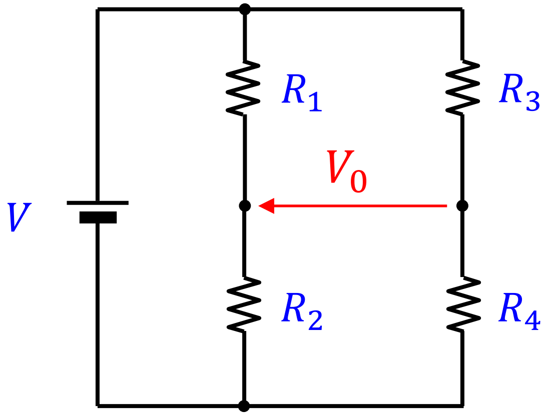
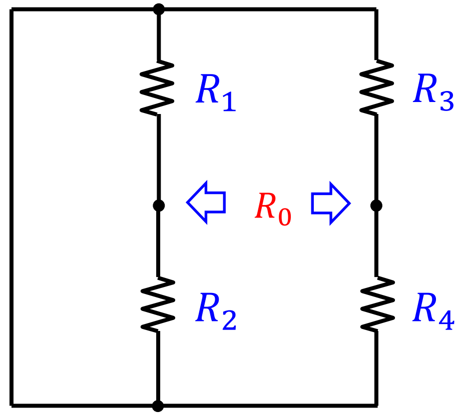
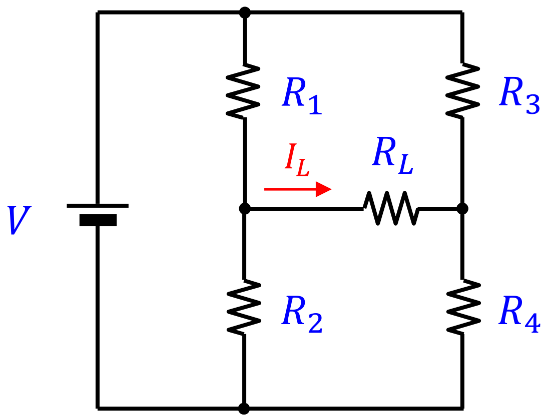


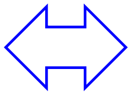
$$V_0 = \frac{R_3}{R_1 + R_3} V - \frac{R_4}{R_2 + R_4} V$$
$$= \frac{R_2 R_3 - R_2 R_4}{(R_1 + R_3)(R_2 + R_4)} V$$

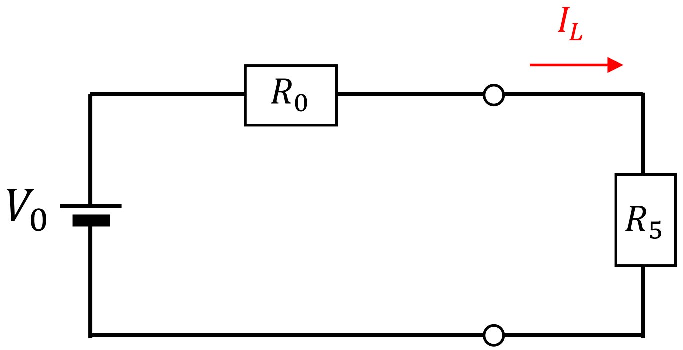


$$R_0 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3}} + \frac{1}{\frac{1}{R_2} + \frac{1}{R_4}}$$
$$= \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

Exercise 3.5 (Answer)







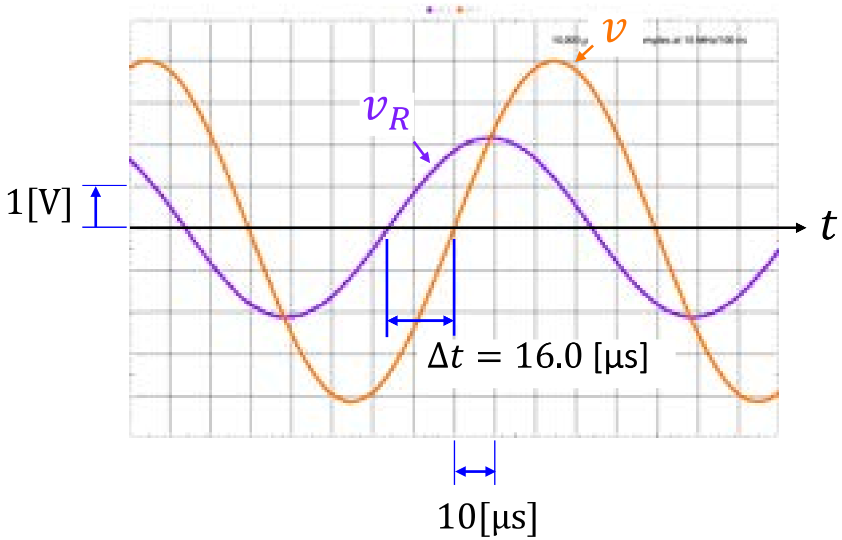
if $R_2R_3 = R_2R_4$,

then $V_0 = 0$ and $I_L = 0$

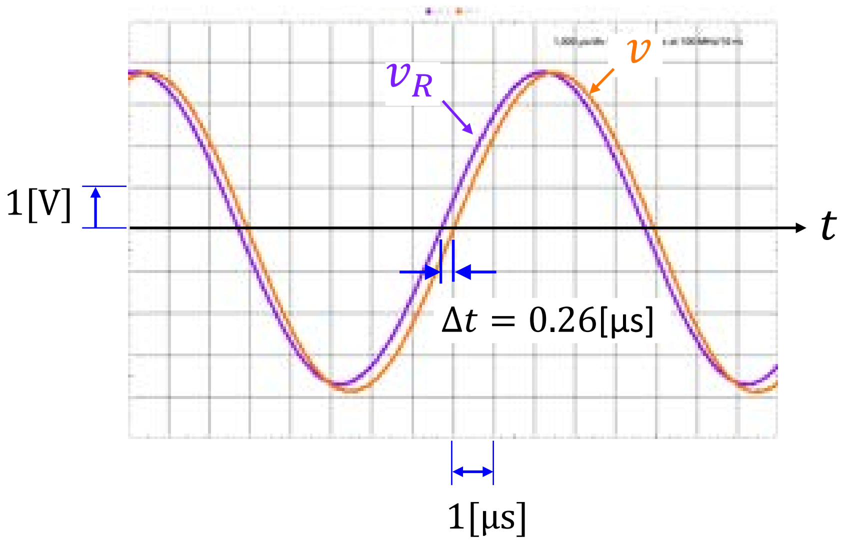
Exercise 4.4.1 (Homework)

Exercise 4.4.1 (Homework)

Exercise 4.4.1 (Homework)



(a) f =10[kHz]



(b) f =100[kHz]

Calculate the phase shifts θ_s of v relative to v_R in (a) and (b). Answer in [°] and [rad].

Exercise 4.4.1 (Answers)

Exercise 4.4.1 (Answers)

Exercise 4.4.1 (Answers)

(a) f =10[kHz]

$$T = \frac{1}{f} = \frac{1}{10000} = 100\ [\mu s]$$

$$\theta_s = -\frac{\Delta t}{T} \times 360\ [°] = -\frac{16.0\ [\mu s]}{100\ [\mu s]} \times 360\ [°] = -57.6\ [°]$$

$$\theta_s = -\frac{\Delta t}{T} \times 2\pi\ [\mathrm{rad}] = -\frac{16.0\ [\mu s]}{100\ [\mu s]} \times 2\pi\ [\mathrm{rad}] = -0.320\pi\ [\mathrm{rad}]$$

(b) f =100[kHz]

$$T = \frac{1}{f} = \frac{1}{100000} = 10\ [\mu s]$$

$$\theta_s = -\frac{\Delta t}{T} \times 360\ [°] = -\frac{0.26\ [\mu s]}{10\ [\mu s]} \times 360\ [°] = -9.4\ [°]$$

$$\theta_s = -\frac{\Delta t}{T} \times 2\pi\ [\mathrm{rad}] = -\frac{0.26\ [\mu s]}{10\ [\mu s]} \times 2\pi\ [\mathrm{rad}] = -0.052\pi\ [\mathrm{rad}]$$

Exercise 4.4.2 (Homework)

Exercise 4.4.2 (Homework)

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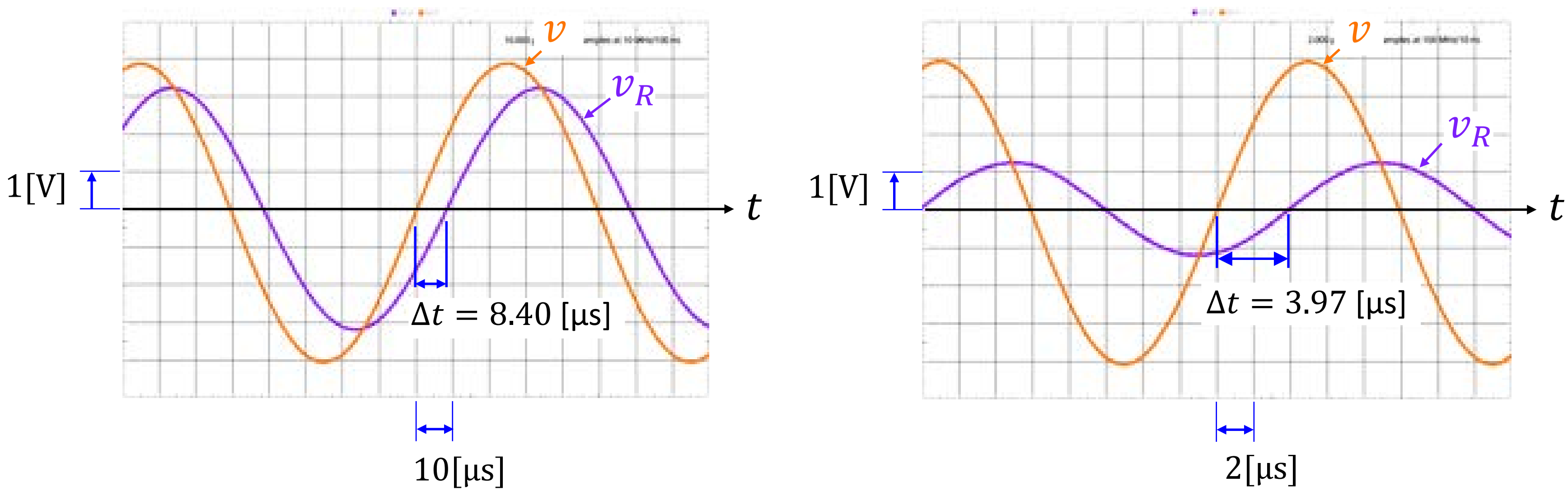


(a) f =10[kHz]

(b) f =50[kHz]

Calculate the phase shifts θ_s of v relative to v_R in (a) and (b). Answer in [°] and [rad].

Exercise 4.4.2 (Answers)

Exercise 4.4.2 (Answers)

Exercise 4.4.2 (Answers)

(a) f =10[kHz]

$$T = \frac{1}{f} = \frac{1}{10000} = 100\ [\mu\text{s}]$$

$$\theta_s = \frac{\Delta t}{T} \times 360\ [^\circ] = \frac{8.40\ [\mu\text{s}]}{100\ [\mu\text{s}]} \times 360\ [^\circ] = 30.2\ [^\circ]$$

$$\theta_s = \frac{\Delta t}{T} \times 2\pi\ [\text{rad}] = \frac{8.40\ [\mu\text{s}]}{100\ [\mu\text{s}]} \times 2\pi\ [\text{rad}] = 0.168\pi\ [\text{rad}]$$

(b) f =50[kHz]

$$T = \frac{1}{f} = \frac{1}{50000} = 20\ [\mu\text{s}]$$

$$\theta_s = \frac{\Delta t}{T} \times 360\ [^\circ] = \frac{3.97\ [\mu\text{s}]}{20\ [\mu\text{s}]} \times 360\ [^\circ] = 71.5\ [^\circ]$$

$$\theta_s = \frac{\Delta t}{T} \times 2\pi\ [\text{rad}] = \frac{3.97\ [\mu\text{s}]}{20\ [\mu\text{s}]} \times 2\pi\ [\text{rad}] = 0.397\pi\ [\text{rad}]$$

Exercise 5.1.1 (Homework)

Exercise 5.1.1 (Homework)

Exercise 5.1 (Homework)

AC current

$$i = I_m \sin \omega t$$

I_m: Amplitude

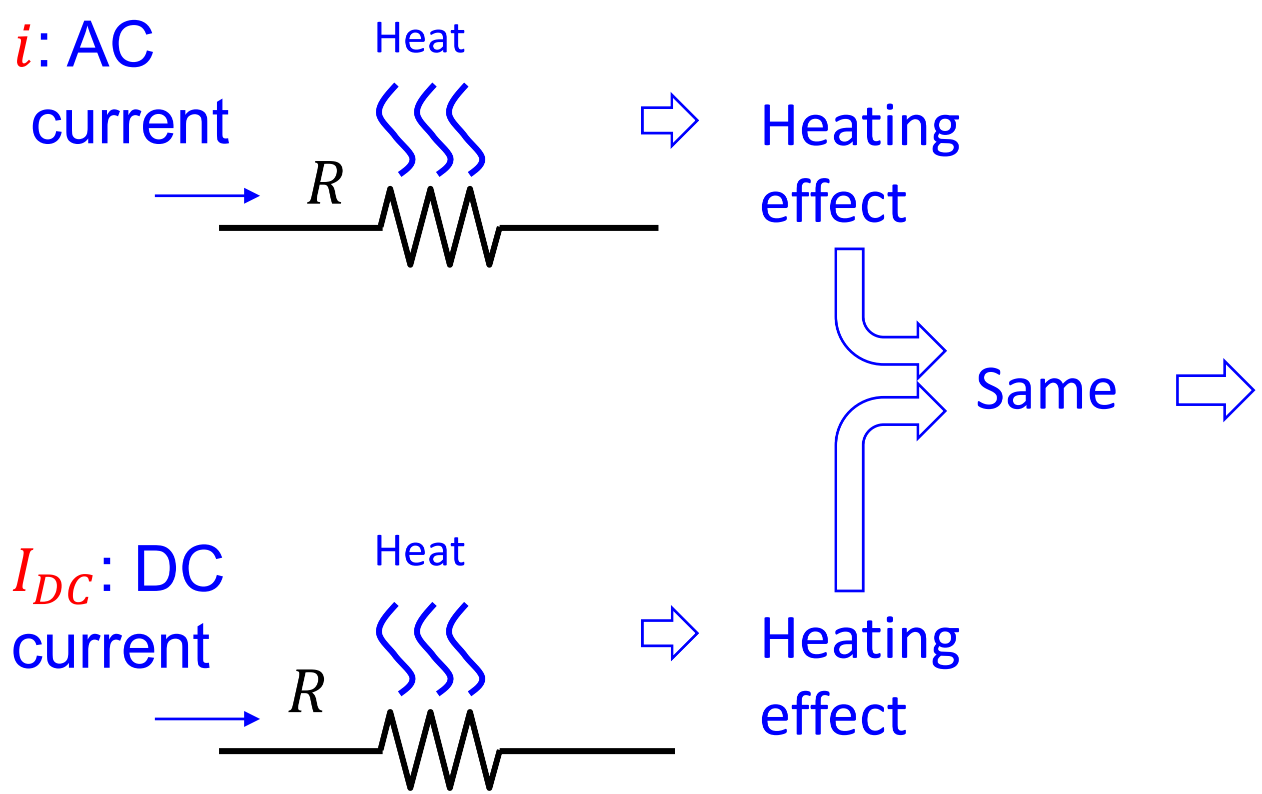
Show that

$$I_e = \frac{I_m}{\sqrt{2}}$$

I_e: Effective value of AC current

An AC current is expressed with $i = I_m \sin \omega t$, where I_m is the amplitude. Show that the effective value of the AC current $I_e = I_m/\sqrt{2}$.

Exercise 5.1 (Homework)



I_{DC}: Equivalent current of AC power

$$I_e \equiv I_{DC}$$

I_e: Effective value of AC current

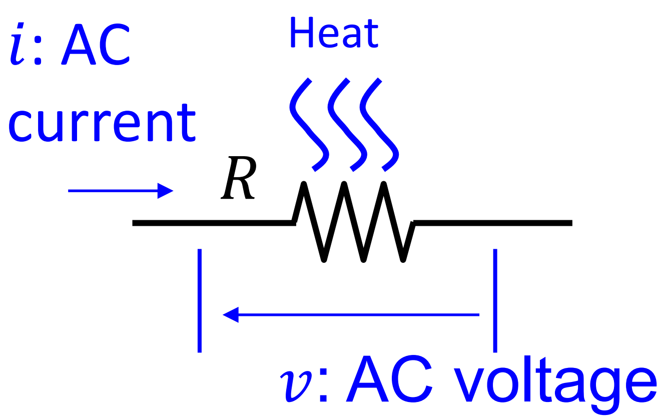
The effective value of AC current is defined as the DC current that gives the same heating effect as the AC current.

Exercise 5.1 (Answer)

Exercise 5.1 (Answer)

Exercise 5.1(Answer)

Heating effect = Average power consumed by the resistor P_{ave}



$$P_{ave} = \frac{1}{T}\int_0^T vi\,dt \quad \Leftarrow \quad v = Ri$$

$$= \frac{1}{T}\int_0^T Ri^2\,dt \quad \Leftarrow \quad i = I_m \sin\omega t$$

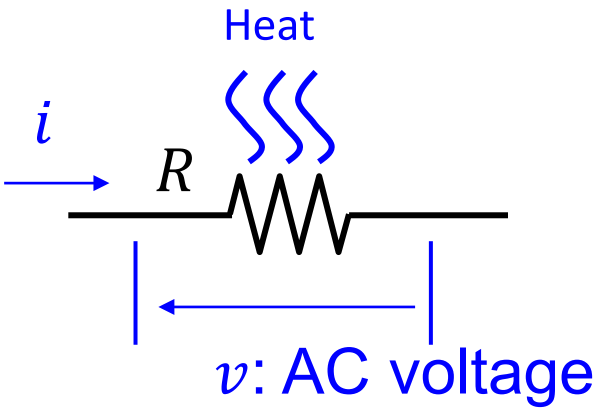
$$= \frac{RI_m^2}{T}\int_0^T \sin^2\omega t\,dt \quad \Leftarrow \quad \cos 2\theta = 1 - 2\sin^2\theta$$

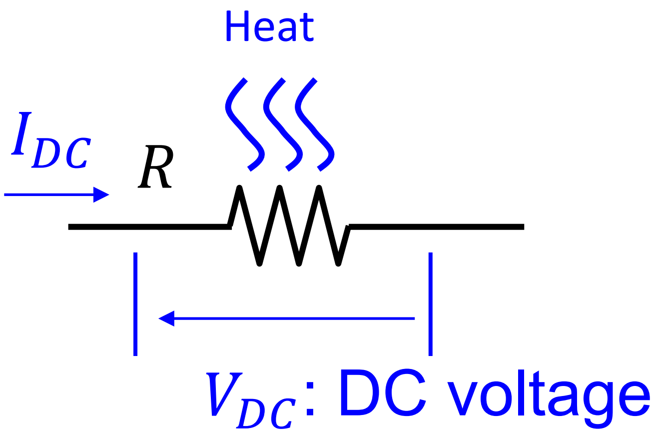
$$= \frac{RI_m^2}{T}\int_0^T \frac{1-\cos 2\omega t}{2}\,dt$$

$$= \frac{RI_m^2}{T}\frac{T}{2}$$

$$= \frac{Ri_m^2}{2}$$

Exercise 5.1 (Answer)





$$P_{ave} = \frac{RI_m^2}{2}$$

$$P_{DC} = RI_{DC}^2$$

$$RI_{DC}^2 = \frac{RI_m^2}{2} \Rightarrow I_{DC} = \frac{I_m}{\sqrt{2}}$$

$$\equiv$$

$$I_e = \frac{I_m}{\sqrt{2}}$$

Exercise 5.4 (Homework)

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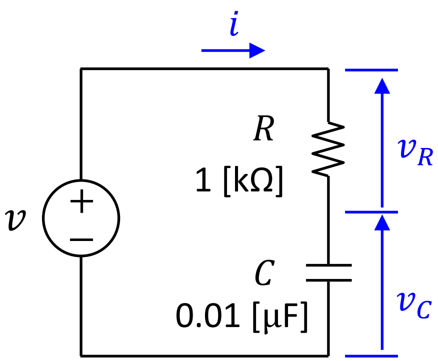
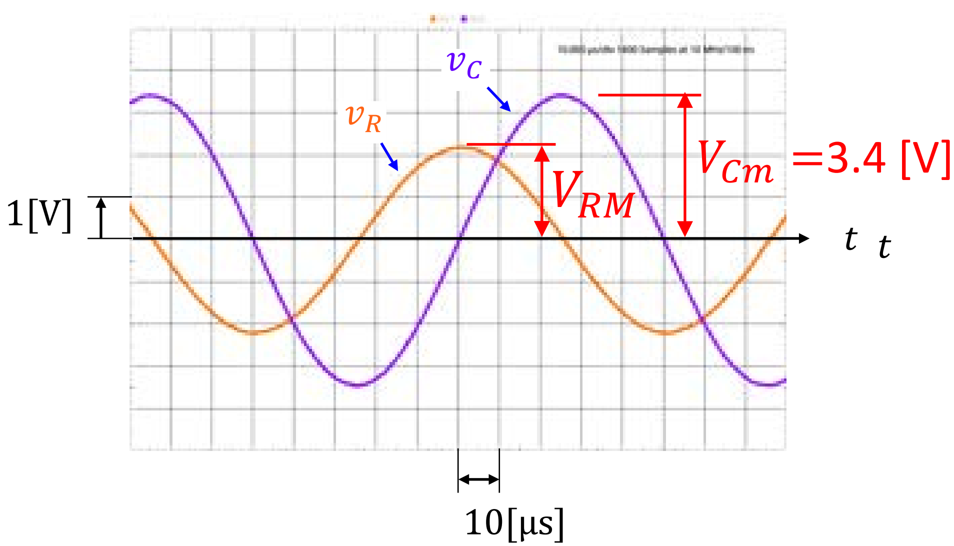


$$v_C = V_{Cm} \sin \omega t$$

$$i = \omega C V_{Lm} \sin\left(\omega t + \frac{\pi}{2}\right)$$



Results in Experiment 5.4.1. $f = 10$ [kHz].

1. Find ωC, where $C = 0.01$ [μF].

2. Find I_m, if $V_{Cm} = 3.4$ [V].

3. Find V_{Rm} using I_m, and compare it to the reading from v_R.

You can round the answers to 2 significant figures

Exercise 5.4 (Answers)

Exercise 5.4 (Answers)

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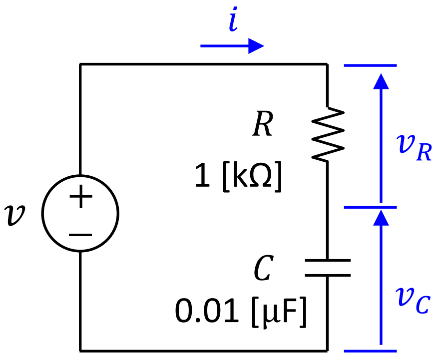

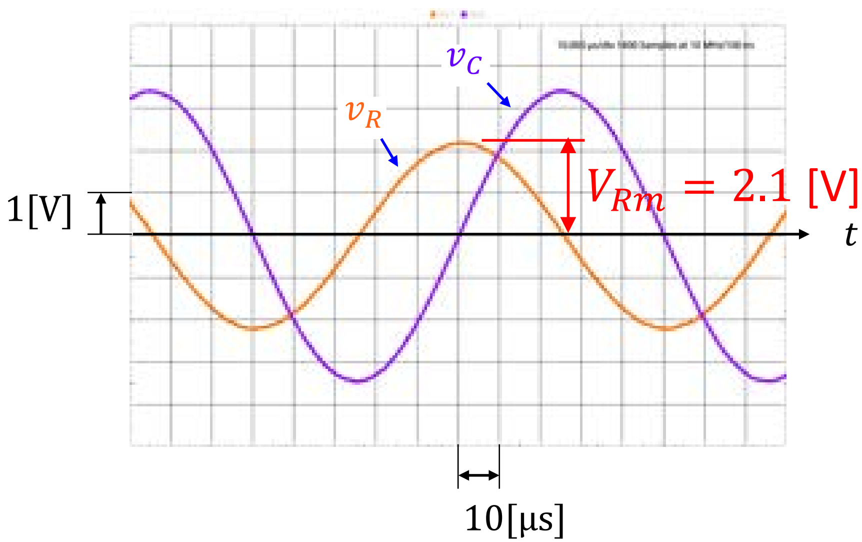

1. Find ωC, where $C = 0.01$ [μF].

$$\omega C = 2\pi f C$$
$$= 2 \times \pi \times 10 \times 10^3 [\text{Hz}] \times 0.01 \times 10^{-6} [\text{H}]$$
$$= 6.3 \times 10^{-4} \ [\Omega]$$

2. Find I_m, if $V_{Cm} = 3.4$ [V].

$$I_m = \omega C V_{Lm} = 6.3 \times 10^{-4} \times 3.4 = 2.1 \times 10^{-3} \ [\text{A}]$$

3. Find V_{Rm} using I_m, and compare it to the reading from v_R.

$$V_{Rm} = R I_m = 10^3 [\Omega] \times 2.1 \times 10^{-3} [\text{A}] = 2.1 \ [\text{V}]$$

Exercise 6.6 (Homework)

Exercise 6.6 (Homework)

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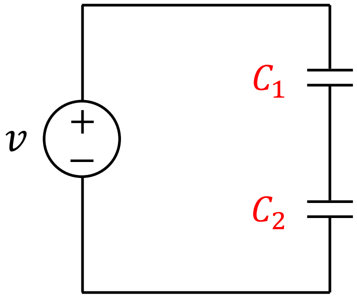
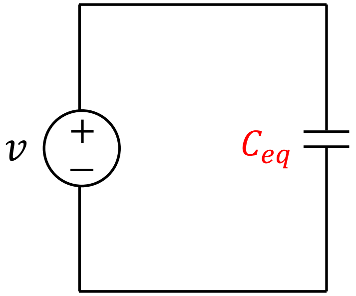


(a) Capacitors in series



(b) Equivalent circuit

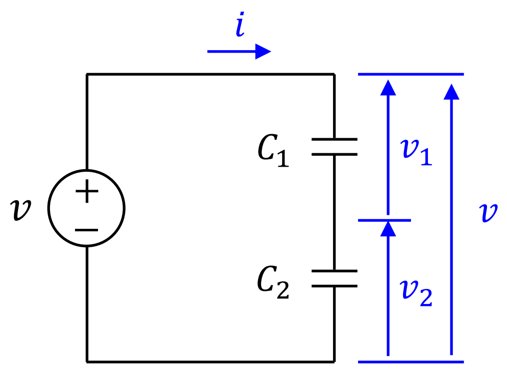
Show that

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

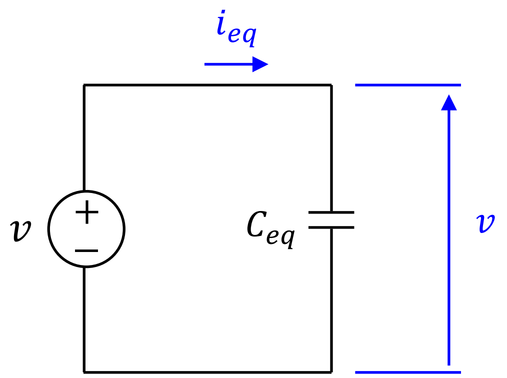
Exercise 6.6 (Answer)

Exercise 6.6 (Answer)

Exercise 6.6 (Answer)



(a) Capacitors in series



(b) Equivalent circuit

$$v_1 = \frac{1}{C_1}\int i\,dt, v_2 = \frac{1}{C_2}\int i\,dt$$

Kirchhoff's voltage Law

$$v = v_1 + v_2$$

$$= \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int i\,dt$$

$$v = \frac{1}{C_{eq}}\int i_{eq}\,dt$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

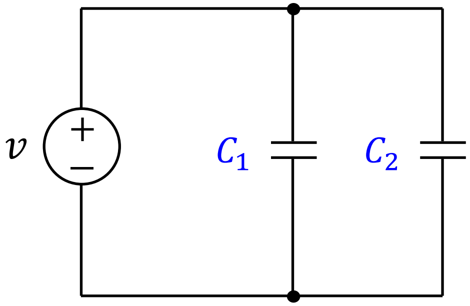
C_{eq}: Combined capacitance

C_{eq} is obtained as $1/(1/C_1 + 1/C_2)$.

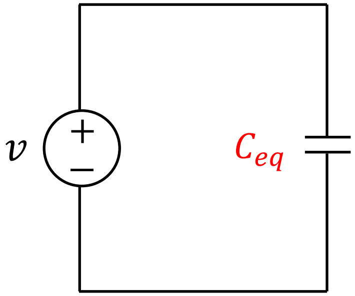
Exercise 6.7 (Homework)

Exercise 6.7 (Homework)

Exercise 6.7



(a) Capacitors in parallel



(b) Equivalent circuit

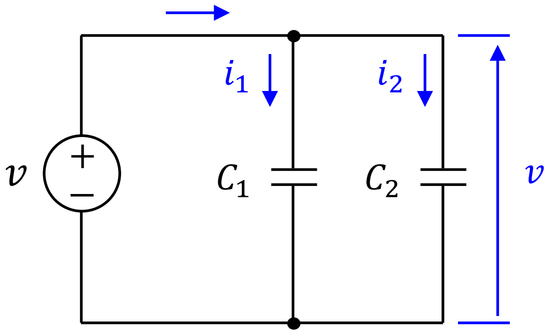
Show that

$$C_{eq} = C_1 + C_2$$

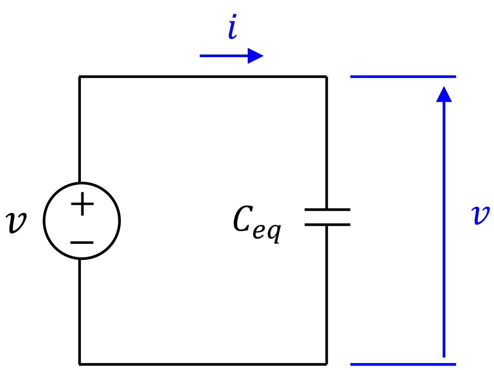
Exercise 6.7 (Answer)

Exercise 6.7 (Answer)

Exercise 6.7 (Answer)



(a) Capacitors in parallel



(b) Equivalent circuit

$$i_1 = C_1 \frac{dv}{dt}$$

$$i_2 = C_2 \frac{dv}{dt}$$

Kirchhoff's current Law

$$\begin{aligned} i &= i_1 + i_2 \\ &= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} \\ &= (C_1 + C_2) \frac{dv}{dt} \end{aligned}$$

$$i = C_{eq} \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2$$

C_{eq}: Combined Capacitance

Exercise 7.1 (Homework)

Exercise 7.1 (Homework)

Exercise 7.1 (Homework)

1. $Z_1 = \frac{\sqrt{3}}{2} + j\frac{1}{2}, Z_2 = \frac{1}{2} + j\frac{\sqrt{3}}{2}.$
   (1) Convert Z_1 and Z_2 into polar form.
   (2) Find $Z_1 + Z_2$ in polar form.
   (3) Plot Z_1, Z_2, and $Z_1 + Z_2$ in complex plane.

2. $Z_1 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}, Z_2 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}.$
   (1) Convert Z_1 and Z_2 into polar form.
   (2) Find $Z_1 \times Z_2$ in polar form.
   (3) Plot Z_1, Z_2, and $V_1 \times V_2$ in complex plane.

3. $Z_1 = \frac{1}{2} + j\frac{\sqrt{3}}{2}, Z_2 = \sqrt{3} + j.$
   (1) Convert Z_1 and Z_2 into polar form.
   (2) Find $\frac{Z_1}{Z_2}$ in polar form.
   (3) Plot Z_1, Z_2, and $\frac{Z_1}{Z_2}$ in complex plane.

Exercise 7.1 (Answers)

Exercise 7.1 (Answers)

Exercise 7.1 (Homework)

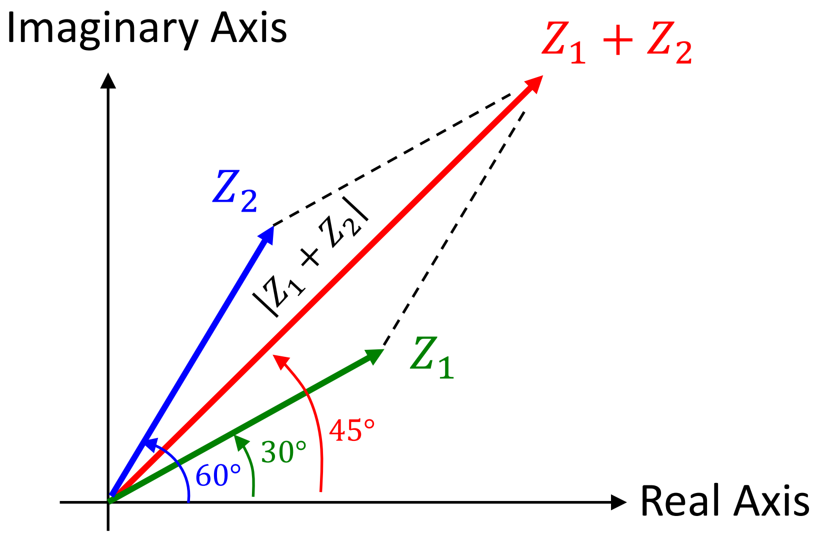
1. $Z_1 = \frac{\sqrt{3}}{2} + j\frac{1}{2}, Z_2 = \frac{1}{2} + j\frac{\sqrt{3}}{2}.$

(1) $Z_1 = \sqrt{\left(\sqrt{3}/2\right)^2 + (1/2)^2}(\cos 30° + j\sin 30°) = \cos 30° + j\sin 30°$

$Z_2 = \sqrt{(1/2)^2 + \left(\sqrt{3}/2\right)^2}(\cos 60° + j\sin 60°) = \cos 60° + j\sin 60°$

(2) $Z_1 + Z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} + j\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)(1 + j) = \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)(\cos 45° + j\sin 45°)$

(3)



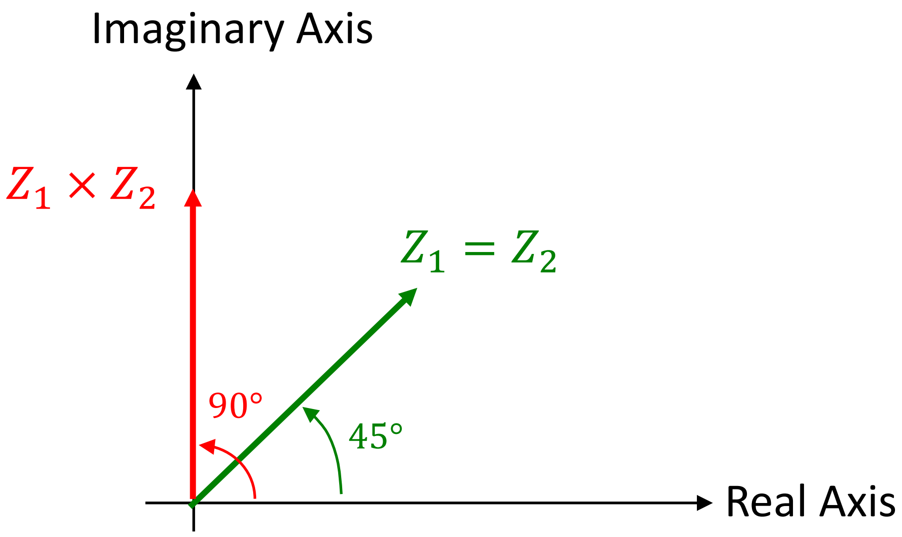
Exercise 7.1 (Homework)

2. $Z_1 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}, Z_2 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}.$

(1) $Z_1 = Z_2 = \sqrt{\left(1/\sqrt{2}\right)^2 + \left(1/\sqrt{2}\right)^2}(\cos 45° + j \sin 45°) = \cos 45° + j \sin 45°$

(2) $Z_1 \times Z_2 = \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \frac{1}{2} - \frac{1}{2} + j\left(\frac{1}{2} + \frac{1}{2}\right) = j = \cos 90° + j \sin 90°$

(3)



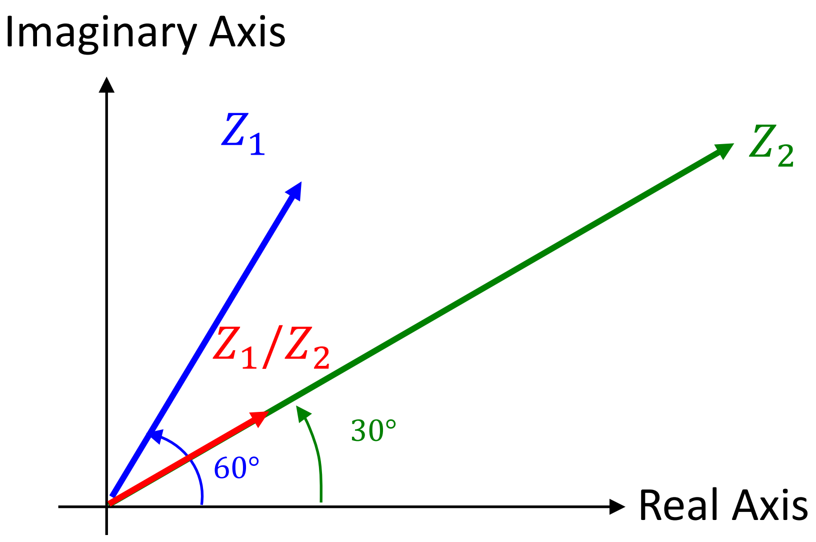
Exercise 7.1 (Homework)

3. $Z_1 = \frac{1}{2} + j\frac{\sqrt{3}}{2}, Z_2 = \sqrt{3} + j.$

(1) $Z_1 = \cos 60° + j\sin 60°,\ Z_2 = 2(\cos 30° + j\sin 30°)$

(2) $$\frac{Z_1}{Z_2} = \frac{\frac{1}{2} + j\frac{\sqrt{3}}{2}}{\sqrt{3} + j} = \frac{\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(\sqrt{3} - j)}{(\sqrt{3} + j)(\sqrt{3} - j)} = \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + j\left(\frac{3}{2} - \frac{1}{2}\right)}{3 + 1} = \frac{1}{2}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \frac{1}{2}(\cos 30° + j\sin 30°)$$

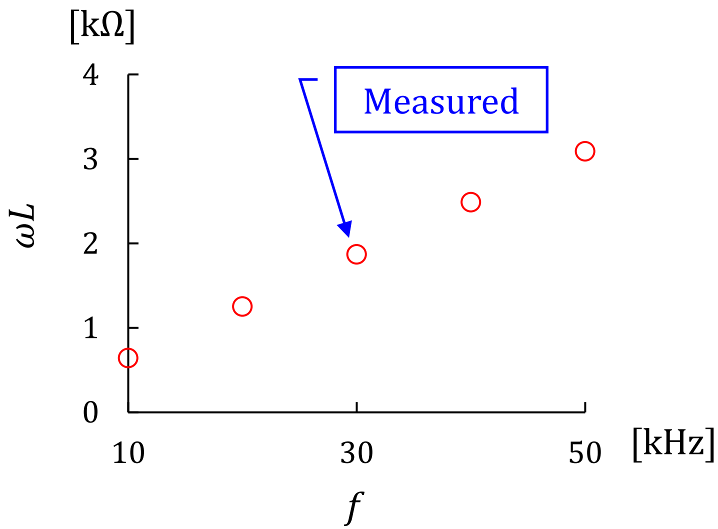
(3)



Exercise 7.4 (Homework)

Exercise 7.4 (Homework)

Exercise 7.4 (Homework)



Inductive Reactance

$L = 10$ [mH]

1. Calculate ωL at $f = 10, 20, 30, 40, 50$ [kHz].

2. Draw a line of ωL.

Exercise 7.4 (Answers)

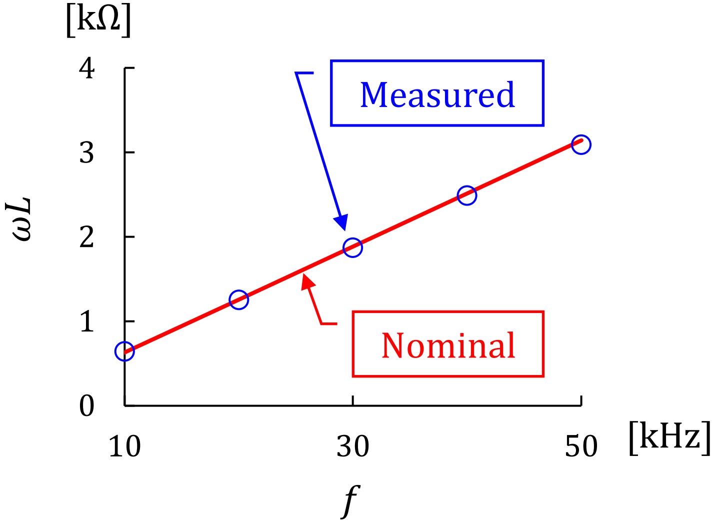
Exercise 7.4 (Answers)

$$f = 10\ [\text{kHz}]$$

$$\omega L = 2\pi f L = 2 \times \pi \times 10^4 [\text{Hz}] \times 0.01 [\text{H}] = 628\ [\Omega]$$

Nominal values of ωL

f [kHz]	ωL [kΩ]
10	0.63
20	1.26
30	1.88
40	2.51
50	3.14

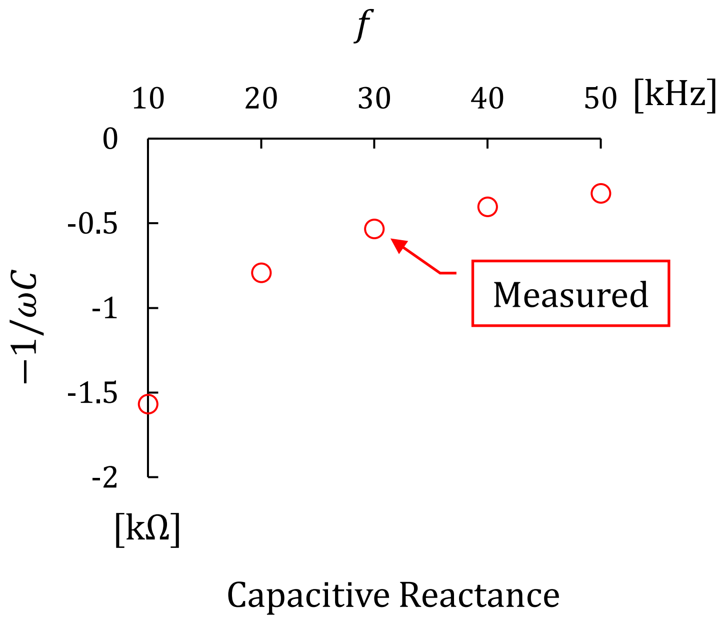


Inductive Reactance

Exercise 7.5 (Homework)

Exercise 7.5 (Homework)

Exercise 7.5 (Homework)



The nominal value of capacitance $C = 0.01$ [μF]

1. Calculate the nominal values of capacitive reactance $-1/\omega C$ at $f = 10, 20, 30, 40, 50$ [kHz].

2. Draw a line that interpolates the nominal values of $-1/\omega C$.

Exercise 7.5 (Answers)

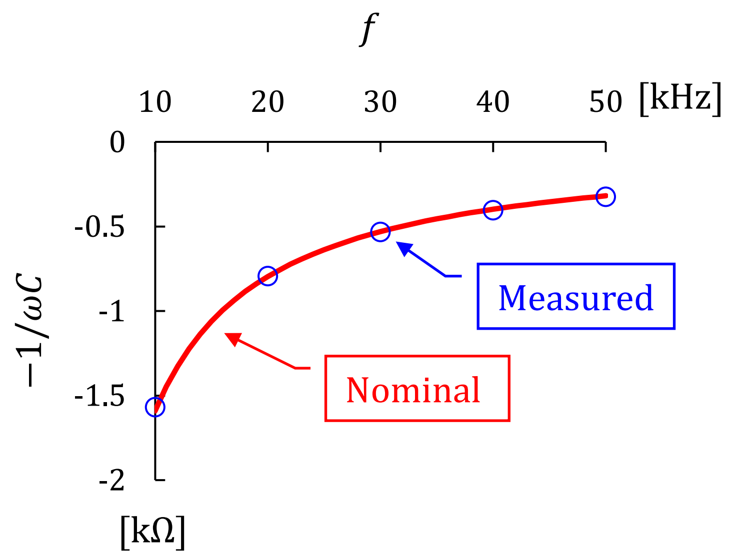
Experiment 7.5 (Answer)

$$f = 10 \text{ [kHz]}$$

$$-\frac{1}{\omega C} = -\frac{1}{2 \times \pi \times 10^4 \text{[Hz]} \times 0.01 \times 10^{-6} \text{[F]}} = -1.59 \text{ [k}\Omega\text{]}$$

Nominal Values of $-1/\omega C$

f [kHz]	$-1/\omega C$ [kΩ]
10	-1.59
20	-0.796
30	-0.531
40	-0.398
50	-0.318

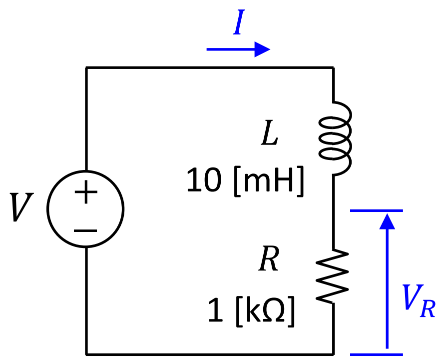


Capacitive Reactance

Exercise 8.5.1 (Homework)

Exercise 8.5.1 (Homework)

Exercise 8.5.1 (Homework)



$$Z = |Z|(\cos\psi + j\sin\psi)$$

$$|Z| = \sqrt{R^2 + (\omega L)^2} \qquad \psi = \tan^{-1}\frac{\omega L}{R}$$

If $V = V_e(\cos\theta + j\sin\theta)$, then

$$I = \frac{V}{Z}$$

$$= \frac{V_e(\cos\theta + j\sin\theta)}{|Z|(\cos\psi + j\sin\psi)}$$ ⬅ Eq. (7.4)

$$= \frac{V_e}{|Z|}\{\cos(\theta - \psi) + j\sin(\theta - \psi)\}$$

If $f = 10$ [kHz], $V_e = 2.96$ [V], then $V_{Re} = ?$

If $f = 10$ [kHz], $V_e = 2.96$ [V] as in Experiment 8.5, then what is V_{Re}?

Exercise 8.5.1 (Answer)

Exercise 8.5.1 (Answer)

Exercise 8.5.1 (Answer)

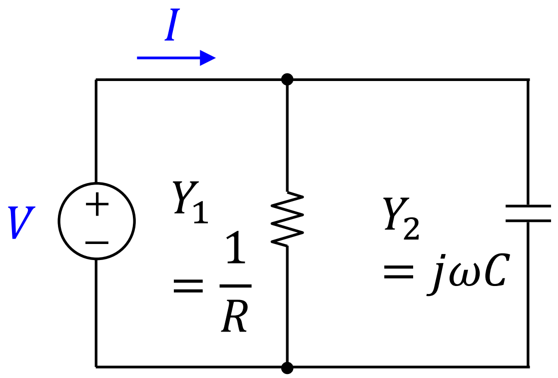
$$V_R = RI$$
$$= R\frac{V}{Z}$$
$$= R\frac{V_e}{|Z|}\{\cos(\theta-\psi)+j\sin(\theta-\psi)\}$$

$$V_{Re} = R\frac{V_e}{|Z|}$$
$$= R\frac{V_e}{\sqrt{R^2+(\omega L)^2}}$$
$$= 10^3[\Omega]\times\frac{2.96\ [\mathrm{V}]}{\sqrt{(10^3[\Omega])^2+(2\times\pi\times10^4[\mathrm{Hz}]\times0.01[\mathrm{H}])^2}}$$
$$= 2.51\ [\mathrm{V}]$$

The result In Experiment 8.5 was that $V_{Re} = 2.40$ [V] at $f = 10$ [kHz]. This means that the amount of actual current is smaller than the nominal value. This is because the magnitude of actual impedance was larger than the nominal value.

Exercise 8.10 (Homework)

Exercise 8.10 (Homework)



This is an $R - C$ parallel circuit. The admittance of the capacitor is $j\omega C$, and that of the resistor is $1/R$.

1. Write the composite admittance Y of this circuit in the polar form.

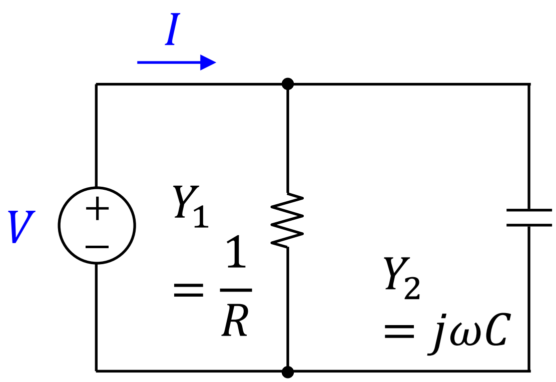
2. If $R = 1\ [k\Omega], C = 0.01\ [\mu\text{F}]$, and $f = 15.9$ [kHz], find the admittance Y in the polar form.

You can round the answer to 2 significant figures.

Exercise 8.10 (Answers)

Exercise 8.10 (Answers)

Exercise 8.10 (Answers)



1. Composite Admittance

$$Y = Y_1 + Y_2 = \frac{1}{R} + j\omega C = |Y|(\cos\varphi + j\sin\varphi)$$

$$|Y| = \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2} \qquad \varphi = \tan^{-1}\omega CR$$

2. $R = 1\ [\mathrm{k\Omega}], C = 0.01\ [\mu\mathrm{F}]$, and $f = 15.9$ [kHz]

$$|Y| = \sqrt{\left(\frac{1}{1000}\right)^2 + (2\times\pi\times 15.9\times 10^3 \times 0.01\times 10^{-6})^2} = \sqrt{(10^{-3})^2 + (1.0\times 10^{-3})^2}$$
$$= 1.4\times 10^{-3}$$

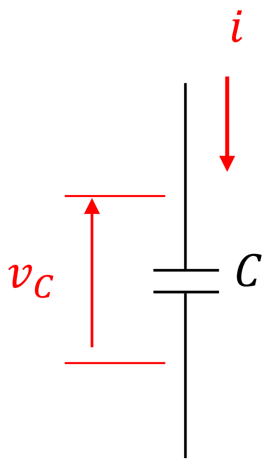
$$\psi = \tan^{-1}2\times\pi\times 15.9\times 10^3\times 0.01\times 10^{-6}\times 10^3 = \tan^{-1}1.0 = 45°$$

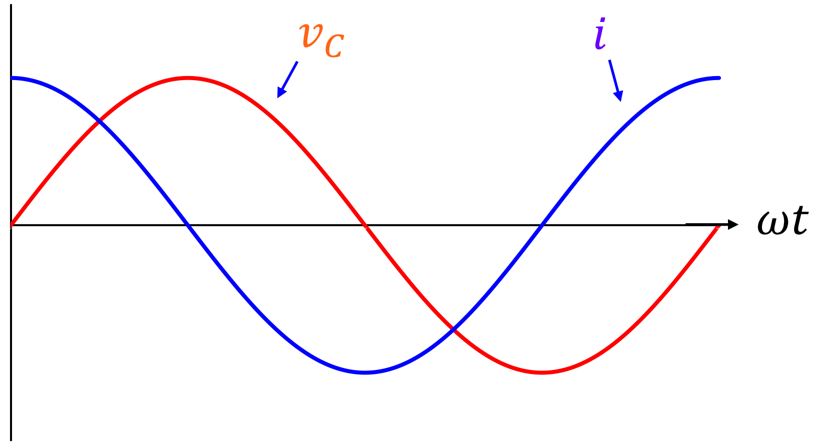
$$Y = 1.4(\cos 45° + j\sin 45°)\ [\mathrm{mS}]$$

Exercise 9.3 (Homework)

Exercise 9.3 (Homework)

Exercise 9.3





1. The voltage v_C and the current i are given by

$$v_C = \sqrt{2}V_{Ce}\sin\omega t$$
$$i = \sqrt{2}I_e\cos\omega t.$$

   Develop the power $p = v_C i$.

2. Draw the waveform of p.

3. Find the average power of p.

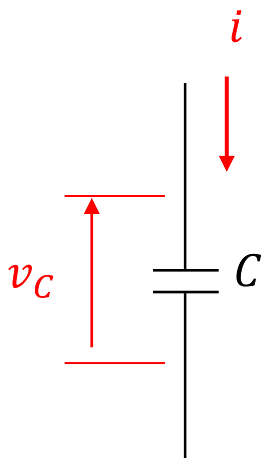
4. The amplitude of p is denoted by P_{Cm} and $P_{Cm} = V_{Ce}I_e$. The unit of P_{Cm} is [Var]. Show that P_{Cm} is

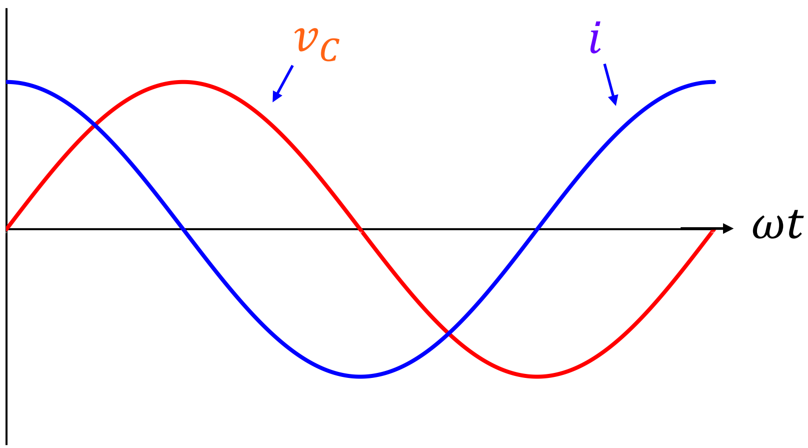
$$P_{Cm} = \frac{I_e^2}{\omega C} = \omega C V_{Ce}^2$$

Exercise 9.3 (Homework)

Exercise 9.3 (Homework)

Exercise 9.3





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2. Draw the waveform of p.

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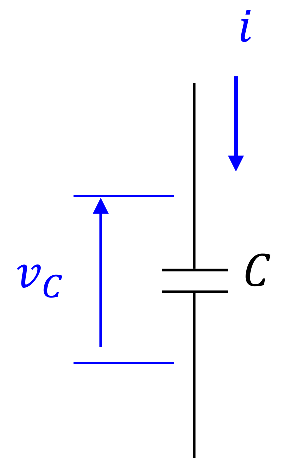
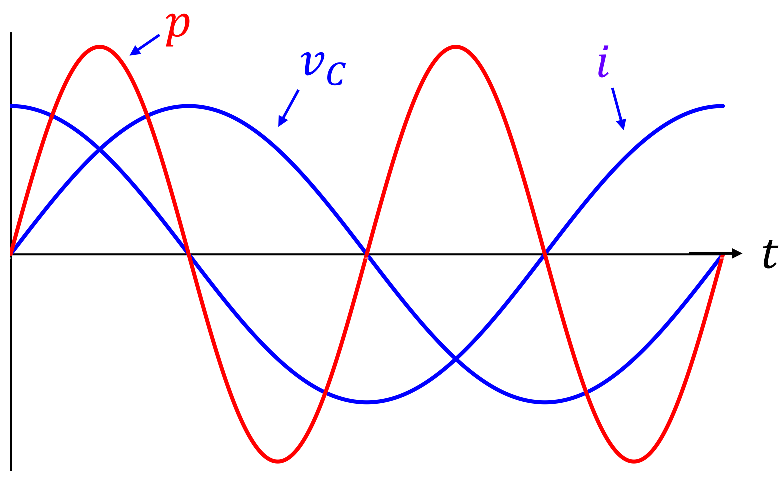
$$P_{Cm} = \frac{I_e^2}{\omega C} = \omega C V_{Ce}^2$$

Exercise 9.3 (Answer)

Exercise 9.3 (Answer)





1.

$$p = v_C i$$
$$= \sqrt{2}V_{Ce} \sin \omega t \times \sqrt{2} I_e \cos \omega t$$
$$= 2V_{Ce} I_e \sin \omega t \cos \omega t$$
$$= V_{Ce} I_e \sin 2\omega t$$

3.

$$P_C = \frac{1}{2\pi}\int_0^{2\pi} p d\omega t$$
$$= \frac{1}{2\pi}\int_0^{2\pi} V_{Ce} I_e \sin 2\omega t \, d\omega t$$
$$= 0$$

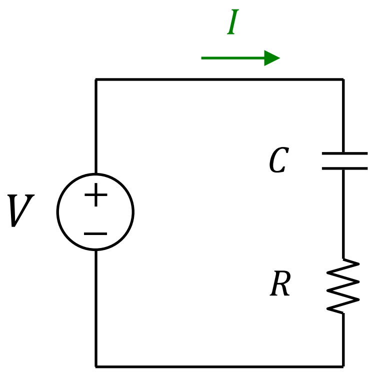
4.

$$P_{Cm} = V_{Ce} I_e \quad \Leftarrow \quad V_C = \frac{I_e}{\omega C}$$
$$= \frac{I_e^2}{\omega C} \quad \Leftarrow \quad I_e = \omega C V_{Ce}$$
$$= \omega C V_{Ce}^2$$

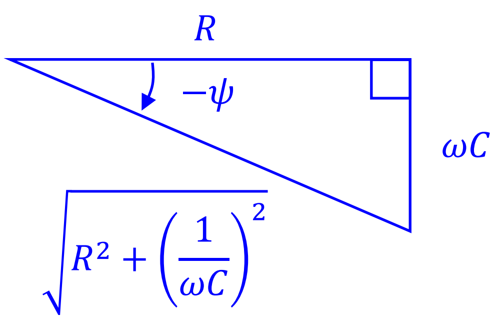
Exercise 9.6.1 (Homework)

Exercise 9.6.1 (Homework)

Exercise 9.6.1 (Homework)



$$\psi = -\tan^{-1}\frac{1}{\omega CR}$$

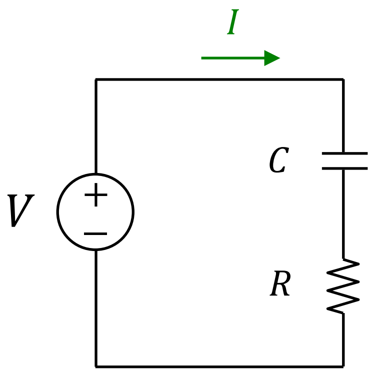


$$\cos\psi = \frac{R}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

$$\sin\psi = -\frac{\frac{1}{\omega C}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

In the case of $R - C$ series circuit, $\cos\psi$ and $\sin\psi$ are given by these equations.

Exercise 9.6.1 (Homework)



Show that the active power P_{act} and the reactive power P_r are given by the following equations:

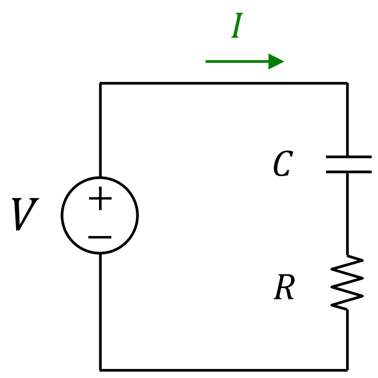
$$P_{act} = \begin{cases} R I_e^2 \\ \dfrac{R}{R^2 + (1/\omega C)^2} V_e^2 \end{cases}$$

$$P_r = \begin{cases} -(1/\omega C) I_e^2 \\ -\dfrac{1/\omega C}{R^2 + (1/\omega C)^2} V_e^2 \end{cases}$$

Exercise 9.6.1 (Answers)

Exercise 9.6.1 (Answers)

Exercise 9.6.1 (Answers)



$$P_{act} = V_e I_e \cos\psi \quad \Leftarrow \quad \cos\psi = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$$

$$= V_e I_e \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} \quad \Leftarrow \quad V_e = \sqrt{R^2 + (1/\omega C)^2} I_e$$

$$= R I_e^2 \quad \Leftarrow \quad I_e = \frac{V_e}{\sqrt{R^2 + (1/\omega C)^2}}$$

$$= \frac{R}{R^2 + (1/\omega C)^2} V_e^2$$

$$P_r = V_e I_e \sin\psi \quad \Leftarrow \quad \sin\psi = -\frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$$

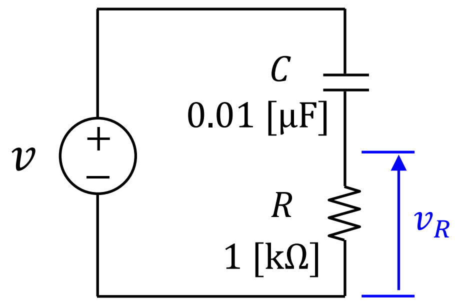
$$= -V_e I_e \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$$

$$= \begin{cases} -(1/\omega C) I_e^2 \\ -\dfrac{1/\omega C}{R^2 + (1/\omega C)^2} V_e^2 \end{cases}$$

Exercise 9.6.2 (Homework)

Exercise 9.6.2 (Homework)

Exercise 9.6.2 (Homework)



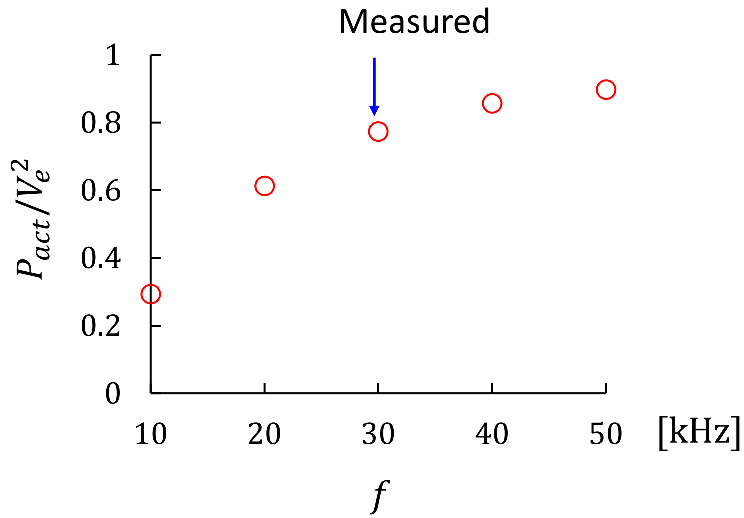
Measured values in Experiment 9.6.1

f [kHz]	P_{act}/V_e^2(Measured)	P_r/V_e^2(Measured)
10	0.293	-0.450
20	0.613	-0.482
30	0.773	-0.412
40	0.856	-0.347
50	0.898	-0.296

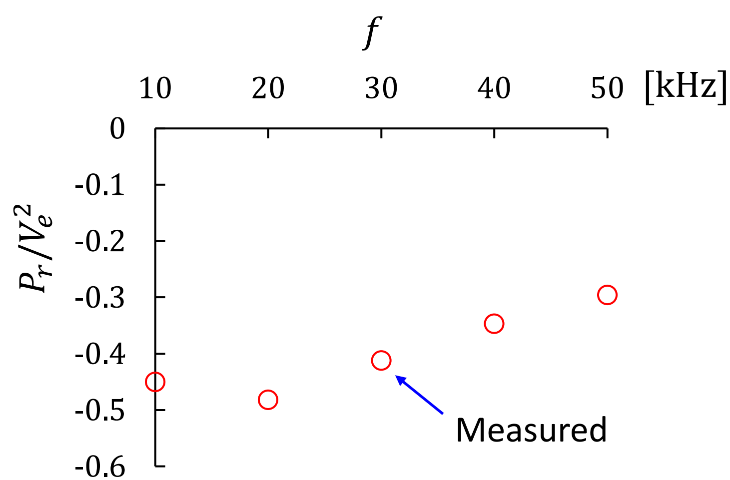
Nominal values:

$$\frac{P_{act}}{V_e^2} = \frac{R}{R^2 + (1/\omega C)^2}$$

$$\frac{P_r}{V_e^2} = -\frac{1/\omega C}{R^2 + (1/\omega C)^2}$$



(a) Active Power

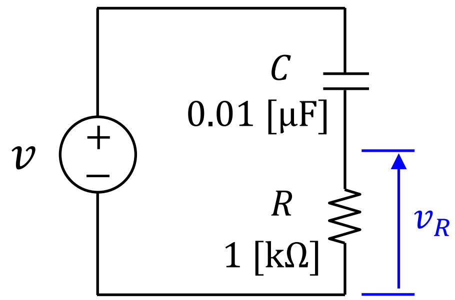


(b) Reactive Power

Find the nominal values of P_{act}/V_e^2 and P_r/V_e^2 using these equations. Then compare the measured values to the nominal ones.

Exercise 9.6.2 (Answers)

Exercise 9.6.2 (Answers)



$f = 10$ [kHz]

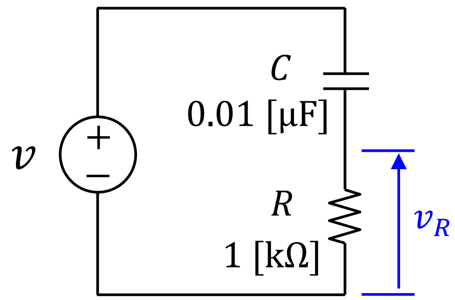
$$\frac{P_{act}}{V_e^2} = \frac{R}{R^2 + (1/\omega C)^2}$$
$$= \frac{1000[\Omega]}{1000^2 + (1/(2\times\pi\times10^4[\text{Hz}]\times0.01\times10^{-6}[\text{F}]))^2}$$
$$= 0.283$$

$$\frac{P_r}{V_e^2} = -\frac{1/\omega C}{R^2 + (1/\omega C)^2}$$
$$= -\frac{1/(2\times\pi\times10^4\times0.01\times10^{-6})}{1000^2 + (1/(2\times\pi\times10^4\times0.01\times10^{-6}))^2}$$
$$= 0.450$$

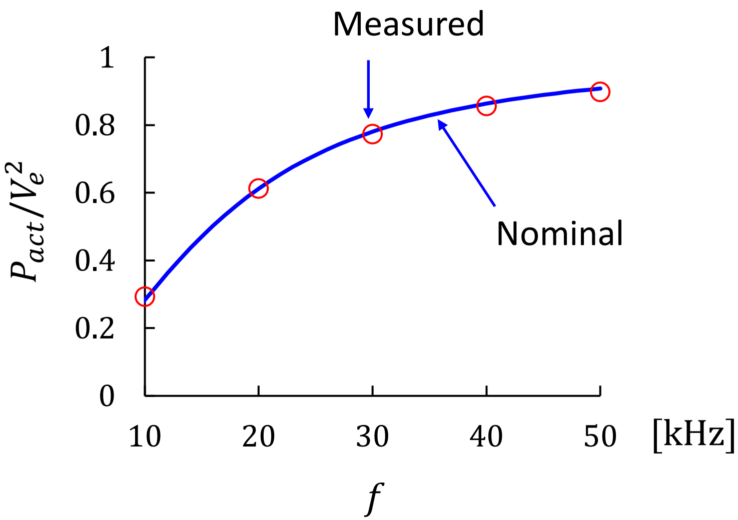
f [kHz]	P_{act}/V_e^2 (Measured)	P_{act}/V_e^2 (Nominal)	P_r/V_e^2 (Measured)	P_r/V_e^2 (Nominal)
10	0.293	0.283	-0.450	-0.450
20	0.613	0.612	-0.482	-0.487
30	0.773	0.780	-0.412	-0.414
40	0.856	0.863	-0.347	-0.344
50	0.898	0.908	-0.296	-0.289

These equations show examples of the calculation at $f = 10$ [kHz]. The table shows the results.

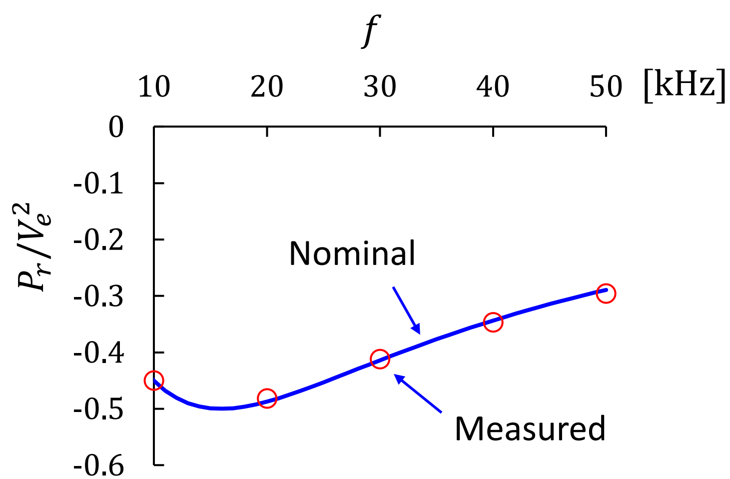
Exercise 9.6.2 (Answers)



f [kHz]	P_{act}/V_e^2 (Measured)	P_{act}/V_e^2 (Nominal)	P_r/V_e^2 (Measured)	P_r/V_e^2 (Nominal)
10	0.293	0.283	-0.450	-0.450
20	0.613	0.612	-0.482	-0.487
30	0.773	0.780	-0.412	-0.414
40	0.856	0.863	-0.347	-0.344
50	0.898	0.908	-0.296	-0.289



(a) Active Power



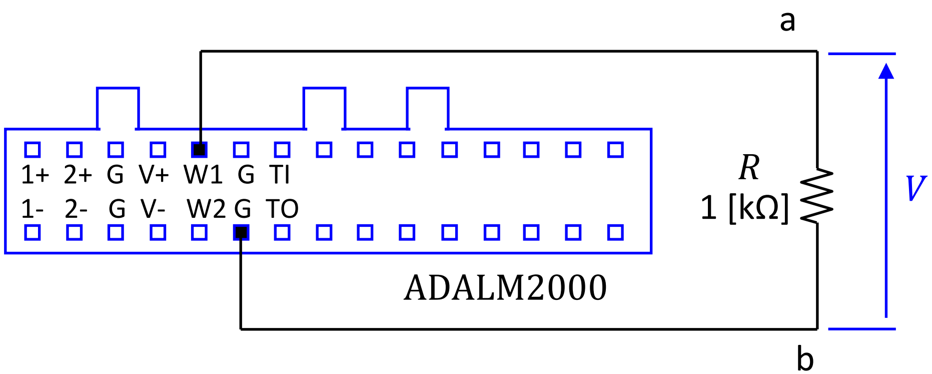
(b) Reactive Power

The measured values met the nominal values well. The resistance component of the capacitor was small.

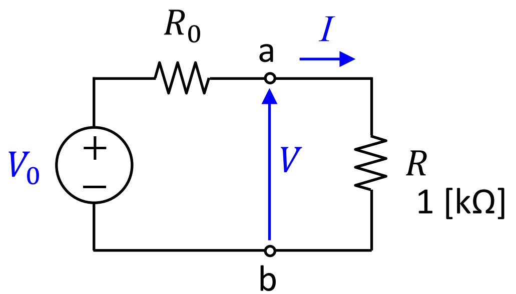
Exercise 10.1.2 (Homework)

Exercise 10.1.2 (Homework)

Exercise 10.1.2 (Homework)



(a) Experimental circuit



(b) Equivalent Circuit of Signal Generator

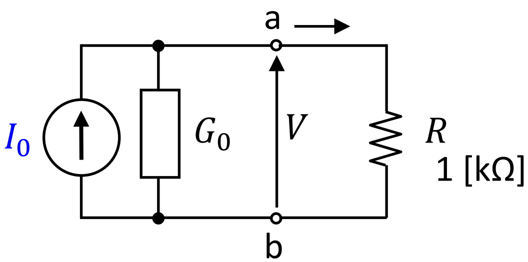
This exercise is to find the internal resistor R_0 of the Signal Generator(SG). (a) shows an experimental circuit to measure R_0. (b) shows its equivalent circuit. The SG is represented by a practical voltage source having the source voltage V_0, and the internal resistance R_0. Assume that the measured effective values of V_0 and V were

V_{0e} [V]	V_e [V]
2.85	2.71

1. Find the internal resistance R_0.

Pause the video and find the internal resistance R_0 of the signal generator.

Exercise 10.1.2 (Homework)



(c) Equivalent Circuit

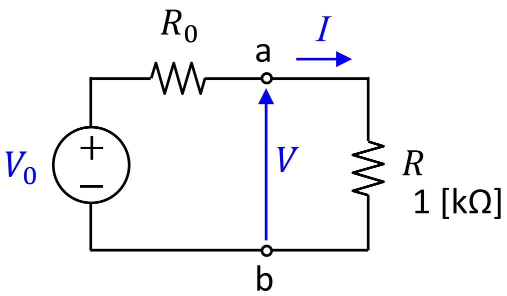
(c) shows another equivalent circuit of the SG. This circuit employs the current source I_0 and the internal conductance G_0.

2. Find the current source I_0, and internal conductance G_0.

Pause the video and find the current source I_0, and internal conductance G_0 .

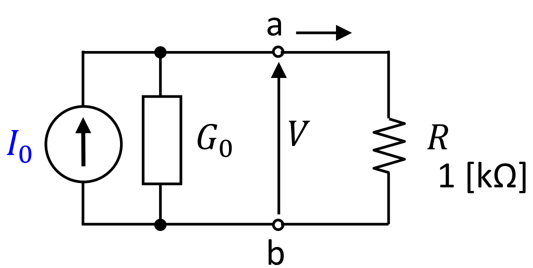
Exercise 10.1.2 (Answers)

Exercise 10.1.2 (Answers)



$$I = \frac{V_0}{R_0 + R}$$

$$V = RI = \frac{R}{R_0 + R} V_0 \quad \Rightarrow \quad R_0 = \frac{1 - \frac{V}{V_0}}{\frac{V}{V_0}} R = \frac{1 - \frac{2.71}{2.85}}{\frac{2.71}{2.85}} \times 1000 = 51.7\ [\Omega]$$

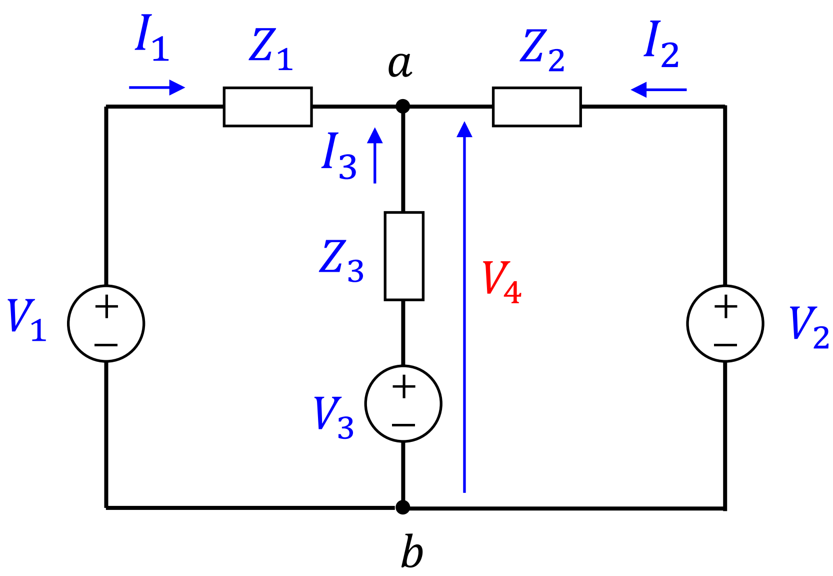


$$I_0 = \frac{V_0}{R_0} = \frac{2.86\ [\mathrm{V}]}{51.7\ [\Omega]} = 55.3 [\mathrm{mA}]$$

$$G_0 = \frac{1}{R_0} = \frac{1}{51.7[\Omega]} = 19.3 [\mathrm{mS}]$$

Exercise 10.2 (Homework)

Exercise 10.2 (Homework)



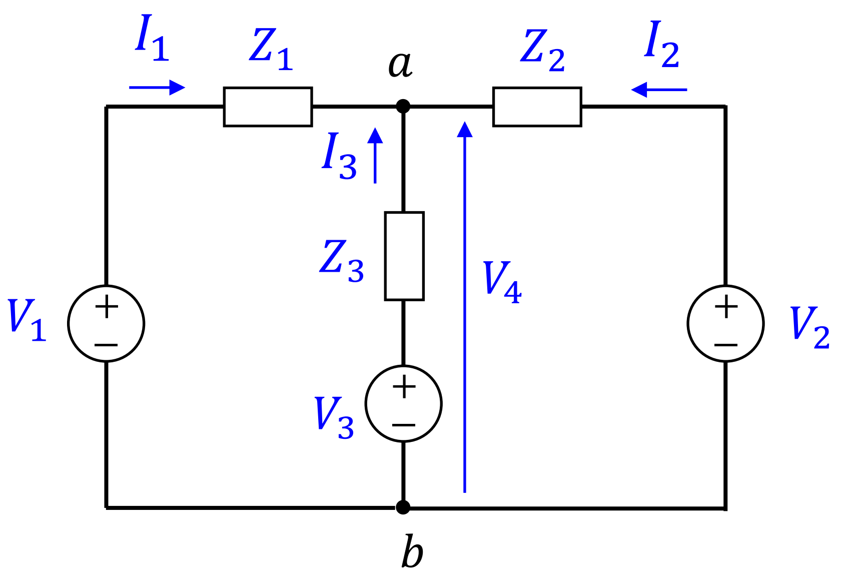
Derive V_4 as a function of Z_1, Z_2, Z_3, V_1, V_2 and V_3.

Derive V_4 as a function of Z_1, Z_2, Z_3, V_1, V_2 and V_3, by applying the Node Voltage Method.

Exercise 10.2 (Answer)

Exercise 10.2 (Answer)

Exercise 10.2 (Answer)



$$I_1 = \frac{V_1 - V_4}{Z_1}$$

$$I_2 = \frac{V_2 - V_4}{Z_2}$$

$$I_3 = \frac{V_3 - V_4}{Z_3}$$

KCL

$$I_1 + I_2 + I_3 = 0$$

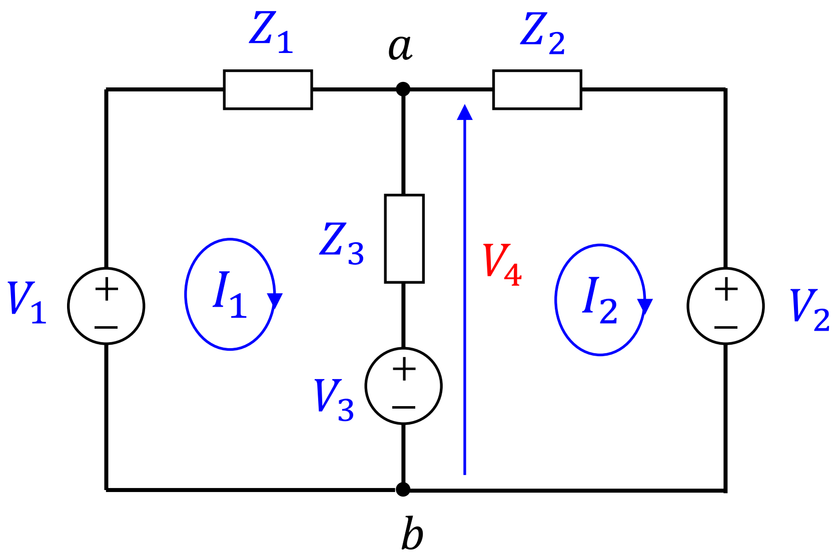
$$\frac{V_1 - V_4}{Z_1} + \frac{V_2 - V_4}{Z_2} + \frac{V_3 - V_4}{Z_3} = 0$$

$$V_4 = \frac{\frac{V_1}{Z_1} + \frac{V_2}{Z_2} + \frac{V_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

Exercise 10.3.2 (Homework)

Exercise 10.3.2 (Homework)

Exercise 10.3.2 (Homework)



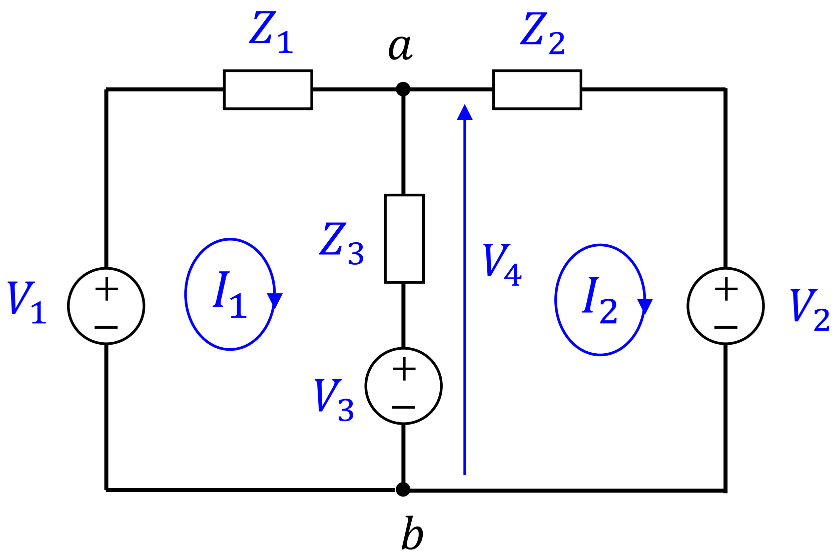
Derive V_4 as a function of Z_1, Z_2, Z_3, V_1, V_2 and V_3.

Derive V_4 as a function of Z_1, Z_2, Z_3, V_1, V_2 and V_3, by applying the Mesh Current Method.

Exercise 10.3.2 (Answer)

Exercise 10.3.2 (Homework)

KVL



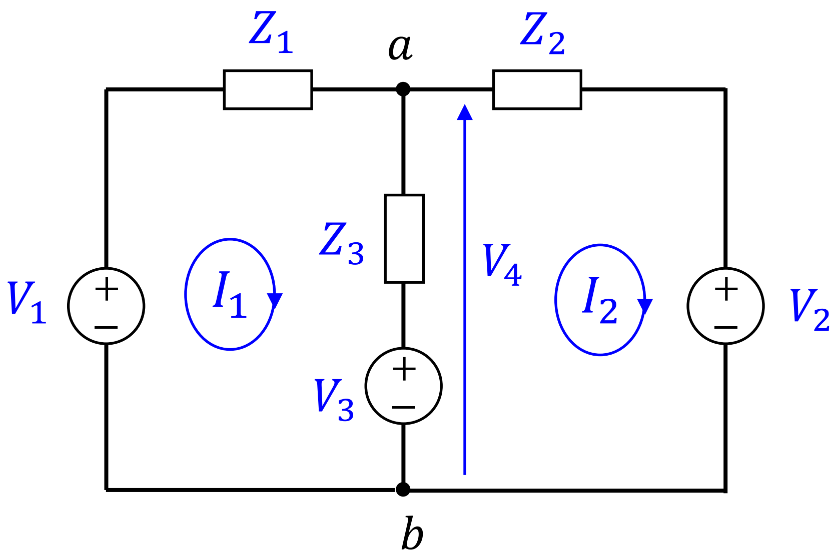
Loop 1: $V_1 - V_3 = (Z_1 + Z_3)I_1 \quad - Z_3 I_2$

Loop 2: $-V_2 + V_3 = \quad -Z_3 I_1 \quad + (Z_2 + Z_3)I_2$

$$\begin{pmatrix} V_1 - V_3 \\ -V_2 + V_3 \end{pmatrix} = \begin{pmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 \end{pmatrix}^{-1} \begin{pmatrix} V_1 - V_3 \\ -V_2 + V_3 \end{pmatrix}$$

Exercise 10.3.2 (Homework)



$$I_1 = \frac{(Z_2 + Z_3)(V_1 - V_3) + Z_3(-V_2 + V_3)}{(Z_1 + Z_3)(Z_2 + Z_3) - Z_3^2}$$

$$= \frac{(Z_2 + Z_3)V_1 - Z_3 V_2 - Z_2 V_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

$$V_4 = V_1 - Z_1 I_1$$

$$= V_1 - Z_1 \frac{(Z_2 + Z_3)V_1 - Z_3 V_2 - Z_2 V_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

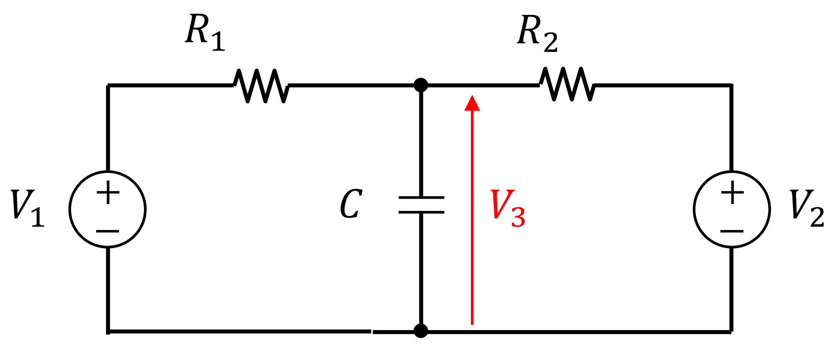
$$= \frac{Z_2 Z_3 V_1 + Z_1 Z_3 V_2 + Z_1 Z_2 V_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

$$= \frac{\frac{V_1}{Z_1} + \frac{V_2}{Z_2} + \frac{V_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

Exercise 10.4.2 (Homework)

Exercise 10.4.2 (Homework)

Exercise 10.4.2 (Homework)



1. Derive the voltage V_3 as a function of R_1, R_2, C, V_1, and V_2.

2. $R_1 = R_2 = R = 1[\mathrm{k}\Omega], C = 0.01\ [\mu\mathrm{F}]$, and

$$v_1 = V_{1m} \sin \omega t$$

$$v_2 = V_{2m} \sin\left(\omega t - \frac{\pi}{2}\right)$$

where V_{1m} =4 [V] ($V_{1,p-p}$ =8 [V]), V_{2m} = 2 [V] ($V_{2,p-p}$ =4 [V]) and f = 15.9 [kHz].

Show that

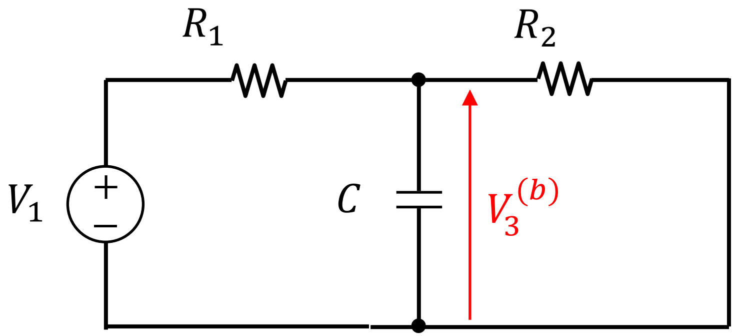
$$V_{3e} = \frac{2}{\sqrt{2}}\ [V], \psi_3 = -53°$$

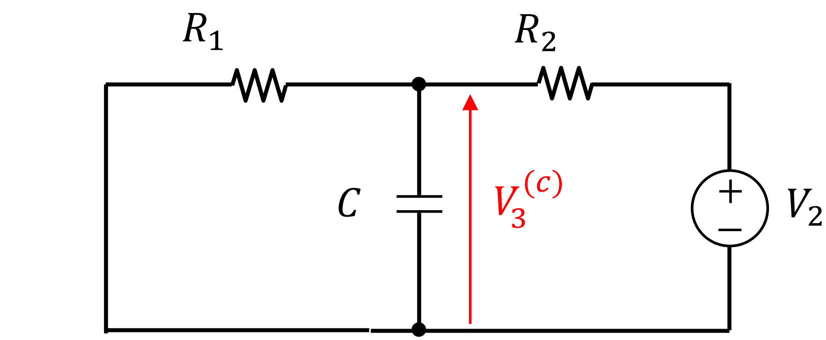
Exercise 10.4.2 (Answers)

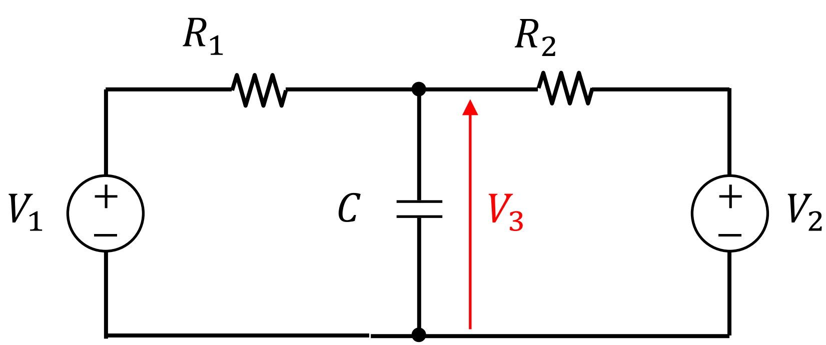
Exercise 10.4.2 (Answers)

The answers to Exercise 10.4.2.

Exercise 10.4.2 (Answers)







1.

$$V_3^{(b)} = \frac{\dfrac{1}{\dfrac{1}{R_2} + j\omega C}}{R_1 + \dfrac{1}{\dfrac{1}{R_2} + j\omega C}} V_1$$

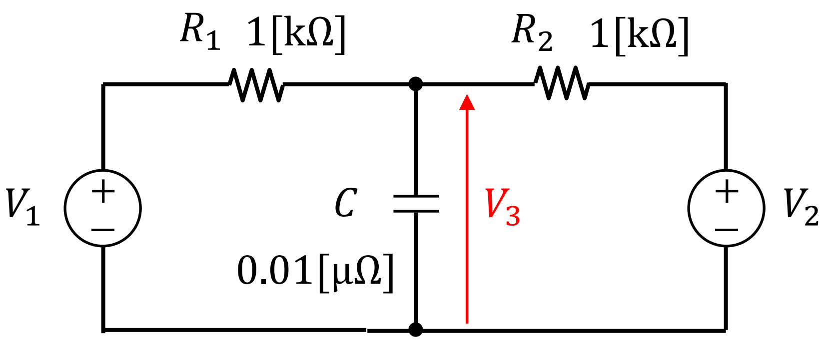
$$= \frac{1}{1 + \dfrac{R_1}{R_2}(1 + j\omega C R_2)} V_1$$

$$V_3^{(c)} = \frac{1}{1 + \dfrac{R_2}{R_1}(1 + j\omega C R_1)} V_2$$

$$V_3 = V_3^{(b)} + V_3^{(C)}$$

$$= \frac{1}{1 + \dfrac{R_1}{R_2}(1 + j\omega C R_2)} V_1 + \frac{1}{1 + \dfrac{R_2}{R_1}(1 + j\omega C R_1)} V_2$$

Exercise 10.4.2 (Answers)



$$V_1 = V_{1e} = \frac{4}{\sqrt{2}}$$

$$V_2 = V_{2e}(\cos 90° - j \sin 90°) = -j\frac{2}{\sqrt{2}}$$

$$\begin{aligned}\omega CR &= 2 \times \pi \times 15.9 \times 10^3[\text{Hz}] \times 0.01 \times 10^{-6}[\text{F}] \times 10^3[\Omega] \\ &= 1.0\end{aligned}$$

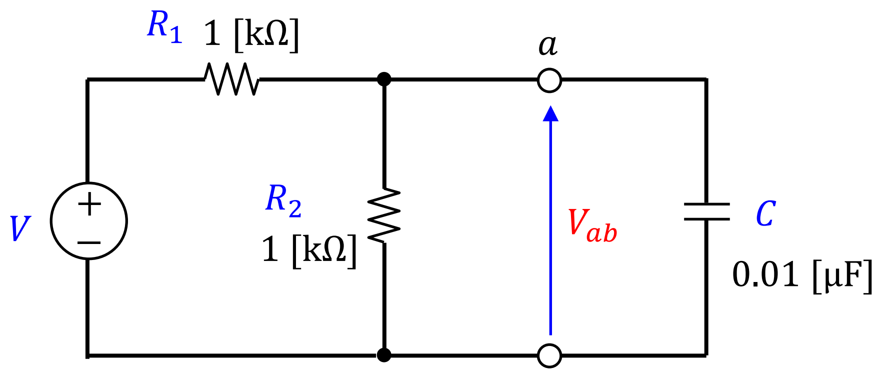
$$\begin{aligned}V_3 &= \frac{1}{1 + \frac{R_1}{R_2}(1 + j\omega CR_2)} V_1 + \frac{1}{1 + \frac{R_2}{R_1}(1 + j\omega CR_1)} V_2 \\ &= \frac{V_1 + V_2}{1 + (1 + j)} \\ &= \frac{2}{\sqrt{2}} \frac{2 - j}{2 + j}\end{aligned}$$

$$V_{3e} = \frac{2}{\sqrt{2}}$$

$$\psi_3 = -\tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{2} = -53°$$

Exercise 10.5.2 (Homework)

Exercise 10.5.2 (Homework)



Original Circuit

1. Derive V_{ab} as a function of R_1, R_2, C, and V by applying Ho-Thévenin's Theorem.

2. $v = V_m \sin \omega t$

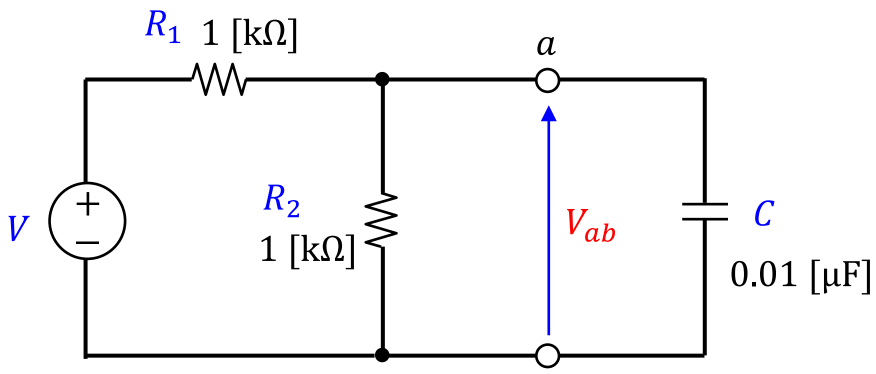
   where V_m =4 [V] (V_{p-p} =8 [V]), f = 20 [kHz].

   Find V_{ab}.

Exercise 10.5.2 (Answer)

Exercise 10.5.2 (Answer)

Exercise 10.5.2 (Answer)



1. $Z_1 = R_1, Z_2 = R_2, Z_L = -j\frac{1}{\omega C}.$

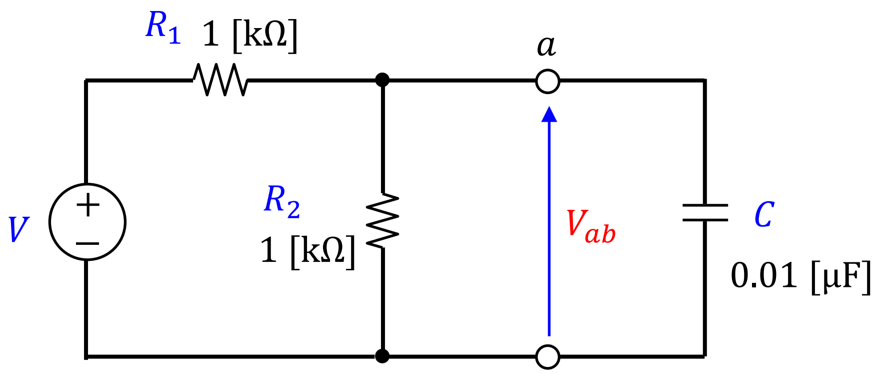
   From the answer to Exercise 10.5.1,

$$V_{ab} = \frac{Z_2 Z_L}{Z_1 Z_2 + Z_2 Z_L + Z_L Z_1} V$$

$$= \frac{-j\frac{R_2}{\omega C}}{R_1 R_2 + (R_1 + R_2)\left(-j\frac{1}{\omega C}\right)} V$$

$$= \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2} V$$

Exercise 10.5.2 (Answer)



2. $V = \dfrac{4}{\sqrt{2}}$ [V]

$$V_{abe} = \frac{R_2}{\sqrt{(R_1 + R_2)^2 + (\omega C R_1 R_2)^2}} |V|$$

$$= \frac{1000}{\sqrt{(2000[\Omega])^2 + (2 \times \pi \times 20 \times 10^3 [\text{Hz}] \times 0.01 \times 10^{-6} [\text{F}] \times 1000[\Omega] \times 1000[\Omega])^2}} \frac{4}{\sqrt{2}}$$

$$= 1.20 \text{ [V]}$$

$$\psi_{ab} = -\tan^{-1} \frac{\omega C R_1 R_2}{R_1 + R_2}$$

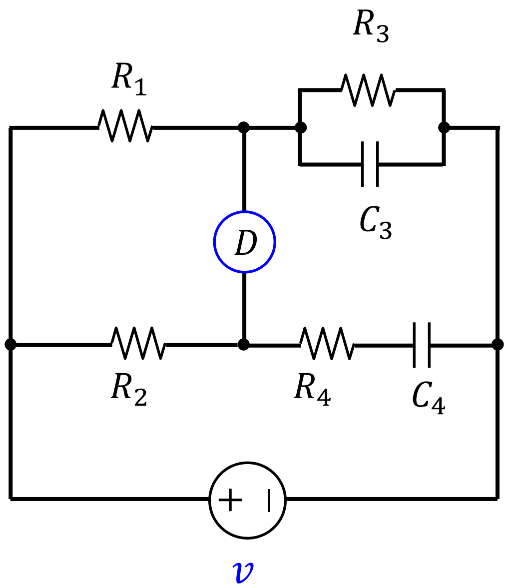
$$= -\tan^{-1} \frac{2 \times \pi \times 20 \times 10^3 \times 0.01 \times 10^{-6} \times 1000 \times 1000}{2000}$$

$$= -32.1 \text{ [°]}$$

Exercise 10.6 (Homework)

Exercise 10.6 (Homework)

Exercise 10.6 (Homework)

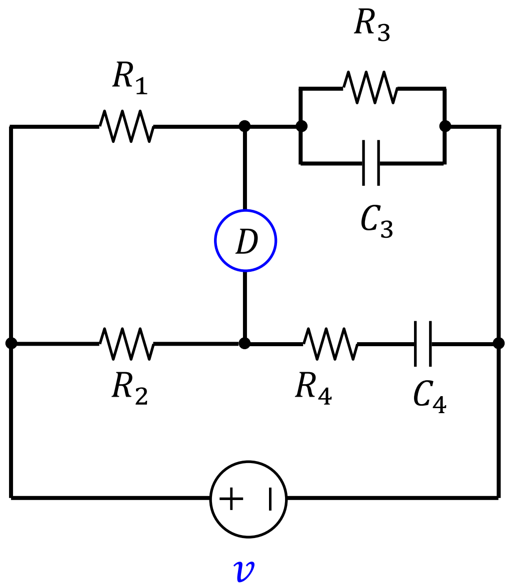


Derive the balanced condition of this bridge circuit.

Exercise 10.6 (Answer)

Exercise 10.6 (Answer)

Exercise 10.6 (Answer)



Balanced condition:

$$R_1\left(R_4 - j\frac{1}{\omega C_4}\right) = R_2 \frac{1}{\frac{1}{R_3} + j\omega C_3}$$

$$\left(R_4 - j\frac{1}{\omega C_4}\right)\left(\frac{1}{R_3} + j\omega C_3\right) = \frac{R_2}{R_1}$$

$$\frac{R_4}{R_3} + \frac{C_3}{C_4} + j\left(\omega C_3 R_4 - \frac{1}{\omega C_4 R_3}\right) = \frac{R_2}{R_1}$$

Real part:

$$\frac{R_4}{R_3} + \frac{C_3}{C_4} = \frac{R_2}{R_1}$$

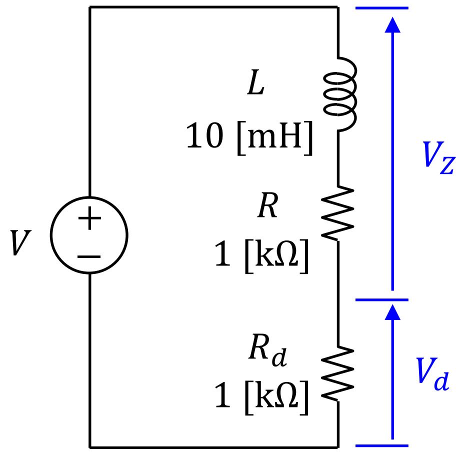
Imaginary part:

$$\omega C_3 R_4 = \frac{1}{\omega C_4 R_3}$$

Exercise 11.2.1 (Homework)

Exercise 11.2.1 (Homework)

Exercise 11.2.1 (Homework)



1. Derive $G_Z = 20\log_{10}|Z|$, and ψ_Z as functions of the frequency f.
   Z is the impedance of $R - L$ series circuit.

2. Plot G_Z and ψ_Z in the range of 0.1 [kHz]≤ $f \leq$ 1000 [kHz].

Exercise 11.2.1 (Answer)

Exercise 11.2.1 (Answer)

Experiment 11.2.1 (Answer)

$$Z = R + j\omega L$$

The unit of Z: [kΩ]

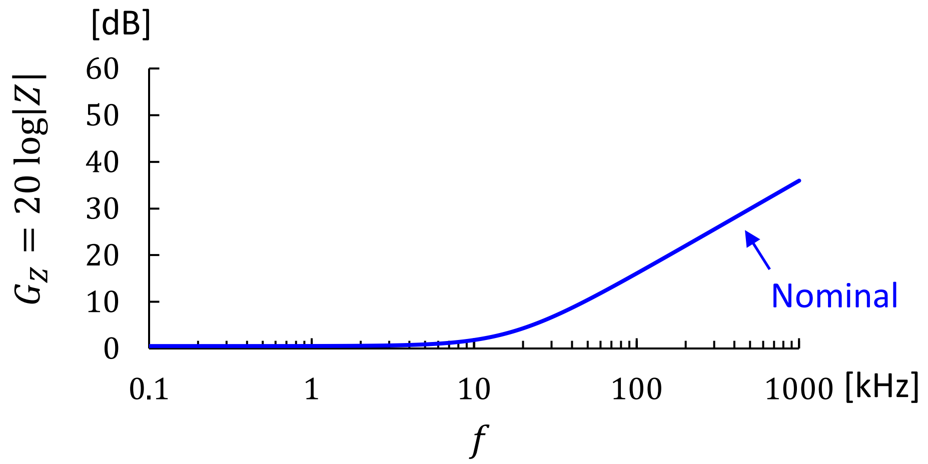
$$20\log_{10}|Z| = 20\log_{10}\sqrt{\left(\frac{R}{1000[\Omega]}\right)^2 + \left(\frac{\omega L}{1000[\Omega]}\right)^2}$$

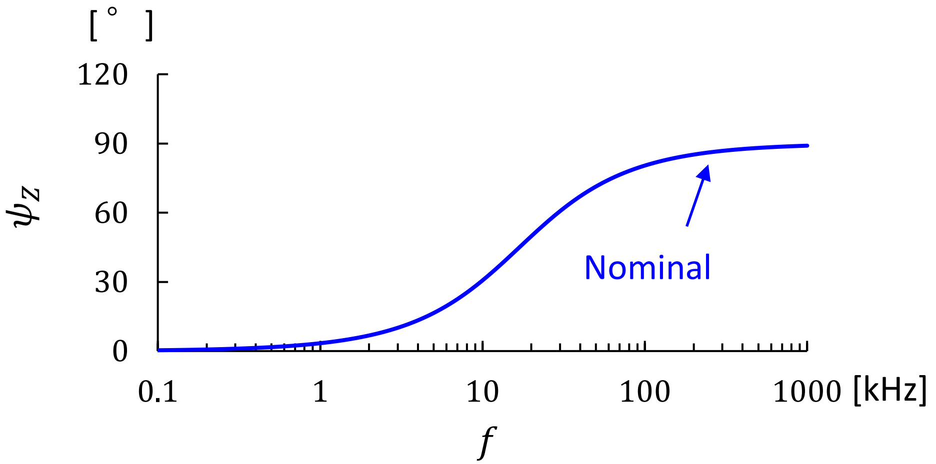
$$= 20\log_{10}\sqrt{\left(\frac{1000[\Omega]}{1000[\Omega]}\right)^2 + \left(\frac{2 \times \pi \times f\,[\mathrm{Hz}] \times 0.01[\mathrm{H}]}{1000[\Omega]}\right)^2} \quad [\mathrm{dB}]$$

$$\psi_Z = \tan^{-1}\frac{\omega L}{R}$$

$$= \tan^{-1}\frac{2 \times \pi \times f\,[\mathrm{Hz}] \times 0.01[\mathrm{H}]}{1000\,[\Omega]} \quad [^\circ\ ]$$

Experiment 11.2.1 (Answer)

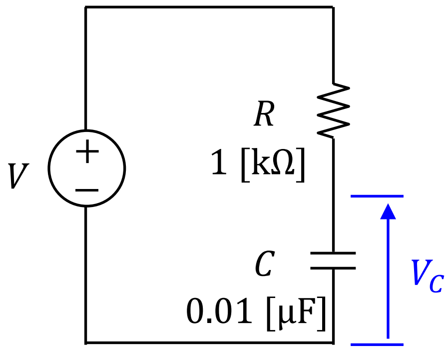




Exercise 11.2.2 (Homework)

Exercise 11.2.2 (Homework)

Exercise 11.2.2 (Homework)



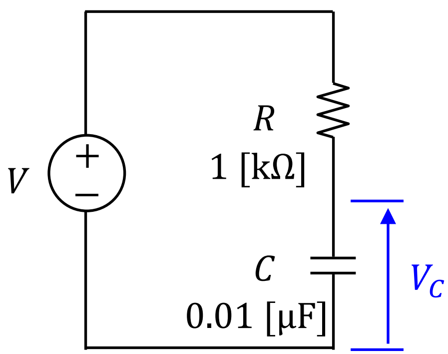
1. Derive $G_Z = 20\log_{10}\left|\frac{V_C}{V}\right|$, and $\psi_{V_C - V}$ as functions of ω.

2. Plot G_Z and ψ_Z in the range of 0.1 [kHz]$\leq f \leq$ 1000 [kHz].

Exercise 11.2.2 (Answer)

Exercise 11.2.2 (Answer)

Exercise 11.2.2 (Answer)



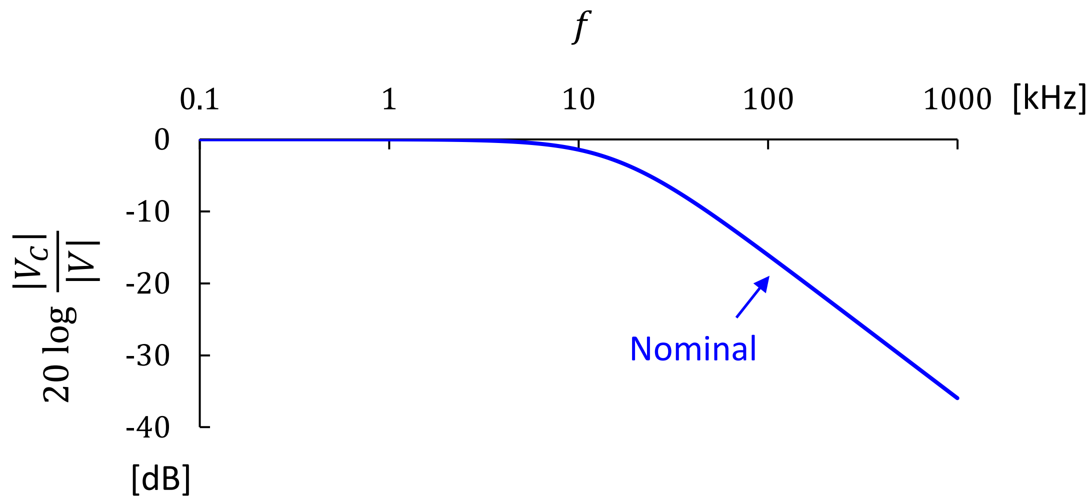
$$V_C = \frac{-j\dfrac{1}{\omega C}}{R - j\dfrac{1}{\omega C}} V$$

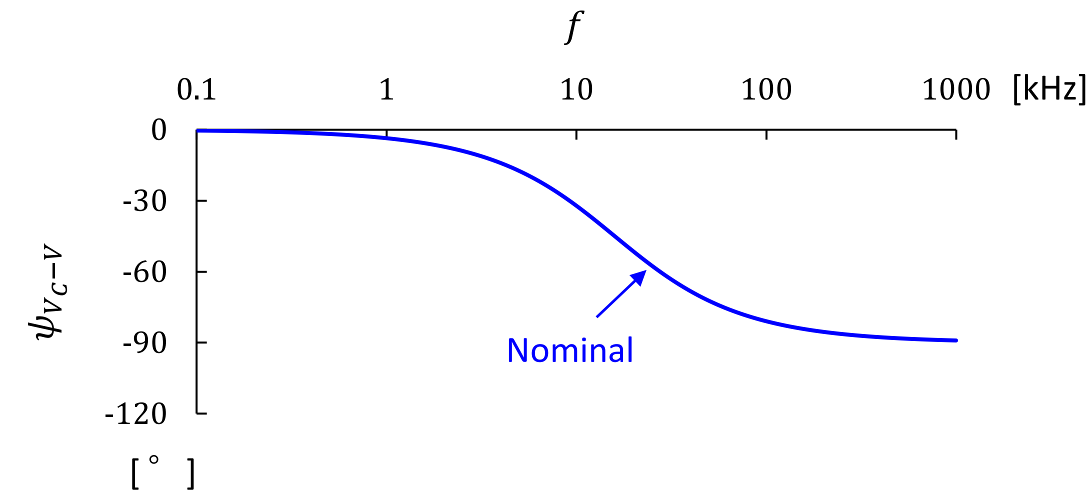
$$= \frac{1}{1 + j\omega CR} V$$

$$20\log_{10}\frac{|V_C|}{|V|} = 20\log_{10}\frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$\psi_{V_C - V} = -\tan^{-1}\omega CR$$

Exercise 11.2.2 (Answer)

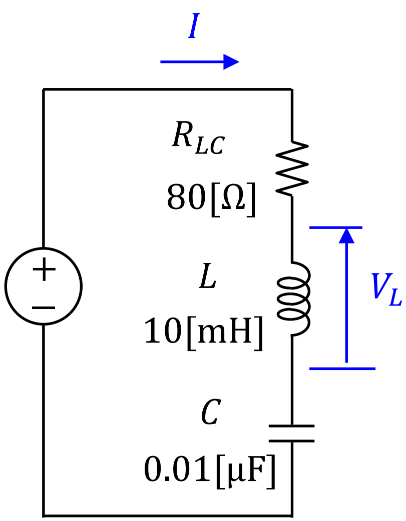




Exercise 12.4 (Homework)

Exercise 12.4 (Homework)

Exercise 12.4 (Homework)



Equivalent Circuit

1. Derive the relationship between the ratio

$$G_{V_L} = 20\log\frac{V_{Le}}{V_e} \quad \text{[dB]}$$

   and the frequency f.

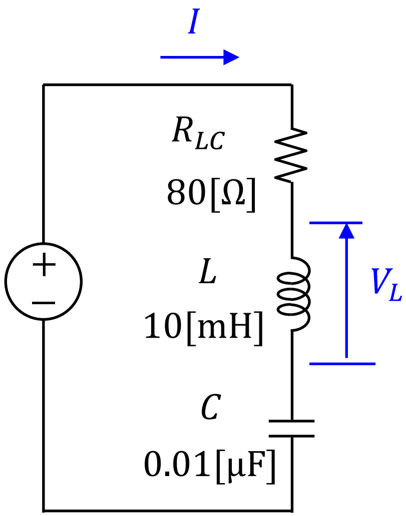
2. Derive the relationship between the phase angle of V_L relative to V, ψ_{V_L-V}, and the frequency f.

3. Draw the plots of G_{V_C} and ψ_{V_L-V} in the range from 1 [kHz] to 100 [kHz].

Exercise 12.4 (Answer)

Exercise 12.4 (Answer)

Exercise 12.4 (Answer)



Equivalent Circuit

$$G_{V_C} = 20\log\frac{V_{Ce}}{V_e}$$

$$V = \left\{R + j\left(\omega L - \frac{1}{\omega C}\right)\right\} I$$

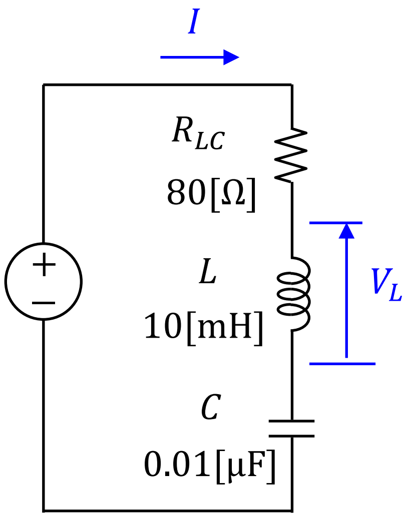
$$V_L = j\omega L I$$

$$V_L = \frac{j\omega L}{R + j\left(\omega L - \frac{1}{\omega C}\right)} V$$

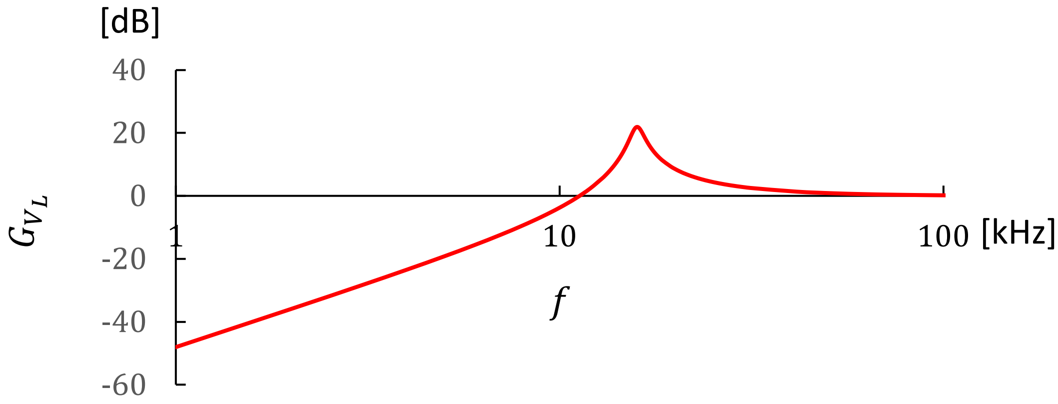
$$G_{V_L} = 20\log\frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\psi_{V_L - V} = \frac{\pi}{2} - \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}$$

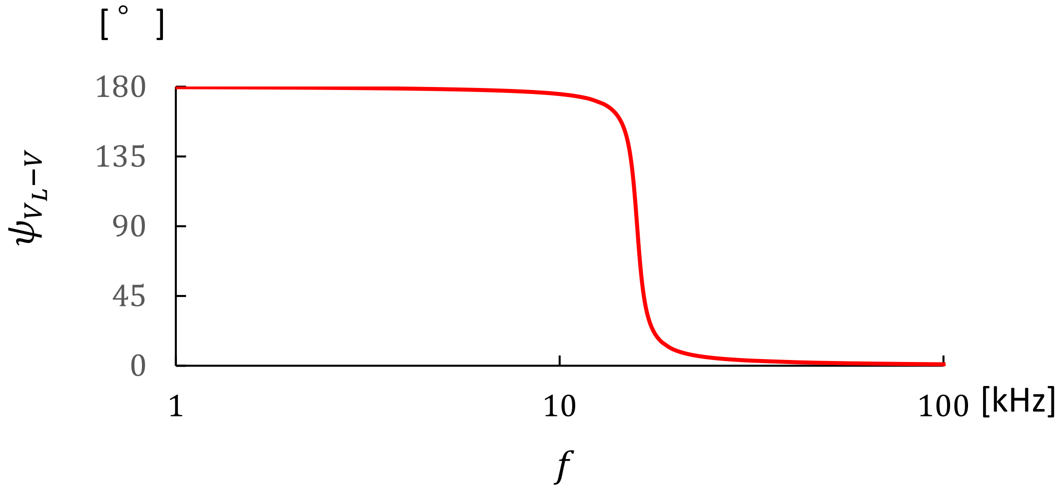
Frequency Characteristics



Equivalent Circuit



(a) G_{V_L}

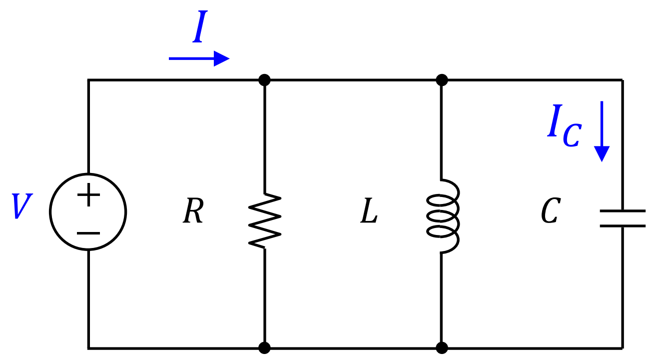


(b) ψ_{V_L-V}

Exercise 12.8.1 (Homework)

Exercise 12.8.1 (Homework)

Exercise 12.8.1 (Homework)



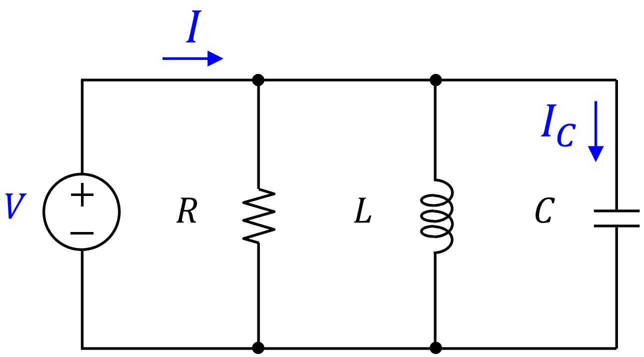
Assume that voltage V is applied to this circuit.

1. Write the relationship between the source current I and the voltage V.
2. Write the relationship between the capacitance current I_C and the voltage V.
3. Write the relationship between the capacitance current I_C and the source current I.
4. Write the relationship between the effective value of capacitance current I_{Ce} and that of the source current I_e.
5. Write the relationship between the ratio $G_{I_C} = 20\log I_{Ce}/I_e$ and the frequency f.
6. Write the phase angle of the capacitance current relative to the source current ψ_{I_C-I} as a function of frequency f.

Exercise 12.8.1 (Answer)

Exercise 12.8.1 (Answer)

Exercise 12.8.1 (Answer)



$$I = \left\{\frac{1}{R} - j\left(\frac{1}{\omega L} - \omega C\right)\right\} V$$

$$I_C = j\omega C V$$

$$I_C = \frac{j\omega C}{\frac{1}{R} - j\left(\frac{1}{\omega L} - \omega C\right)} I \Rightarrow \frac{I_{Ce}}{I_e} = \frac{\omega C}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}}$$

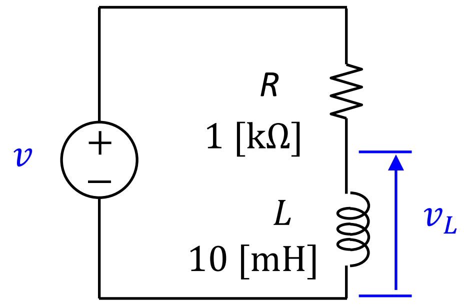
$$G_{I_C} = 20 \log \frac{\omega C}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}}$$

$$\psi_{I_C - I} = \frac{\pi}{2} + \tan^{-1} R\left(\frac{1}{\omega L} - \omega C\right)$$

Exercise 13.2.2 (Homework)

Exercise 13.2.2 (Homework)

Exercise 13.2.2 (Homework)

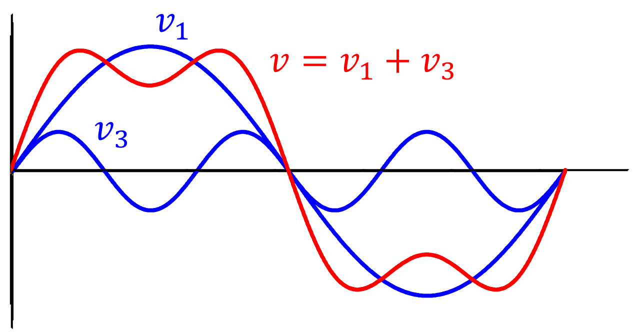


$$v = v_1 + v_3$$

$$v_1 = \sqrt{2} V_e \sin \omega t$$

$$v_3 = \frac{\sqrt{2}}{3} V_e \sin 3\omega t$$

$$f = 15.9 \text{ [kHz]}$$



(1) Derive V_{L1} and V_{L3}.

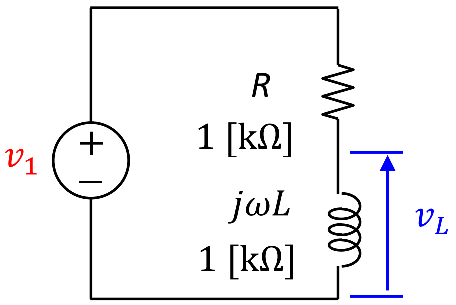
(2) Derive v_{L1} and v_{L3}.

(3) Draw the waveform of v_L $(= v_{L1} + v_{L3})$.

Exercise 13.2.2 (Answer)

Exercise 13.2.2 (Answer)

Exercise 13.2.2 (Answer)



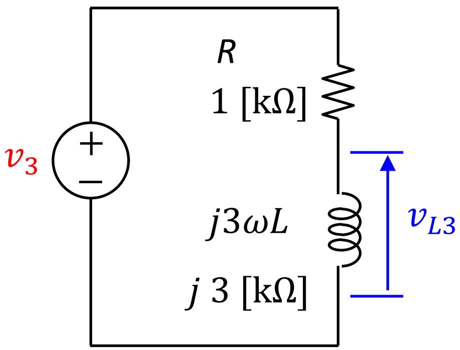
$$v_1 = \sqrt{2} V_e \sin \omega t$$

$$j\omega L = j2\pi \times 15.9 \times 10^3 \times 0.01$$
$$= j\ [\mathrm{k\Omega}]$$

(1) $$V_{L1} = \frac{j\omega L}{R + j\omega L} V_1 = \frac{j}{1+j} V_e$$

(2) $$v_{L1} = \sqrt{2} \frac{1}{\sqrt{2}} V_e \sin\left(\omega t + \frac{\pi}{4}\right)$$
$$= V_e \sin\left(\omega t + \frac{\pi}{4}\right)$$

Exercise 13.2.2 (Answer)



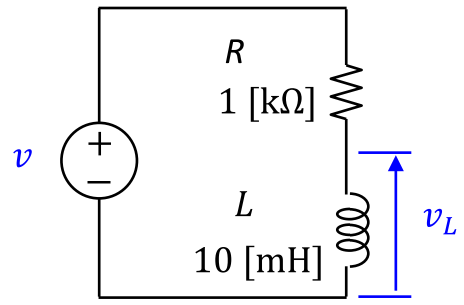
$$v_3 = \frac{\sqrt{2}}{3} V_e \sin 3\omega t$$

$$j3\omega L = j3 \times 2\pi \times 15.9 \times 10^3 \times 0.01$$
$$= j\,3\ [\mathrm{k\Omega}]$$

(1) $$V_{L3} = \frac{j3\omega L}{R + j3\omega L} V_1 = \frac{j\,3}{1 + j\,3} \frac{V_e}{3}$$

(2) $$v_{L3} = \frac{3}{\sqrt{10}} \frac{\sqrt{2}}{3} V_e \sin\left(\omega t + \frac{\pi}{2} - \tan^{-1} 3\right)$$
$$\approx \frac{1}{\sqrt{5}} V_e \sin(\omega t + 0.1\pi)$$

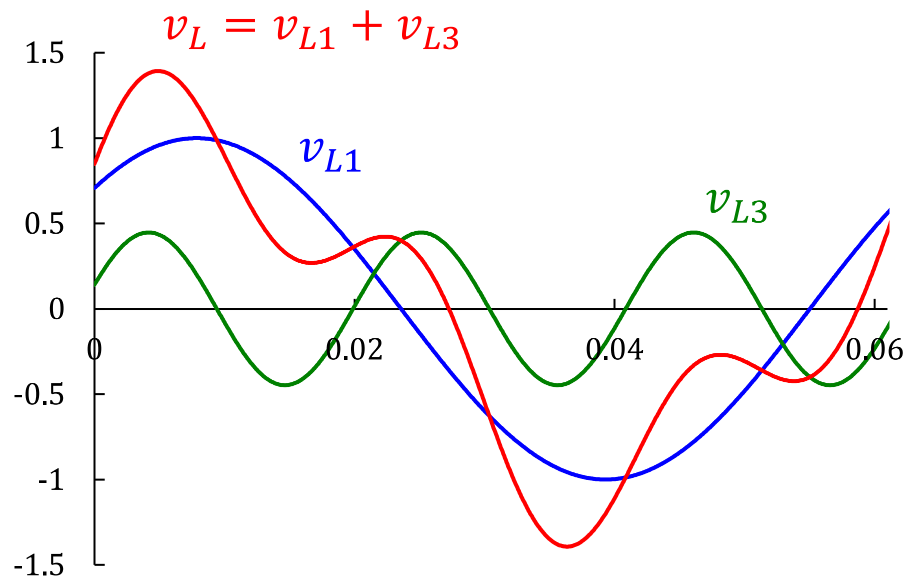
Exercise 13.2.2 (Answer)



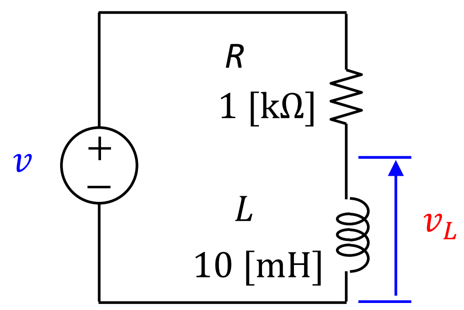
(3) $v_L = v_{L1} + v_{L3}$

$$\approx V_e \sin\left(\omega t + \frac{\pi}{4}\right) + \frac{1}{\sqrt{5}} V_e \sin(\omega t + 0.1\pi)$$

$V_e = 1$



For your reference

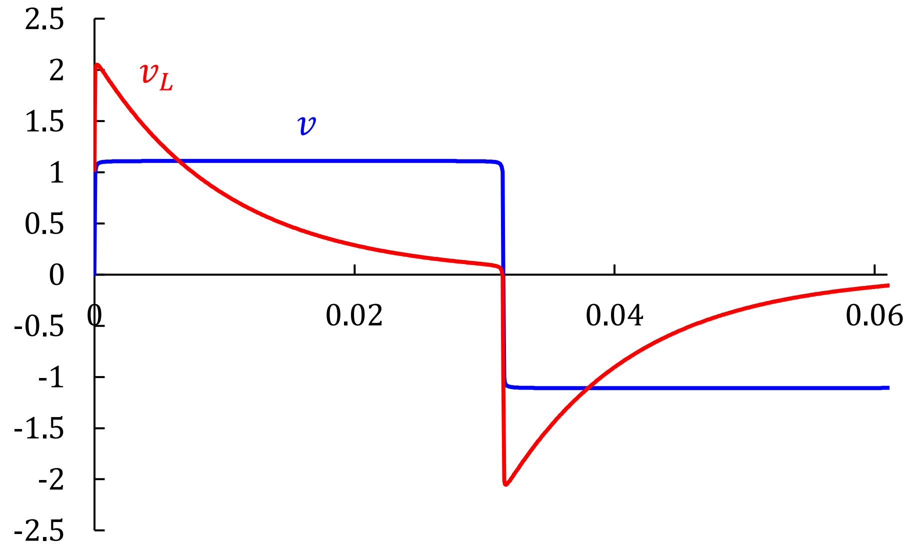


$$v = v_1 + v_3 + \cdots + v_{999}$$

$$v_C = v_{C1} + v_{C3} + \cdots + v_{C999}$$

$$v_{Lk} = \frac{k}{\sqrt{1+k^2}} \frac{\sqrt{2}}{k} V_e \sin\left(\omega t + \frac{\pi}{2} - \tan^{-1} k\right)$$

$$V_e = 1$$

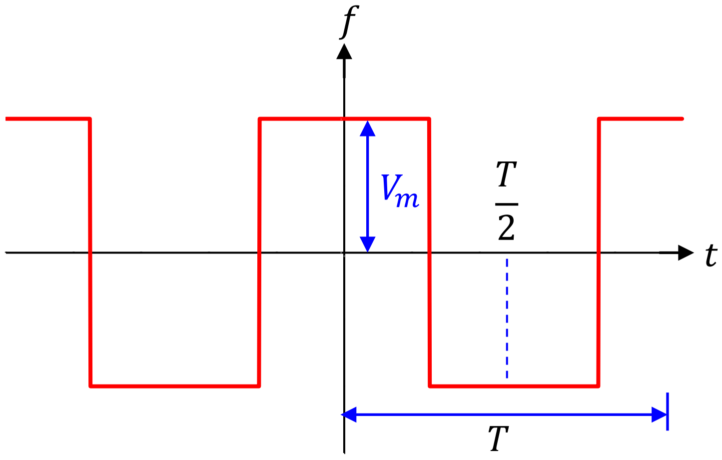


This is for your reference. If the harmonics up to 999th are summed, the inductance voltage v_L becomes like this one.

Exercise 13.3.3 (Homework)

Exercise 13.3.3 (Homework)

Exercise 13.3.3 (Homework)

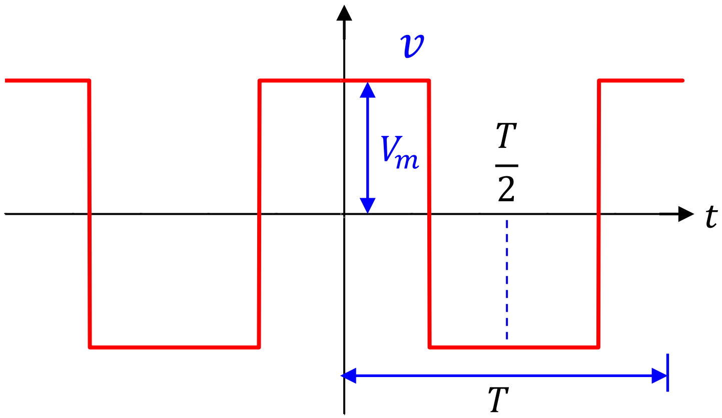


Find the Fourier series for this square wave.

Exercise 13.3.3 (Answer)

Exercise 13.3.3 (Answer)

Exercise 13.3.3 (Answer)



$$a_0 = \frac{1}{T}\int_0^T v dt$$
$$= 0$$

$$a_n = \frac{2}{T}\left(\int_0^{T/4} V_m \cos n\omega t \, dt - \int_{T/4}^{3T/4} V_m \cos n\omega t \, dt + \int_{3T/4}^{T/4} V_m \cos n\omega t \, dt\right)$$
$$= \frac{4}{\pi} V_m \frac{\sin \frac{n\pi}{2}}{n}$$

$$b_n = \frac{2}{T}\int_0^T v \sin n\omega t \, dt$$
$$= 0$$

$$v = V_{m0} \cos \omega t - \frac{1}{3} V_{m0} \cos 3\omega t + \frac{1}{5} V_{m0} \cos 5\omega t - \cdots$$

$$V_{m0} = \frac{4}{\pi} V_m$$

Exercise 13.4.2 (Homework)

Exercise 13.4.2 (Homework)

Exercise 13.4.2 (Homework)

Draw the frequency spectrum of the following two equations:

(1)

$$v = V_{m0} \sin \omega t - \frac{1}{3} V_{m0} \sin 3\omega t + \frac{1}{5} V_{m0} \sin 5\omega t - \cdots$$

$$V_{m0} = \frac{4}{\pi} V_m$$

$$V_m = 1\ [\mathrm{V}], f = 1\ [\mathrm{Hz}].$$

(2)

$$v = V_{m0} \sin \omega t - \frac{1}{2} V_{m0} \sin 2\omega t + \frac{1}{3} V_{m0} \sin 3\omega t - \cdots$$

$$V_{m0} = \frac{2}{\pi} V_m$$

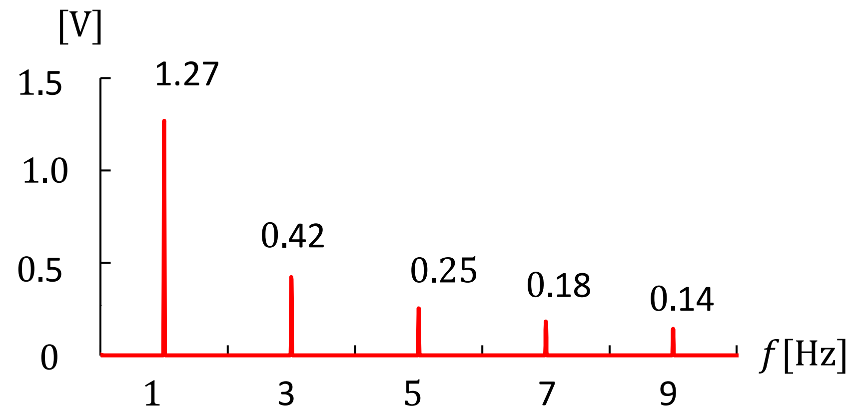
$$V_m = 1\ [\mathrm{V}], f = 1\ [\mathrm{Hz}].$$

Exercise 13.4.2 (Answer)

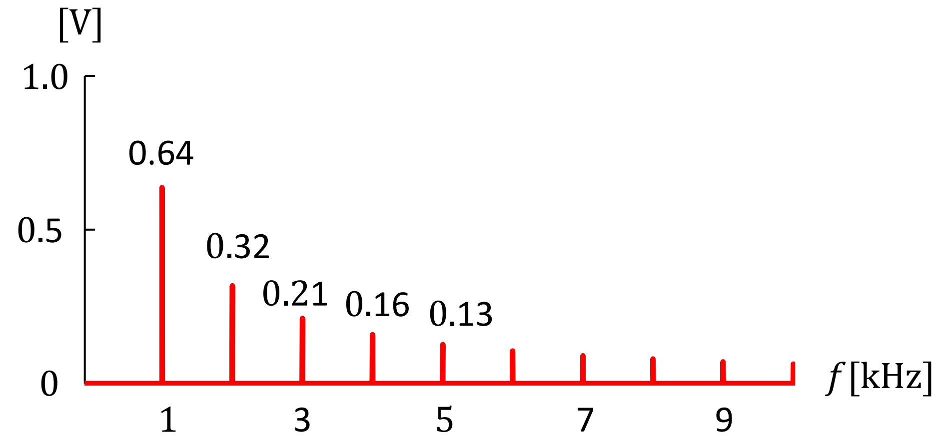
Exercise 13.4.2 (Answer)

Exercise 13.4.2 (Answer)

(1)

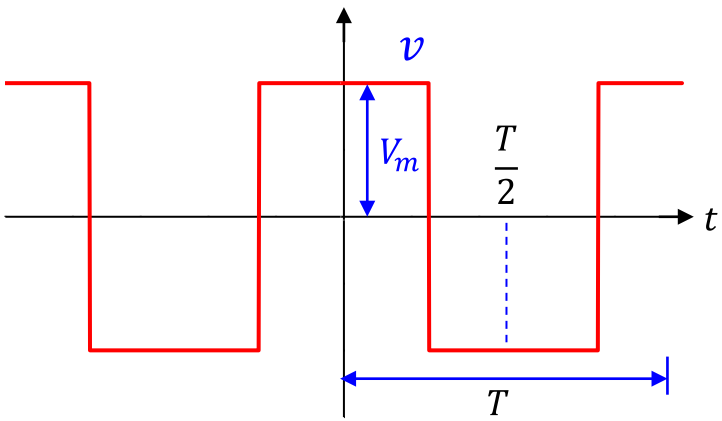


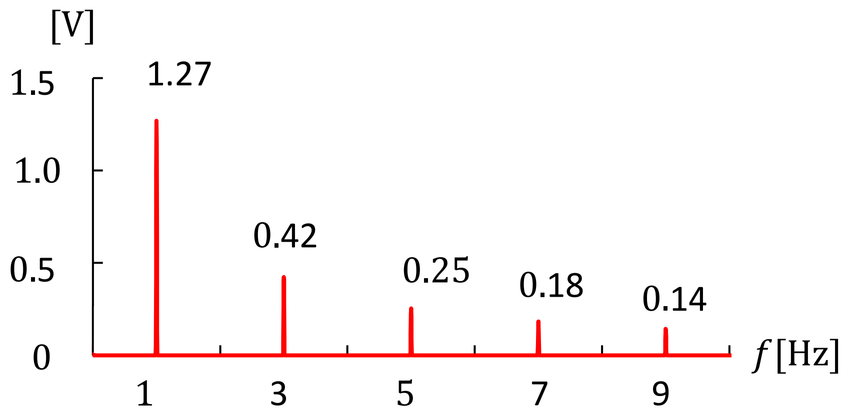
(2)



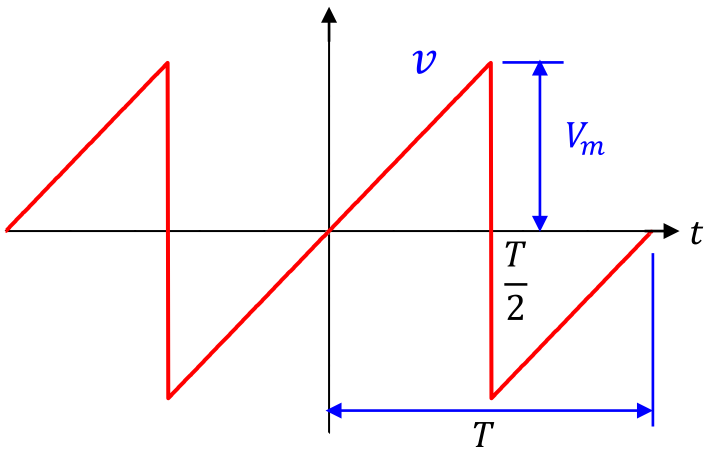
For your reference

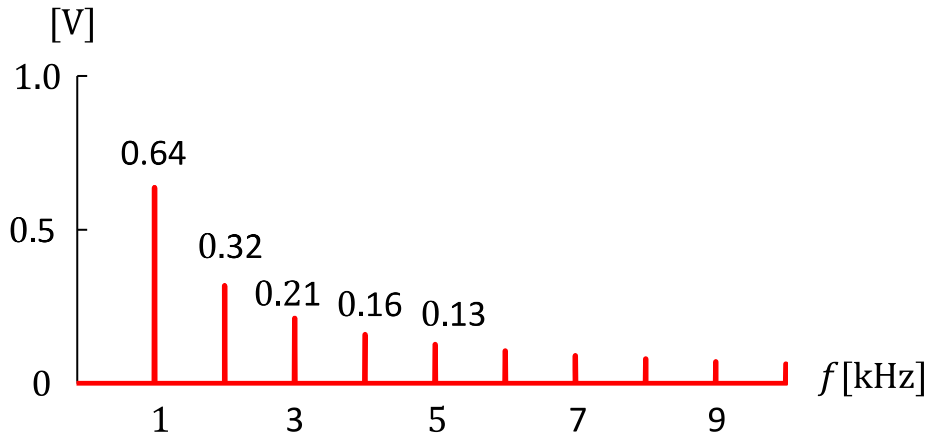
(1)





(2)

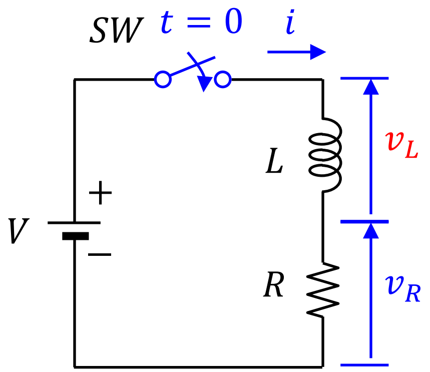




Exercise 14.3 (Homework)

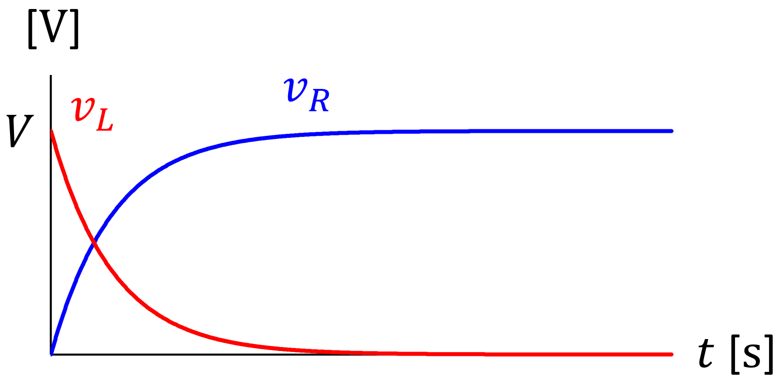
Exercise 14.3 (Homework)

Exercise 14.3 (Homework)



(1) Why do v_R increase and v_L decrease?

(2) Why does $\frac{dv_R}{dt}$ decrease ?

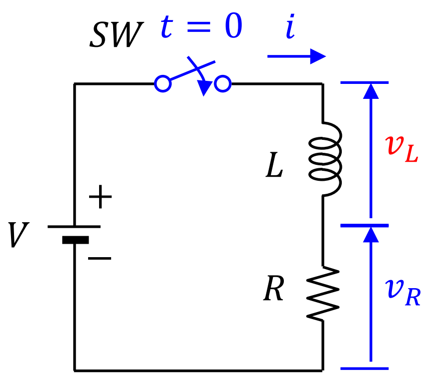


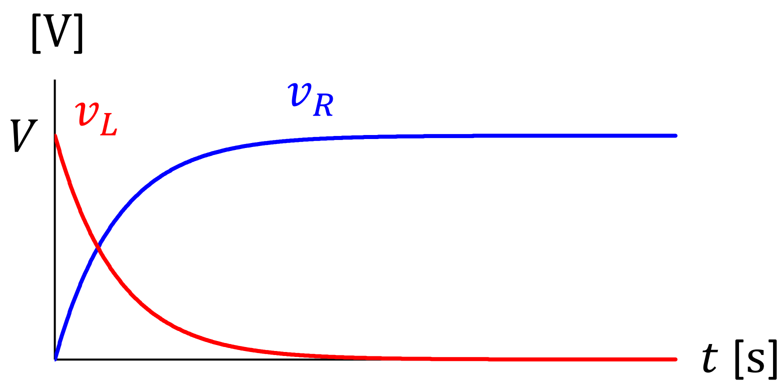
Pause the video, and answer the questions.

Exercise 14.3 (Answer)

Exercise 14.3 (Answer)

Exercise 14.3 (Answer)





(1) Why do v_R increase and v_L decrease?

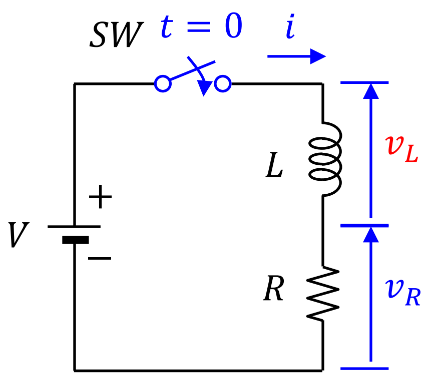
$$\frac{di}{dt} = \frac{v_L}{L}$$

⇨ i increases.

⇨ v_R increases.

⇨ v_L decreases.

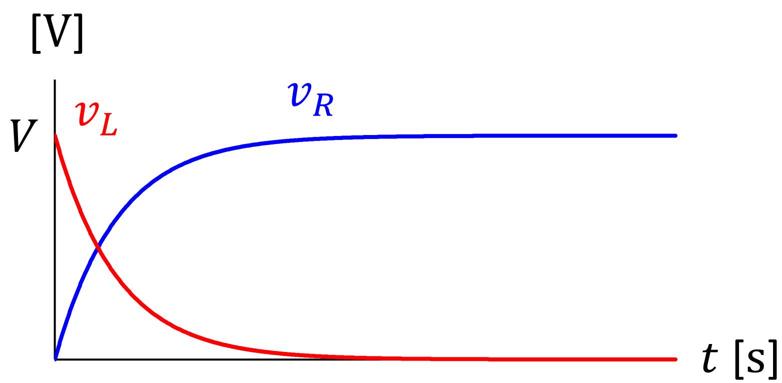
Exercise 14.3 (Answer)



(2) Why does $\dfrac{dv_R}{dt}$ decrease ?

$$\frac{dv_R}{dt} = R\frac{di}{dt} = \frac{R}{L}v_L$$

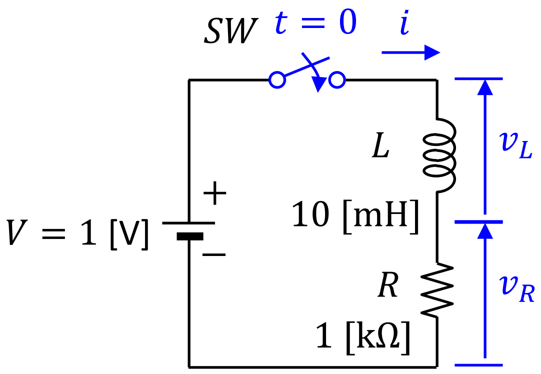
v_L decreases. ⇨ $\dfrac{dv_R}{dt}$ decreases.



Exercise 14.4 (Homework)

Exercise 14.4 (Homework)

Exercise 14.4.1 (Homework)

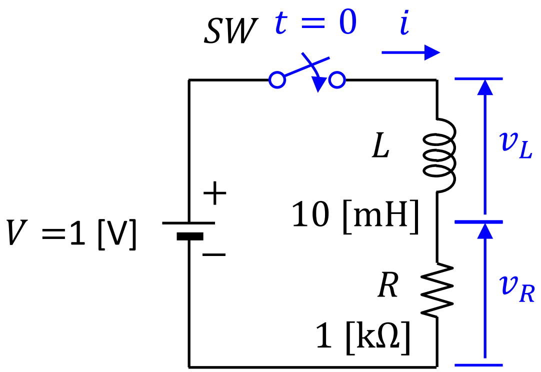


The switch SW is turned on at $t = 0$.

Find v_R, v_L and i at $t = \tau$, where τ is the time constant.

Exercise 14.4.1 (Answer)

Exercise 14.4.1



$(t \geq 0)$

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$v_L = Ve^{-\frac{R}{L}t}$$

$$v_R = V\left(1 - e^{-\frac{R}{L}t}\right)$$

At $t = \tau = \frac{L}{R} = 10\ [\mu s]$

$$i = \frac{V}{R}(1 - e^{-1}) = \frac{1}{1000}(1 - e^{-1}) \approx 0.63\ [\text{mA}]$$

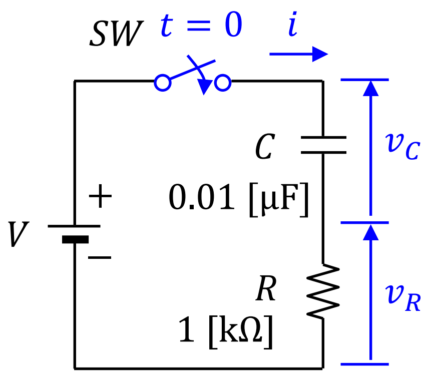
$$v_L = Ve^{-1} = 1e^{-1} \approx 0.37\ [\text{V}]$$

$$v_R = V(1 - e^{-1}) = 1 \times (1 - e^{-1}) \approx 0.63\ [\text{V}]$$

Exercise 14.6 (Homework)

Exercise 14.6 (Homework)

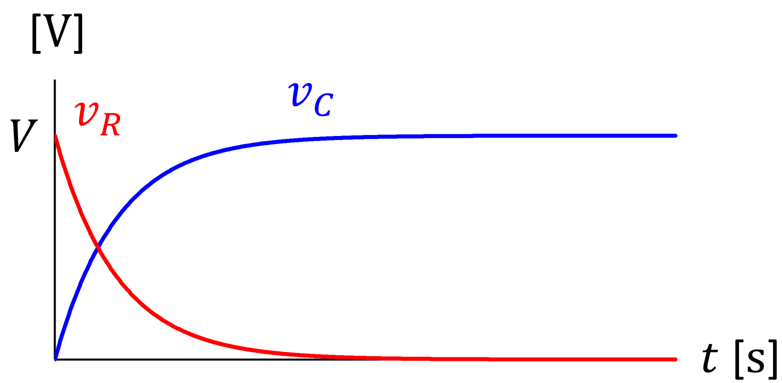
Exercise 14.6 (Homework)



(1) Why are $v_R = V$ [V] and $v_C = 0$ [V] at $t = 0$?

(2) What are the values of $\frac{dq}{dt}, \frac{dv_R}{dt}$, and $\frac{dv_C}{dt}$ at $t = 0$?

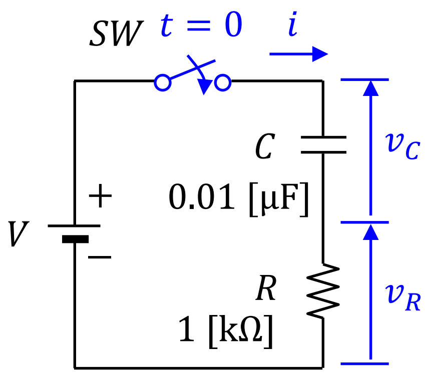
(3) What is time constant of this $R - C$ circuit ?



Exercise 14.6 (Answer)

Exercise 14.6 (Answer)

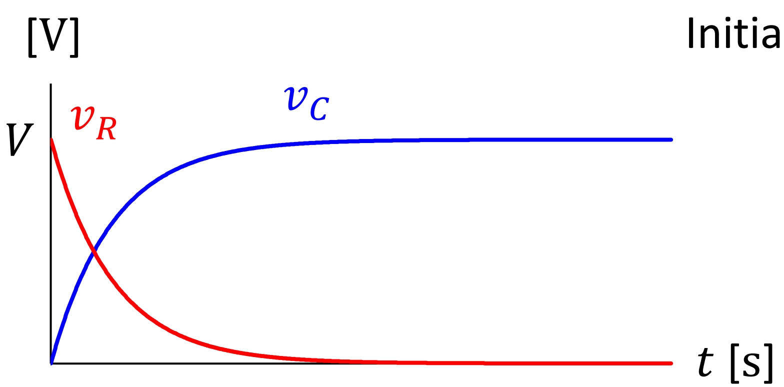
Exercise 14.6 (Answer)



(1) Why are $v_R = V$ [V] and $v_C = 0$ [V] at $t = 0$?

$$v_C = \frac{1}{C}\int i dt \Rightarrow i = C\frac{dv_C}{dt}$$

⇨ v_C cannot change instantaneously.

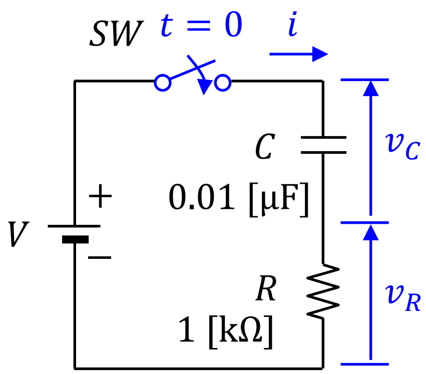


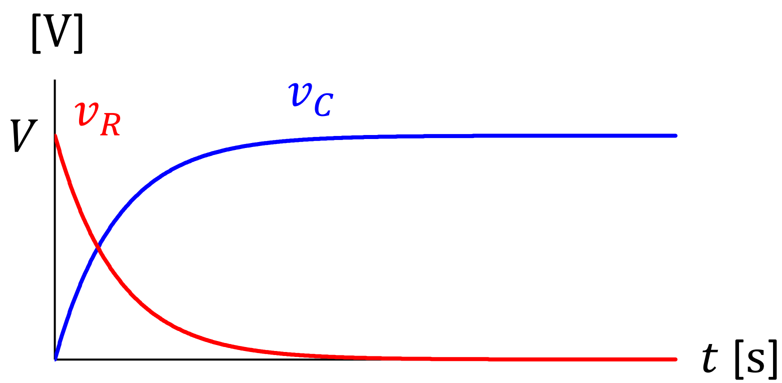
Initial condition: $q = 0$ at $t = 0$

⇨ $v_C = \dfrac{q}{C} = 0$ at $t = 0$.

⇨ $v_R = V$ at $t = 0$.

Exercise 14.6 (Answer)





(2) What are $\dfrac{dq}{dt}, \dfrac{dv_R}{dt}, \dfrac{dv_C}{dt}$ at $t = 0$?

$$q = CV\left(1 - e^{-\frac{1}{RC}t}\right)$$

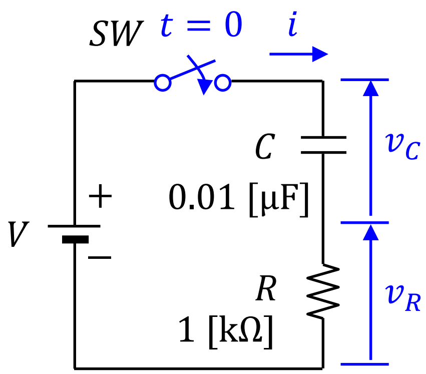
$$\frac{dq}{dt} = \frac{V}{R}e^{-\frac{1}{RC}t}$$

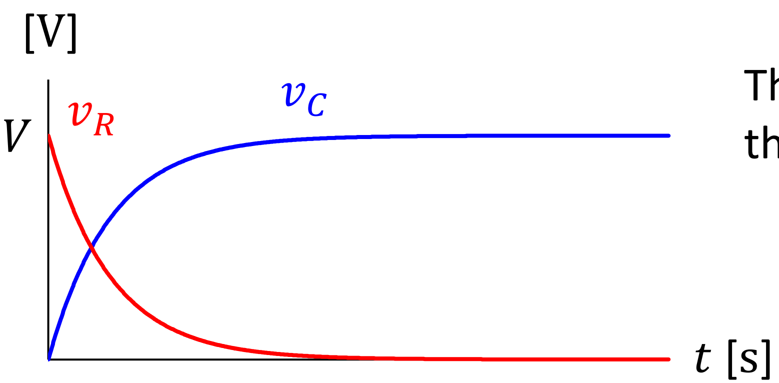
$$i = \frac{dq}{dt}$$

$$\frac{dv_R}{dt} = R\frac{di}{dt} = R\frac{d}{dt}\frac{V}{R}e^{-\frac{1}{RC}t} = -\frac{V}{RC}e^{-\frac{1}{RC}t}$$

$$\frac{dv_C}{dt} = \frac{1}{C}\frac{dq}{dt} = \frac{V}{RC}e^{-\frac{1}{RC}t}$$

Exercise 14.6 (Answer)





(3) What is time constant of this $R - C$ circuit ?

$$v_C = V\left(1 - e^{-\frac{1}{RC}t}\right)$$

The coefficient of t, $1/RC$ determines the rate at which the voltage v_C approaches to V.

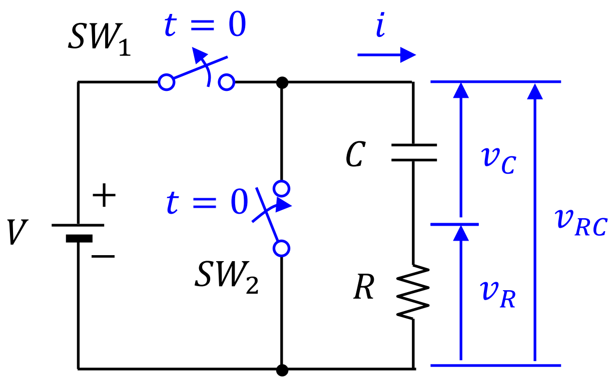
The reciprocal of this coefficient is defined as the time constant of $R - C$ circuit.

$$\tau = RC$$

Exercise 14.7 (Homework)

Exercise 14.7 (Homework)

Exercise 14.7 (Homework)

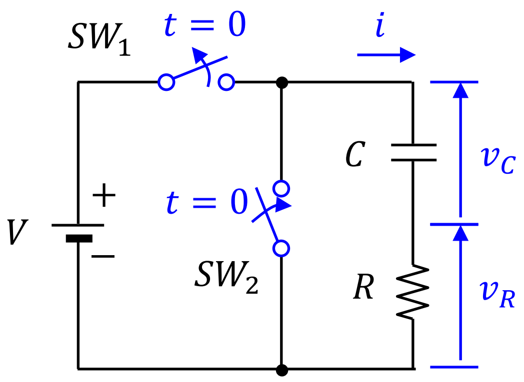


Find v_R and v_C for this circuit under the conditions that for $t \geq 0, v_{RC} = 0$, and. at $t = 0, q = \mathrm{CV}$.

Exercise 14.7 (Answer)

Exercise 14.7 (Answer)

Exercise 14.7 (Answer)



For $t \geq 0$, $v = 0$

$$0 = R\frac{dq}{dt} + \frac{q}{C}$$

Assume that $q = Ae^{-\frac{1}{RC}t} + B$, where A and B are constants.

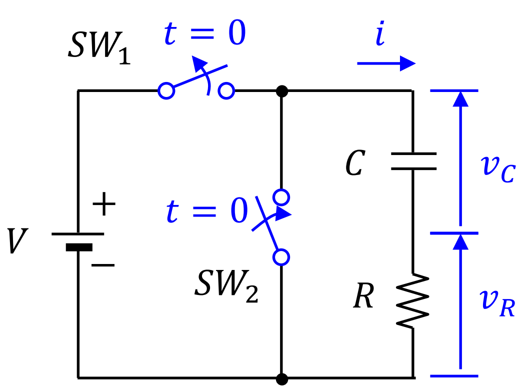
$$0 = R\frac{d}{dt}\left(Ae^{-\frac{1}{RC}t} + B\right) + \frac{1}{C}\left(Ae^{-\frac{1}{RC}t} + B\right)$$
$$= \frac{1}{C}B$$
$$B = 0$$

At $t = 0$, $q = CV$: initial condition

$$CV = A$$

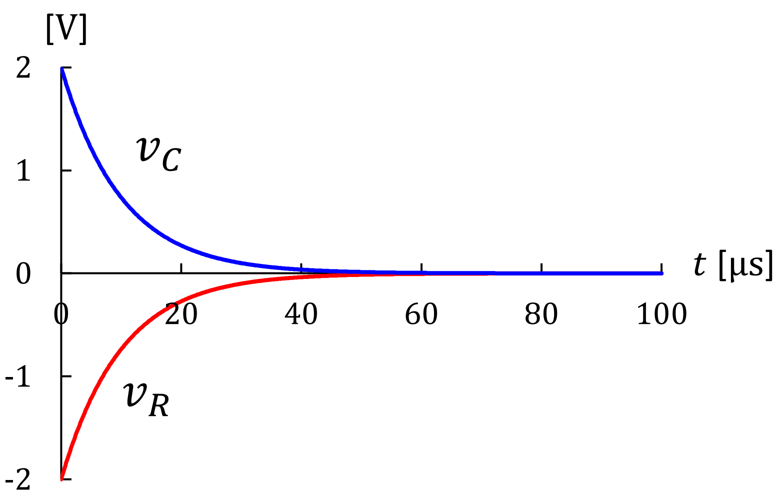
$$q = CVe^{-\frac{1}{RC}t}$$

Exercise 14.7 (Answer)

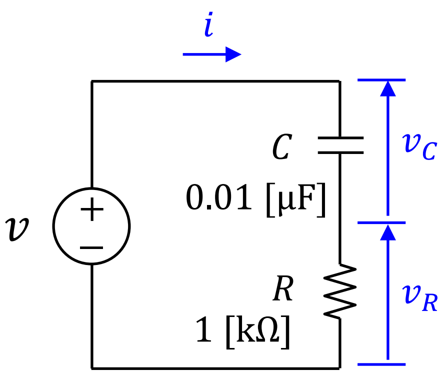


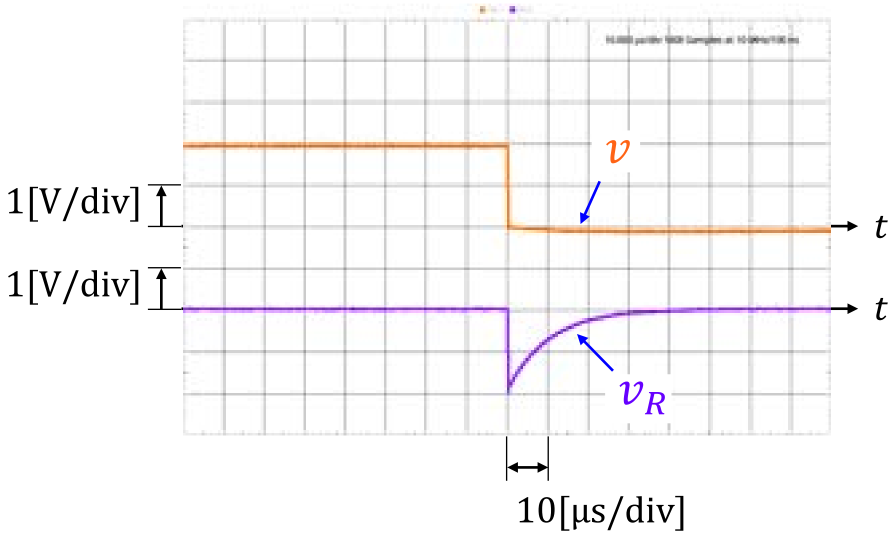
$$
\begin{aligned}
v_R &= Ri \\
&= R\frac{dq}{dt} \\
&= R\frac{d}{dt}\left\{CVe^{-\frac{1}{RC}t}\right\} \\
&= -Ve^{-\frac{1}{RC}t}
\end{aligned}
$$

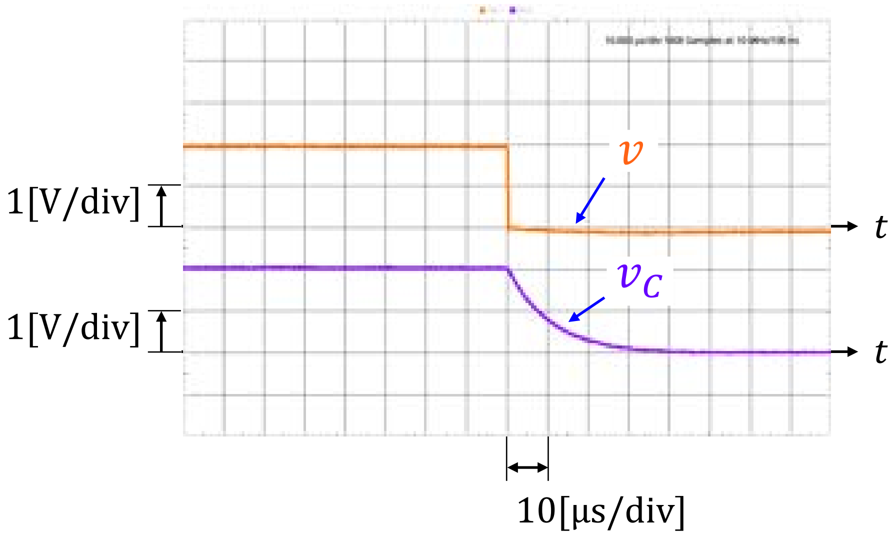
$$
\begin{aligned}
v_C &= \frac{q}{C} \\
&= Ve^{-\frac{1}{RC}t}
\end{aligned}
$$



For your reference



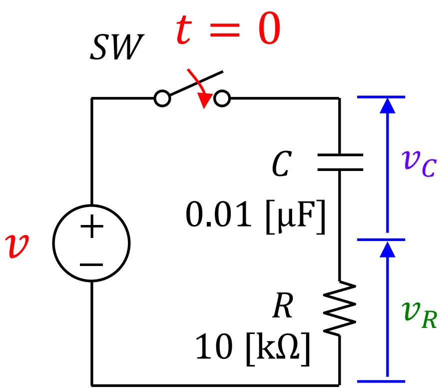




Exercise 14.8 (Homework)

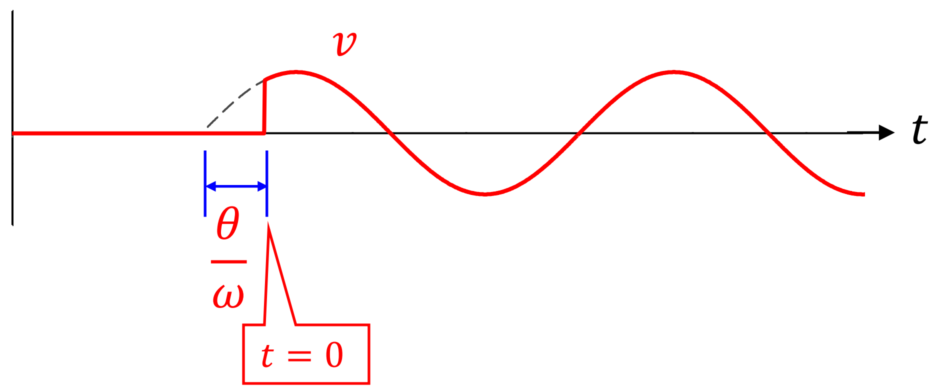
Exercise 14.8 (Homework)

Exercise 14.8 (Homework)



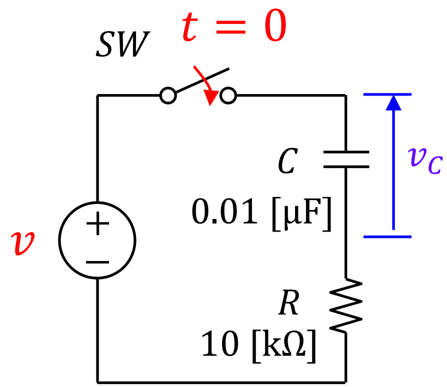
Find the phase angle θ with which no transient response occurs in this circuit.

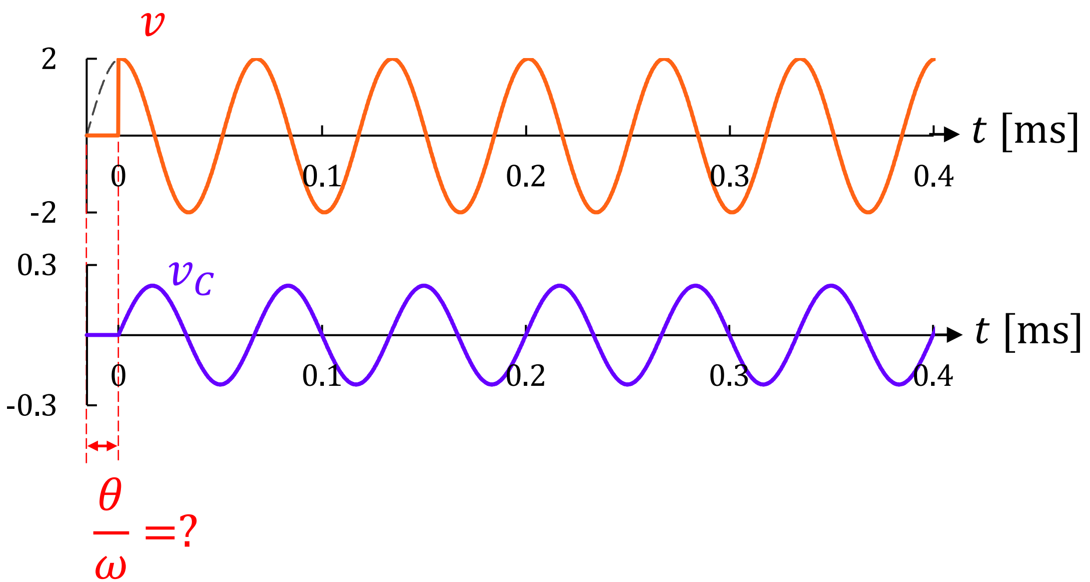
$$v = \begin{cases} 0 & (t < 0) \\ V_m \sin(\omega t + \theta) & (t \geq 0) \end{cases}$$



Initial condition: at $t = 0, q = 0$.

Exercise 14.8 (Homework)

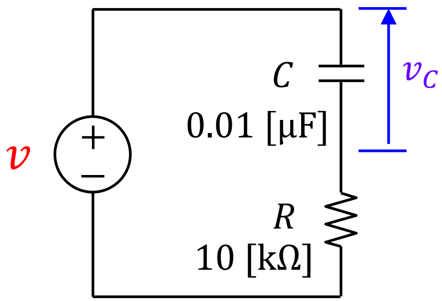




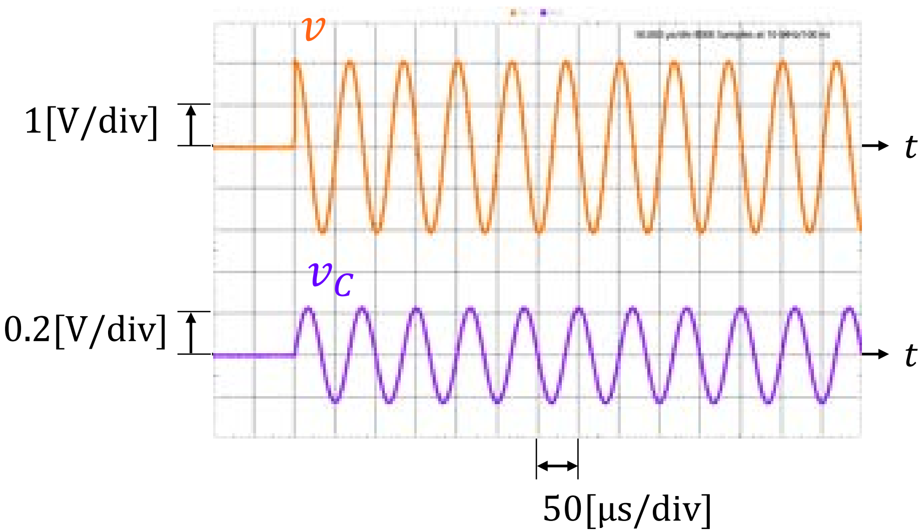
$V_m = 2$ [V]
$f = 15$ [kHz]
$\theta = ?$ [rad]

These are calculated waveforms. For $t \geq 0$, v_C contains only a trigonometric waveform.

Exercise 14.8 (Homework)



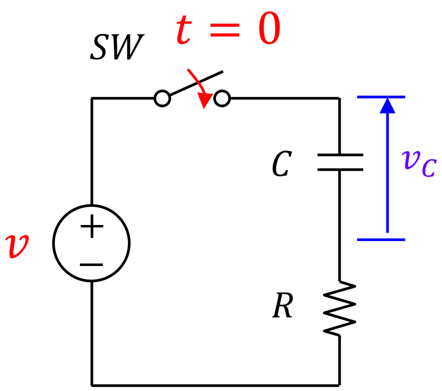
$V_m = 2$ [V]
$f = 15$ [kHz]
$\theta = ?$ [rad]



These are experimental waveforms.

Exercise 14.8 (Answer)

Exercise 14.8 (Answer)



If $\theta = \frac{\pi}{2} - \psi$

$$v_C = \frac{(1/\omega C)V_m}{\sqrt{R^2 + (1/\omega C)^2}} \left\{ \cos(\theta + \psi)\, e^{-\frac{1}{RC}t} - \cos(\omega t + \theta + \psi) \right\}$$

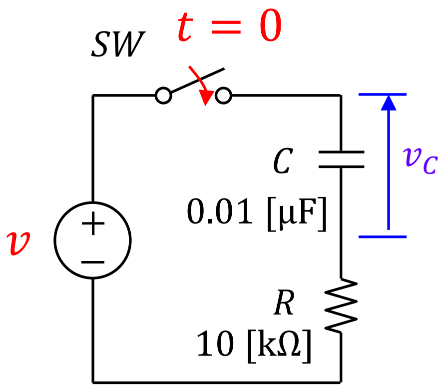
$$= \frac{(1/\omega C)V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin \omega t$$

$\cos(\theta + \psi) = 0$

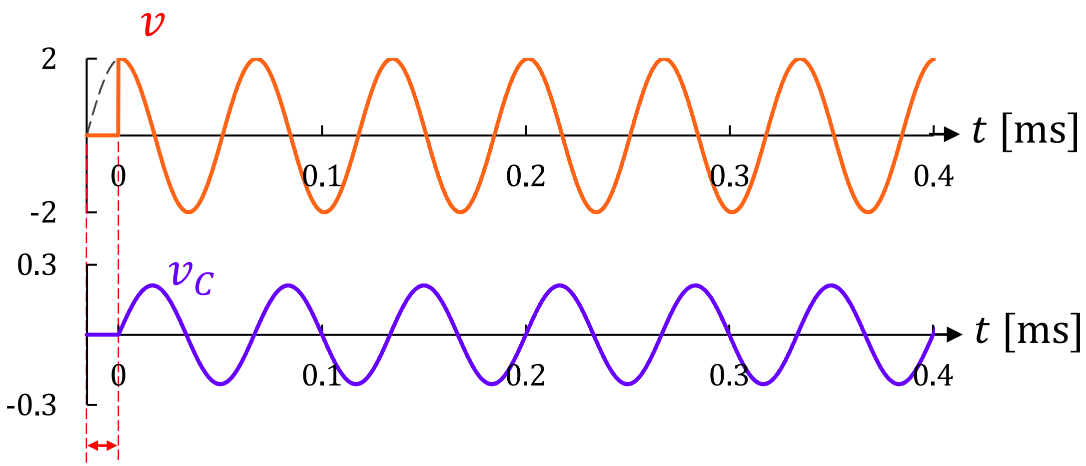
v_C has no transient response.

If $\theta = \frac{\pi}{2} - \psi$, then the term of transient response is 0. Thus, v_C has no transient response.

Exercise 14.8 (Answer)



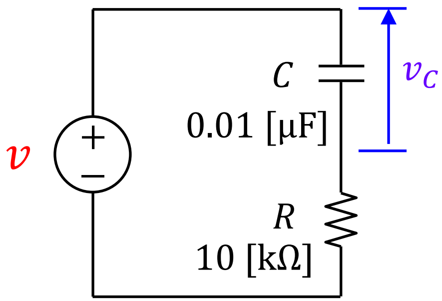
$V_m = 2$ [V]
$f = 15$ [kHz]
$\theta = \dfrac{\pi}{2} - \psi$ [rad]



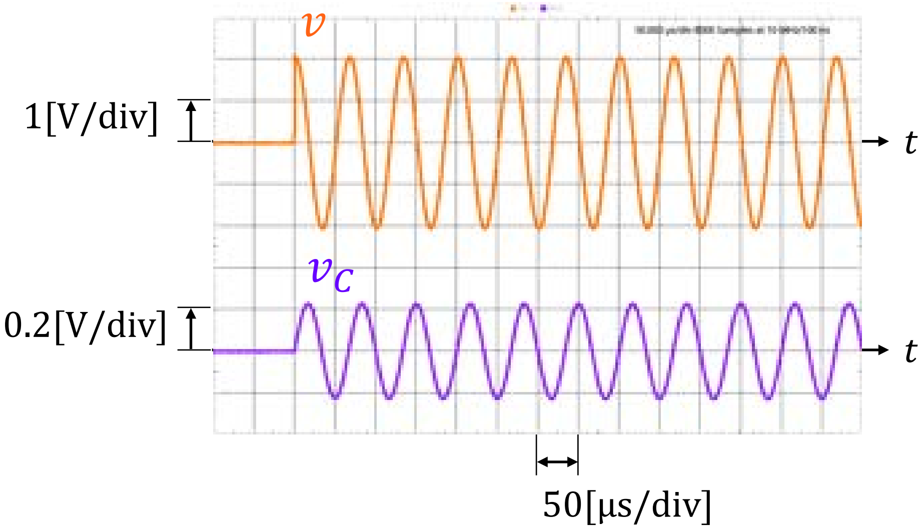
$$\frac{\theta}{\omega} = \frac{\pi/2 - \psi}{\omega}$$

These are calculated waveform.

Exercise 14.8 (Answer)



$V_m = 2$ [V]
$f = 15$ [kHz]
$\theta = \frac{\pi}{2} - \psi$ [rad]



These are experimental waveforms.

Exercise 15.4.2 (Homework)

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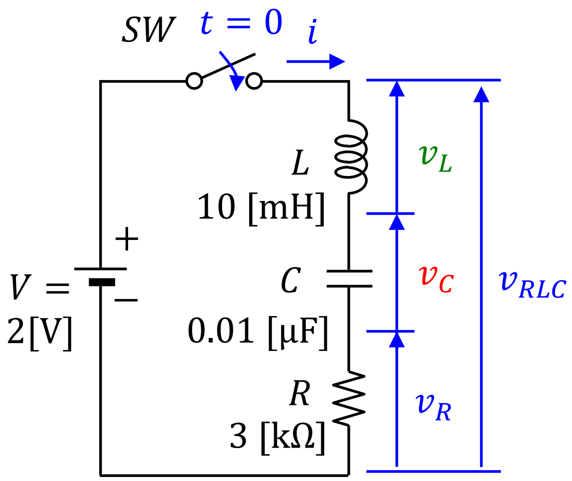


At $t = 0$, the switch SW is turned on.
For $t \geq 0$, $v_{RLC} = V$

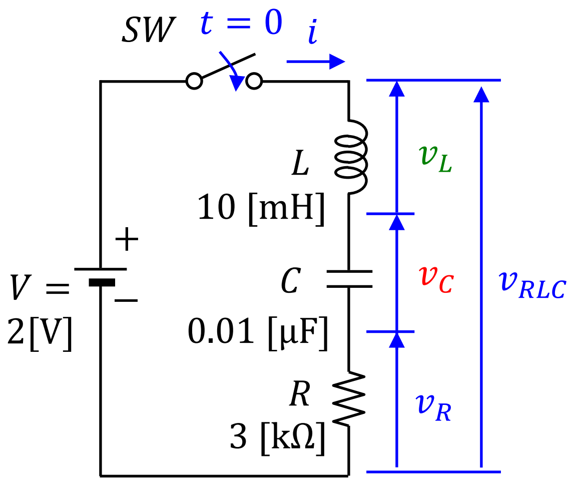
Initial conditions: at $t = 0$, $i = 0$, $v_C = 1$ [V]

1. Obtain v_R, v_L and v_C for $t \geq 0$.

2. Draw the waveforms of v_R, v_L and v_C for $t \geq 0$.

Exercise 15.4.2 (Answers)

Exercise 15.4.2 (Homework)



$$\left(\frac{R}{2L}\right)^2 = \left(\frac{3000}{2\times 0.01}\right)^2 = 2.25\times 10^{10}$$

$$\frac{1}{LC} = \frac{1}{0.01\times 0.01\times 10^{-6}} = 10^{10}$$

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

Assume that

$$q = A_1 e^{p_1 t} + A_2 e^{p_2 t} + B$$

$$\frac{V}{L} = \frac{d^2 q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC}$$

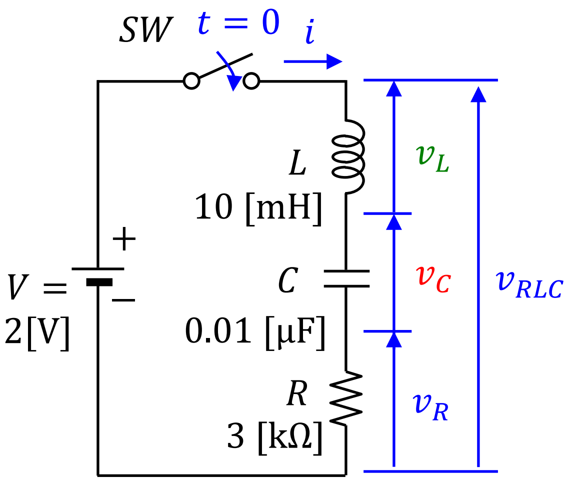
$$= A_1\left(p_1^2 + \frac{R}{L}p_1 + \frac{1}{LC}\right)e^{p_1 t} + A_2\left(p_2^2 + \frac{R}{L}p_2 + \frac{1}{LC}\right)e^{p_2 t} + \frac{B}{LC}$$

$$= \frac{B}{LC}$$

$$B = CV$$

There is no difference in the process to determine B as in the previous case for $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$.

Exercise 15.4.2 (Homework)



Initial conditions: at $t=0, i=0, q=CV_0$

$$q = A_1 e^{p_1 t} + A_2 e^{p_2 t} + B \quad \Leftarrow \quad B = CV$$

$$\frac{dq}{dt} = i = p_1 A_1 e^{p_1 t} + p_2 A_2 e^{p_2 t}$$

$$CV_0 = A_1 + A_2 + \mathrm{CV}$$

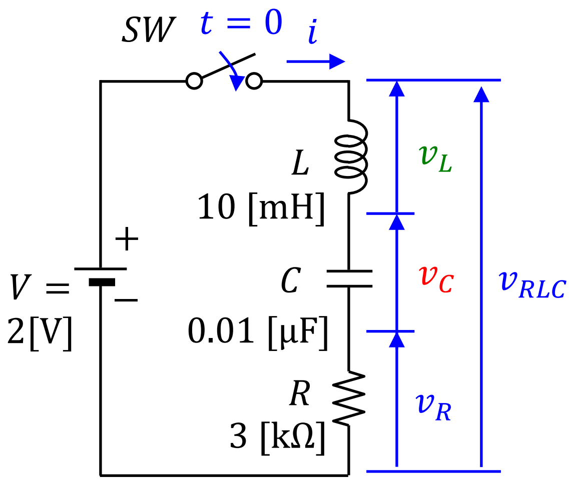
$$0 = p_1 A_1 + p_2 A_2$$

$$A_1 = \frac{p_2}{p_1 - p_2}\mathrm{C}(\mathrm{V} - V_0) = \frac{p_2}{2\beta}\mathrm{C}(\mathrm{V} - V_0)$$

$$A_2 = \frac{-p_1}{p_1 - p_2}\mathrm{C}(\mathrm{V} - V_0) = \frac{-p_1}{2\beta}\mathrm{C}(\mathrm{V} - V_0)$$

The difference is only in the initial condition. At $t=0, q=CV_0$.

Exercise 15.4.2 (Homework)



$$q = \frac{C(\mathrm{V} - V_0)}{2\beta}(p_2 e^{p_1 t} - p_1 e^{p_2 t}) + CV$$

$$i = \frac{dq}{dt}$$

$$= \frac{p_1 p_2}{2\beta}\mathrm{C}(\mathrm{V} - V_0)(e^{p_1 t} - e^{p_2 t})$$

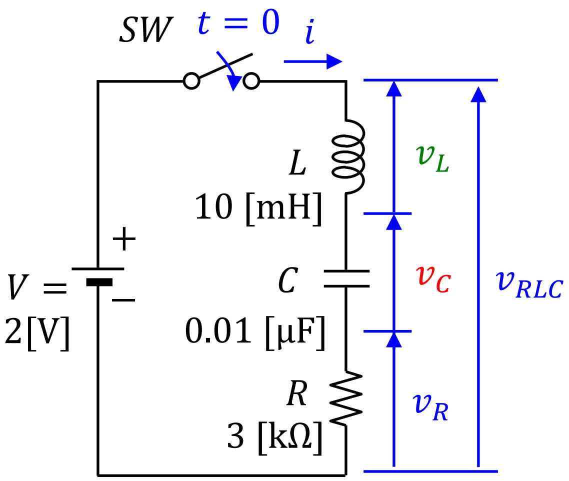
$$= \frac{(\mathrm{V} - V_0)}{2L\beta}(e^{p_1 t} - e^{p_2 t})$$

$$v_L = L\frac{di}{dt} = \frac{(\mathrm{V} - V_0)}{2\beta}(p_1 e^{p_1 t} - p_2 e^{p_2 t})$$

$$v_C = \frac{q}{C} = \frac{(\mathrm{V} - V_0)}{2\beta}(p_2 e^{p_1 t} - p_1 e^{p_2 t}) + V$$

$$v_R = Ri = \frac{\alpha(\mathrm{V} - V_0)}{\beta}(e^{p_1 t} - e^{p_2 t})$$

Exercise 15.4.2 (Homework)



At $t = 0$,

$$v_L = \frac{(V - V_0)}{2\beta}(p_1 - p_2) = \frac{(V - V_0)}{2\beta}2\beta = V - V_0$$

$$v_C = \frac{q}{C} = \frac{(V - V_0)}{2\beta}(p_2 - p_1) + V = \frac{(V - V_0)}{2\beta}(-2\beta) + V$$
$$= V_0$$

$$v_R = \frac{\alpha(V - V_0)}{\beta}(e^{p_1 t} - e^{p_2 t}) = \frac{\alpha(V - V_0)}{\beta}(1 - 1) = 0$$

The major differences are the values of v_L and v_C at $t = 0$.

Exercise 15.4.2 (Homework)

Initial conditions: at $t = 0$, $i = 0$, $v_C = 1$ [V]