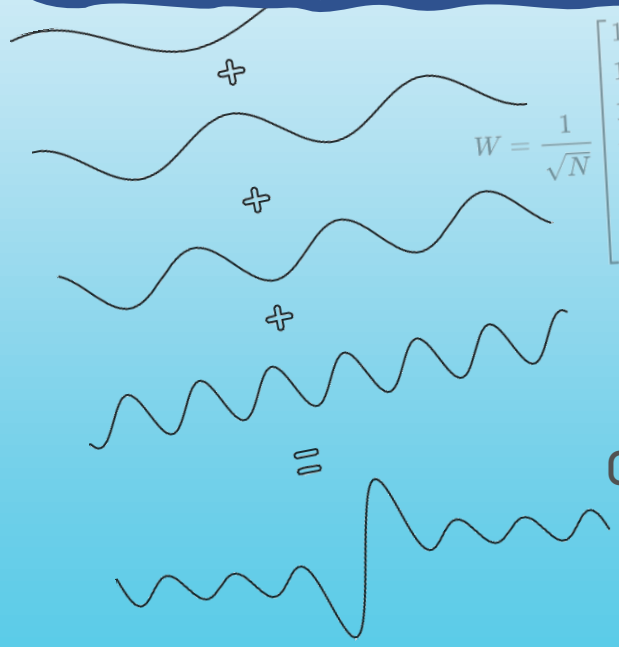
From Playstation to Hospitals

Hidden mathematics in our daily life

Studium Generale – 8th November 2019



$$W = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

$$Rf(\gamma) = \int_\gamma f(x,y)\,ds$$

Henrik Bachmann

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About me

- Born in Hamburg (Germany)

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- Studied mathematics at Hamburg University
- Since last month Associate Professor at Nagoya University in the G30 Program
- Interested in Number theory

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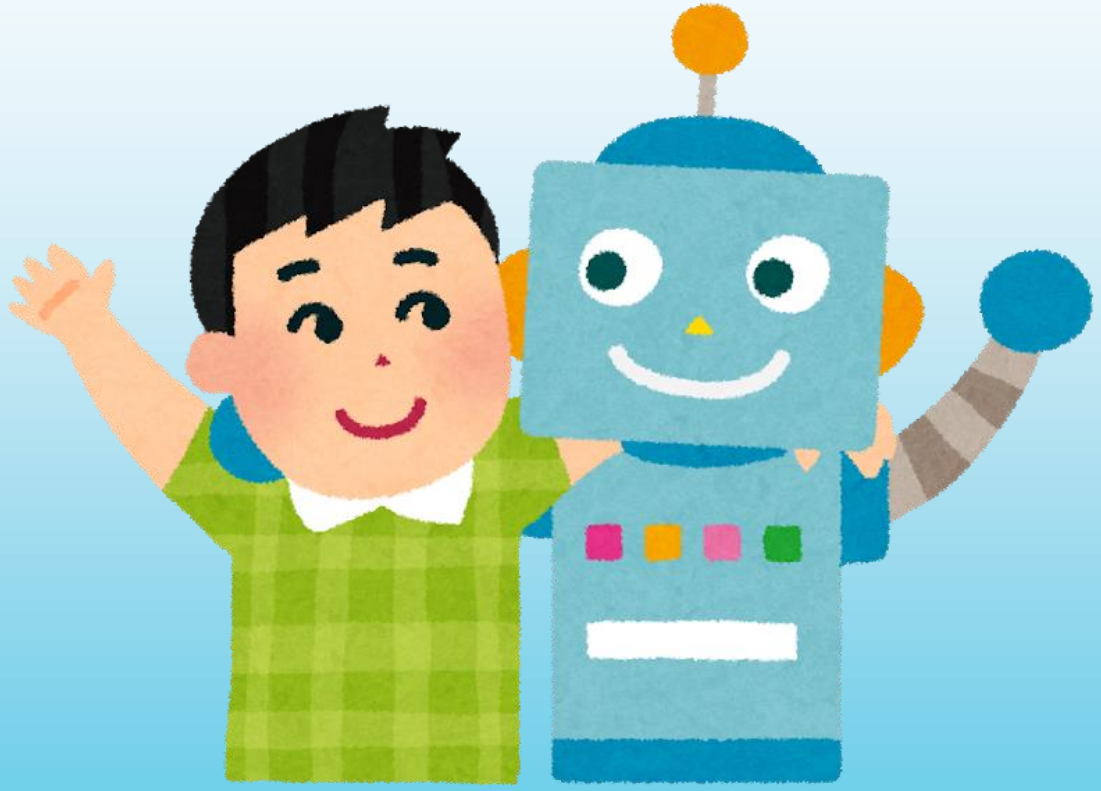
Mathematics….

MATH



MATH IS EVERYWHERE

Do you like mathematics?

Yes

I always liked mathematics in school / university. It is a really cool subject!

No

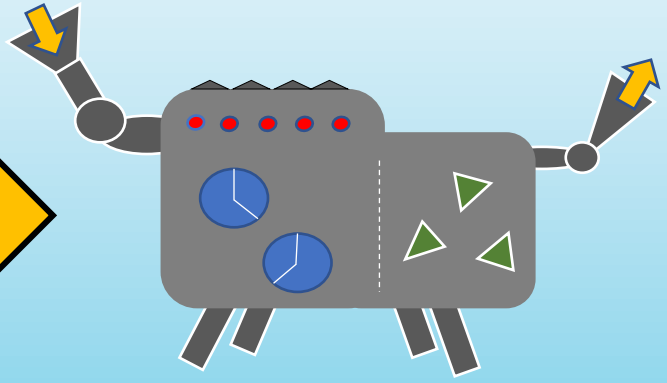
I accept that mathematics might be important, but I was never a big fan of it.

Ein Beispiel / An Example

Heute möchte ich über Mathematik sprechen

Today want I about math talk

„Heute möchte ich über Mathematik sprechen“



Google Translate

“Today I want to talk about math”

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The magic machine

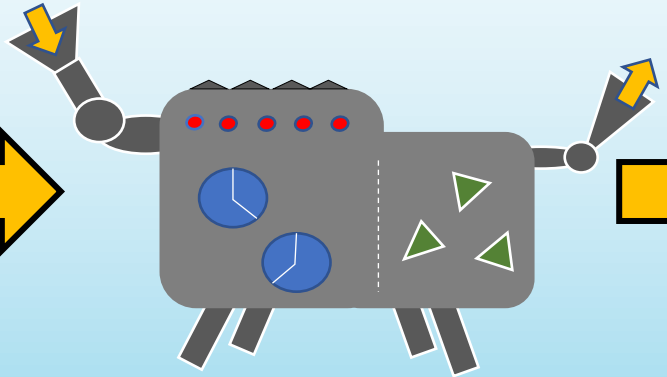
In general we often have something like this….

Input

Data we can not understand

e.g.

- Text in different language
- Something we measure by sensors
- ….



Magic machine

Output

Data we can understand (and manipulate)

Often the „inverse“ machine also exists

Today: Discuss two explicit examples

Karaoke / Playstation SingStar

- Playstation SingStar is a competitive karaoke game.
- You score by singing a song in the correct pitch.

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Also appears on Japanese TV

SingStar: How does it work?

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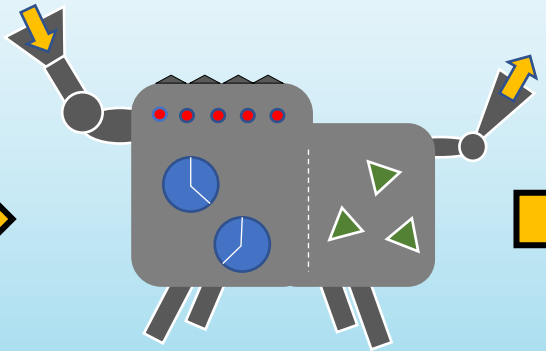
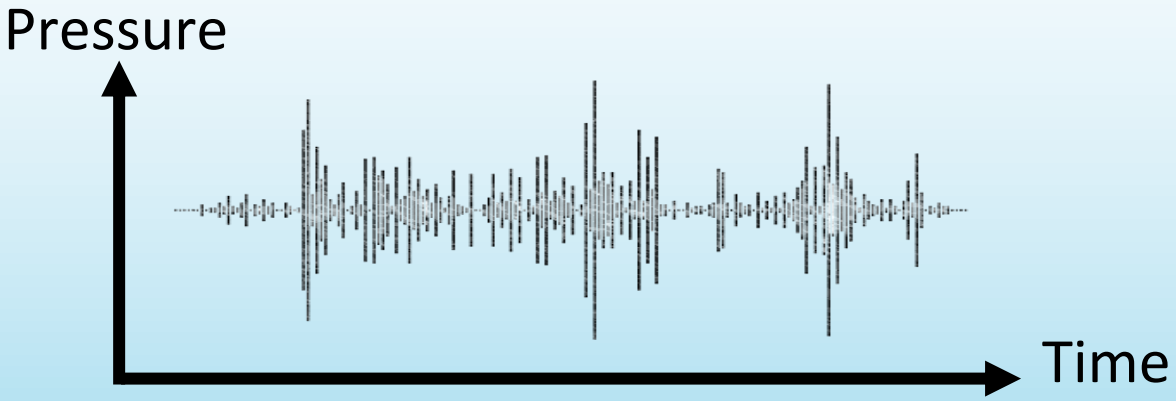


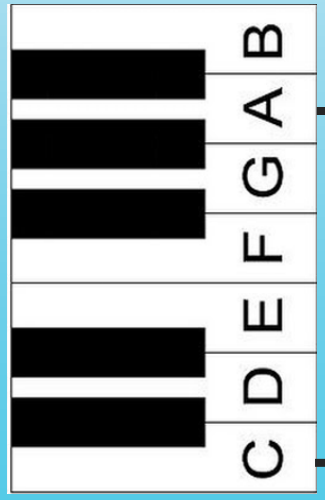
SingStar machine

https://en.wikipedia.org/wiki/SingStar

Soundwaves

Sound is a vibration that typically propagates as an audible wave of pressure, through a transmission medium such as a gas, liquid or solid.

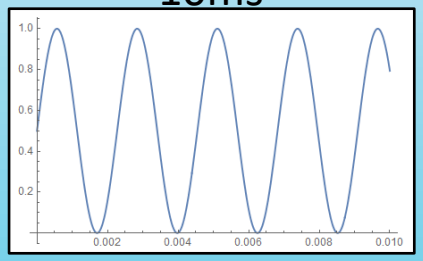


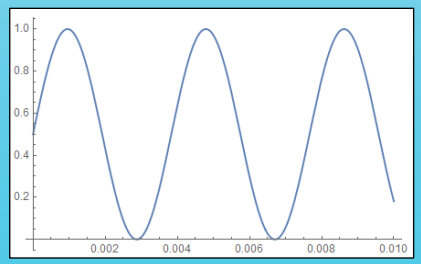


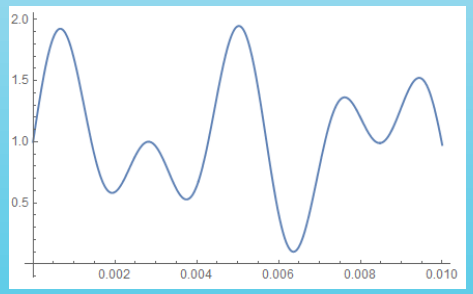
A : 440Hz

C : 261Hz

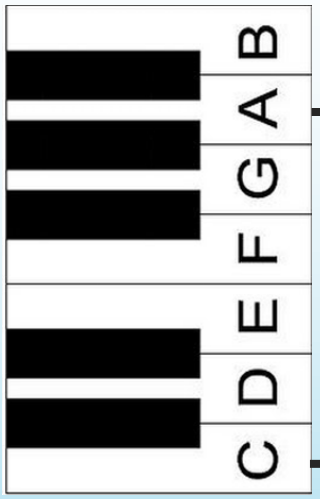
10ms







SingStar: Piano version



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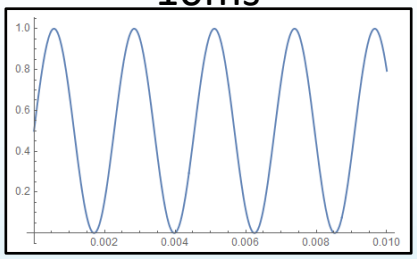
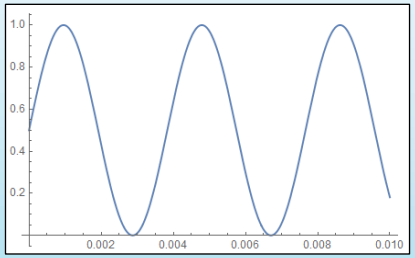
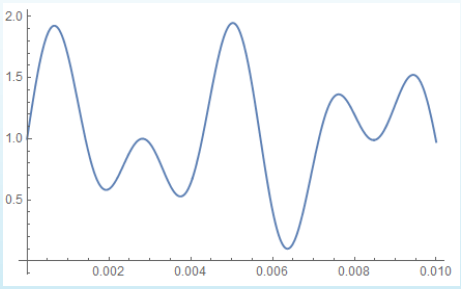
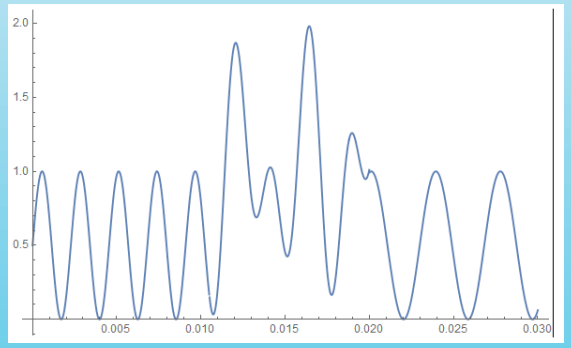
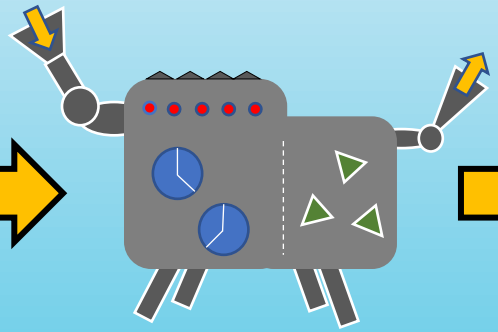




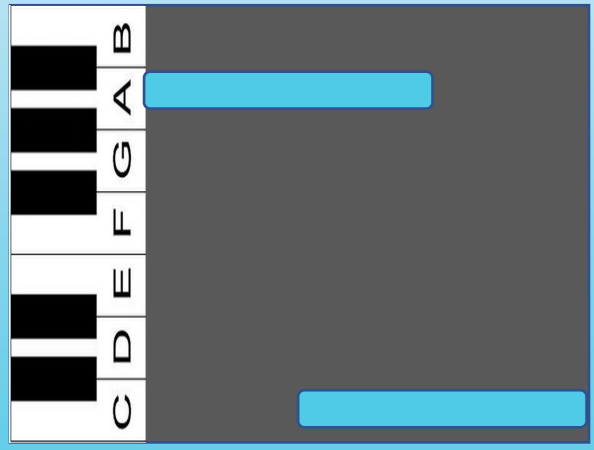




30ms

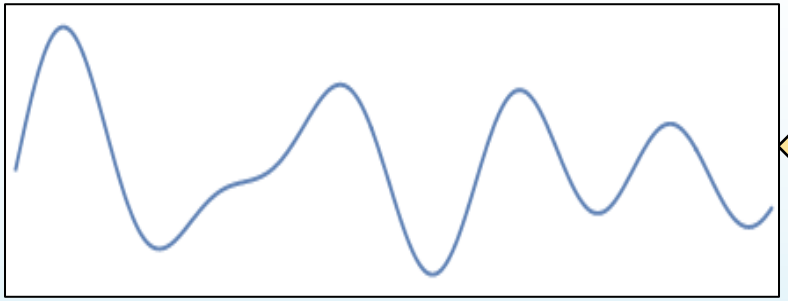


SingStar machine

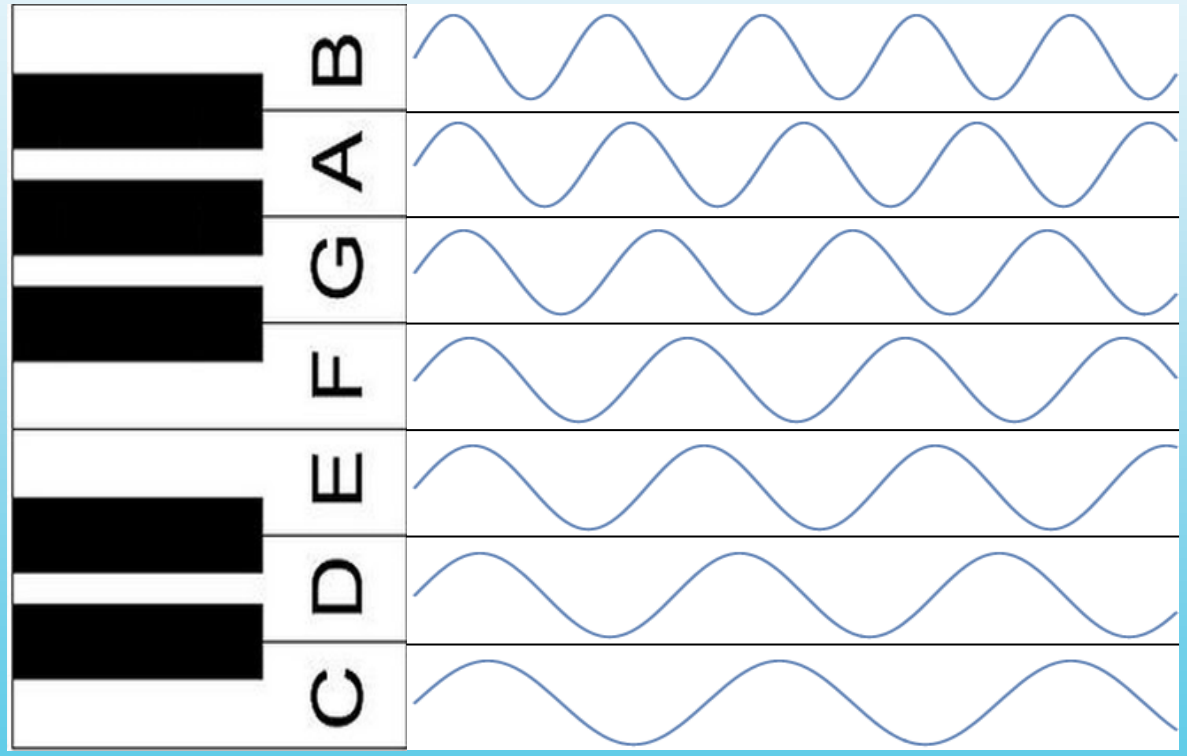


30ms

Can you be a SingStar machine?



Which three keys are played here?



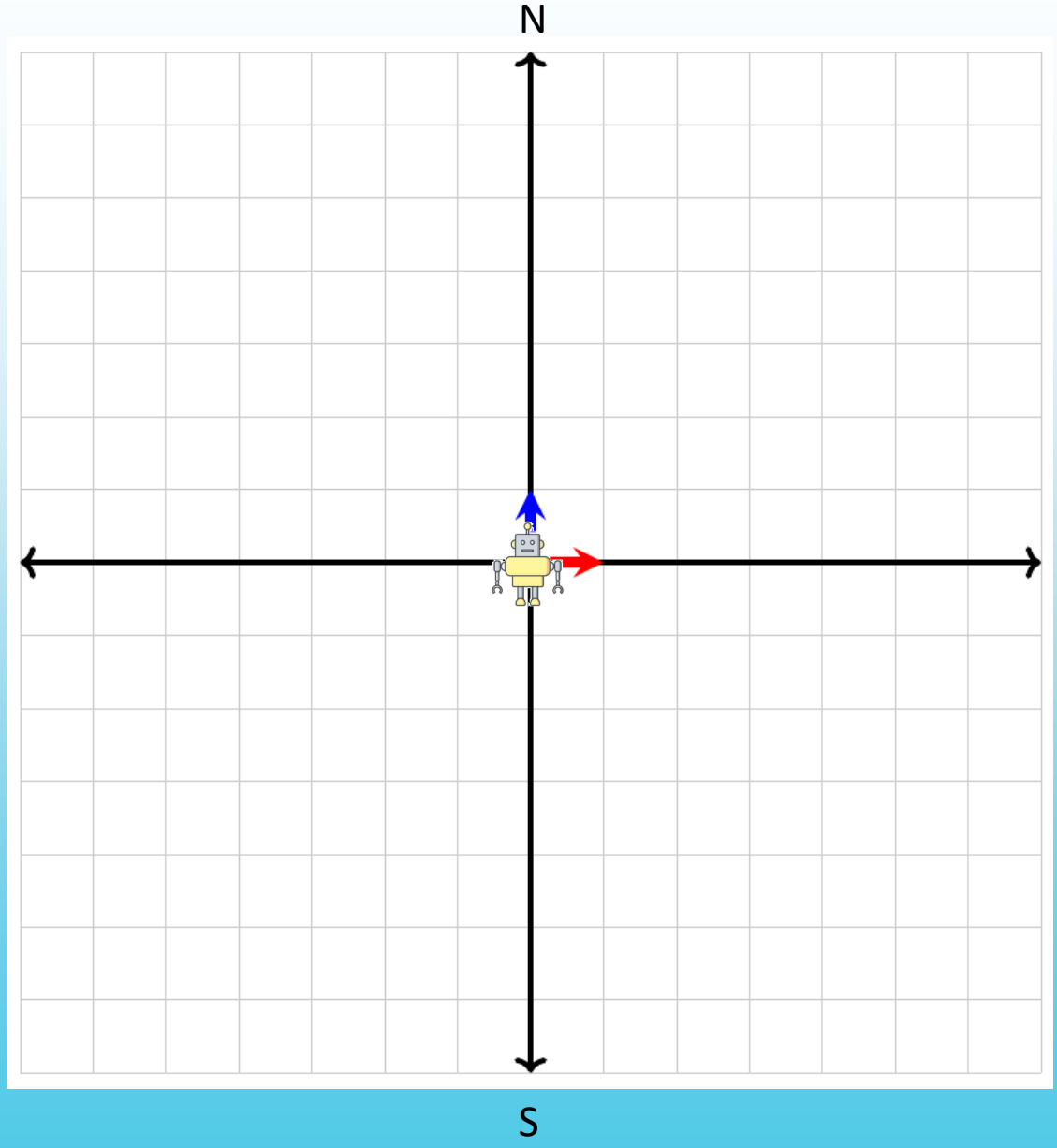
10ms

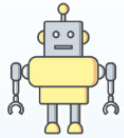
Yes: C + G + B

No: D + F + B

Can you be a SingStar machine?

Some linear algebra…

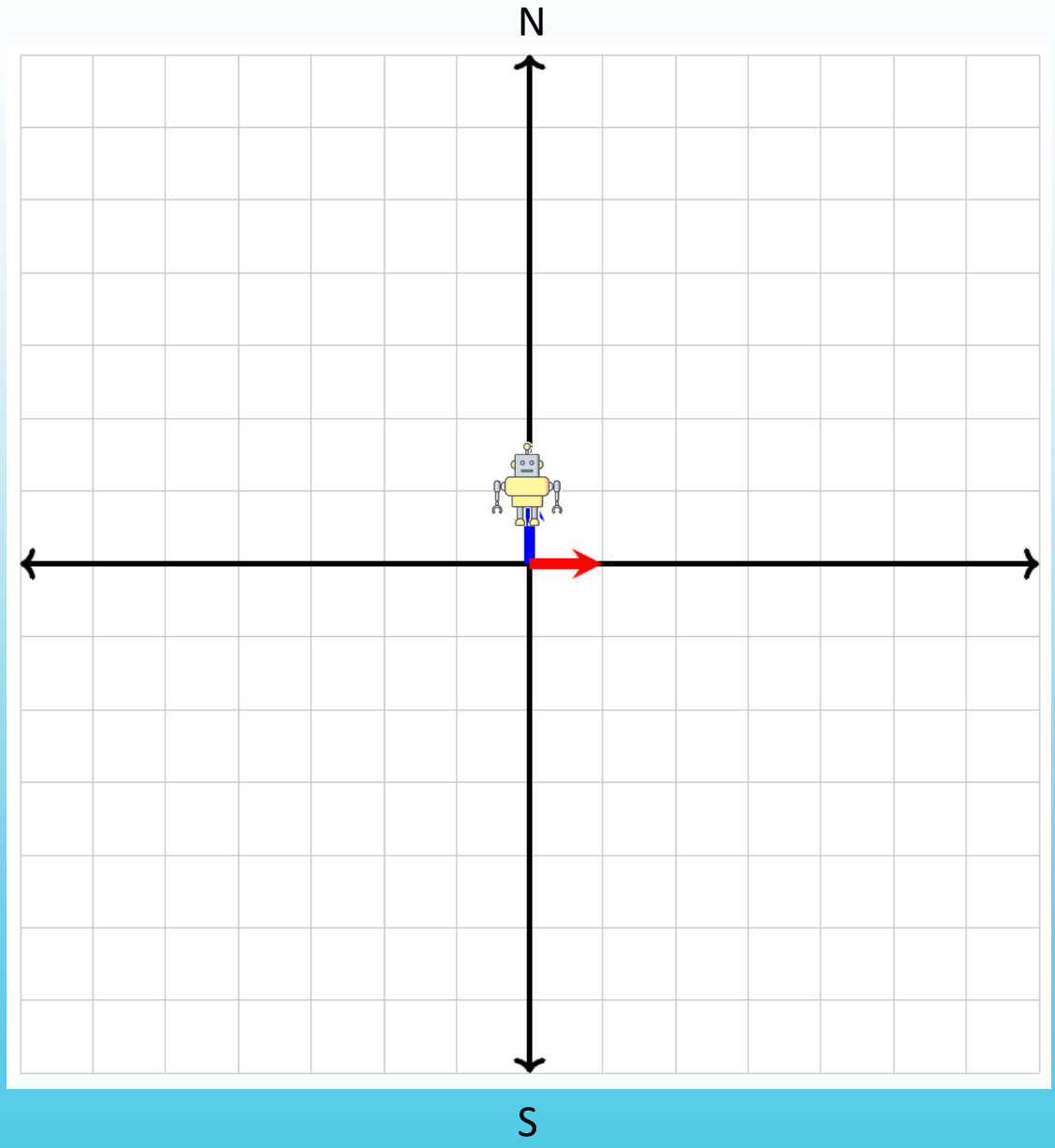


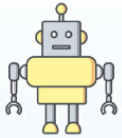


Movement 1 (M1): Go 1 step east

Movement 2 (M2): Go 1 step north

Some linear algebra…





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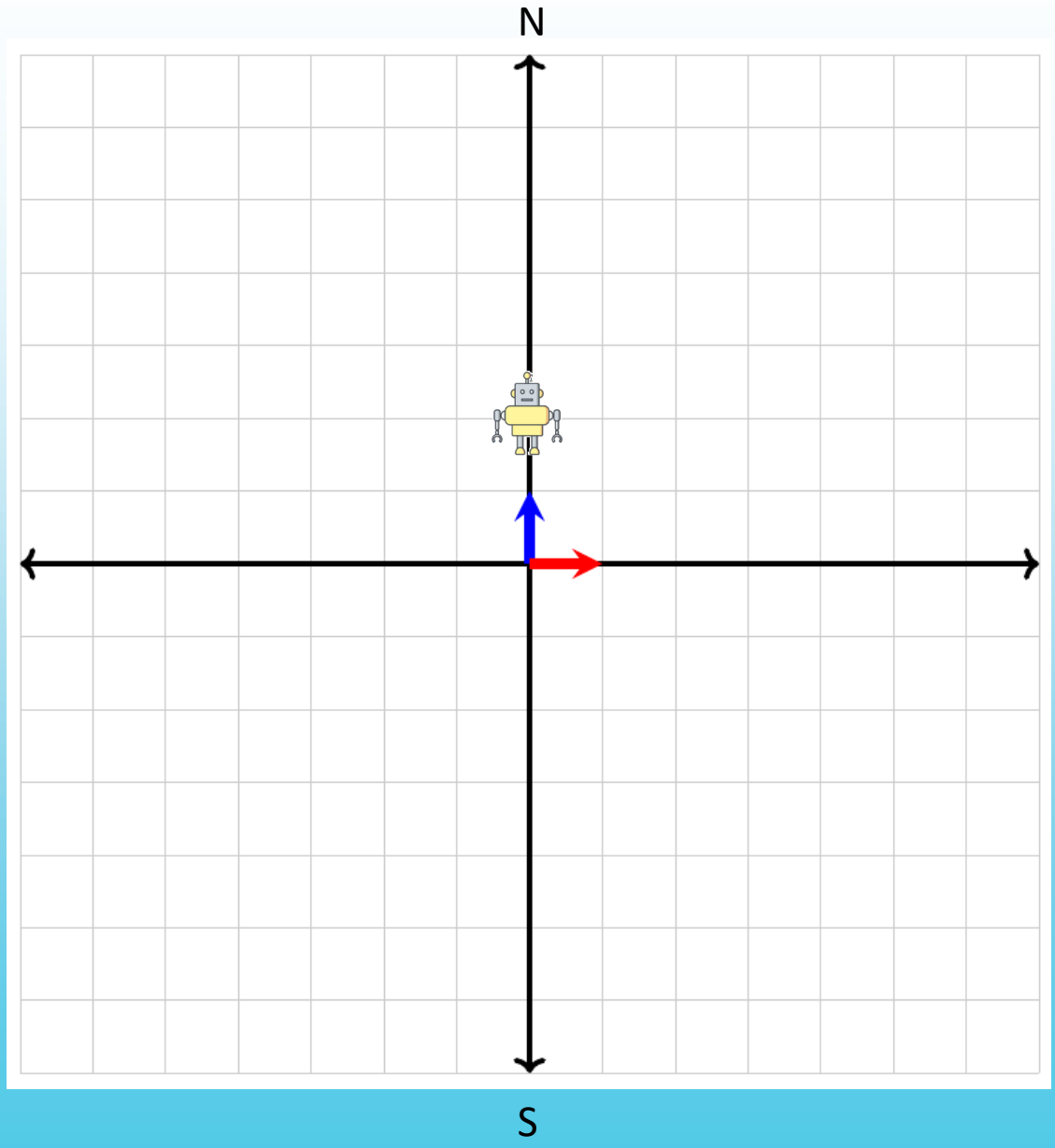

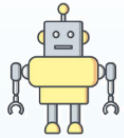

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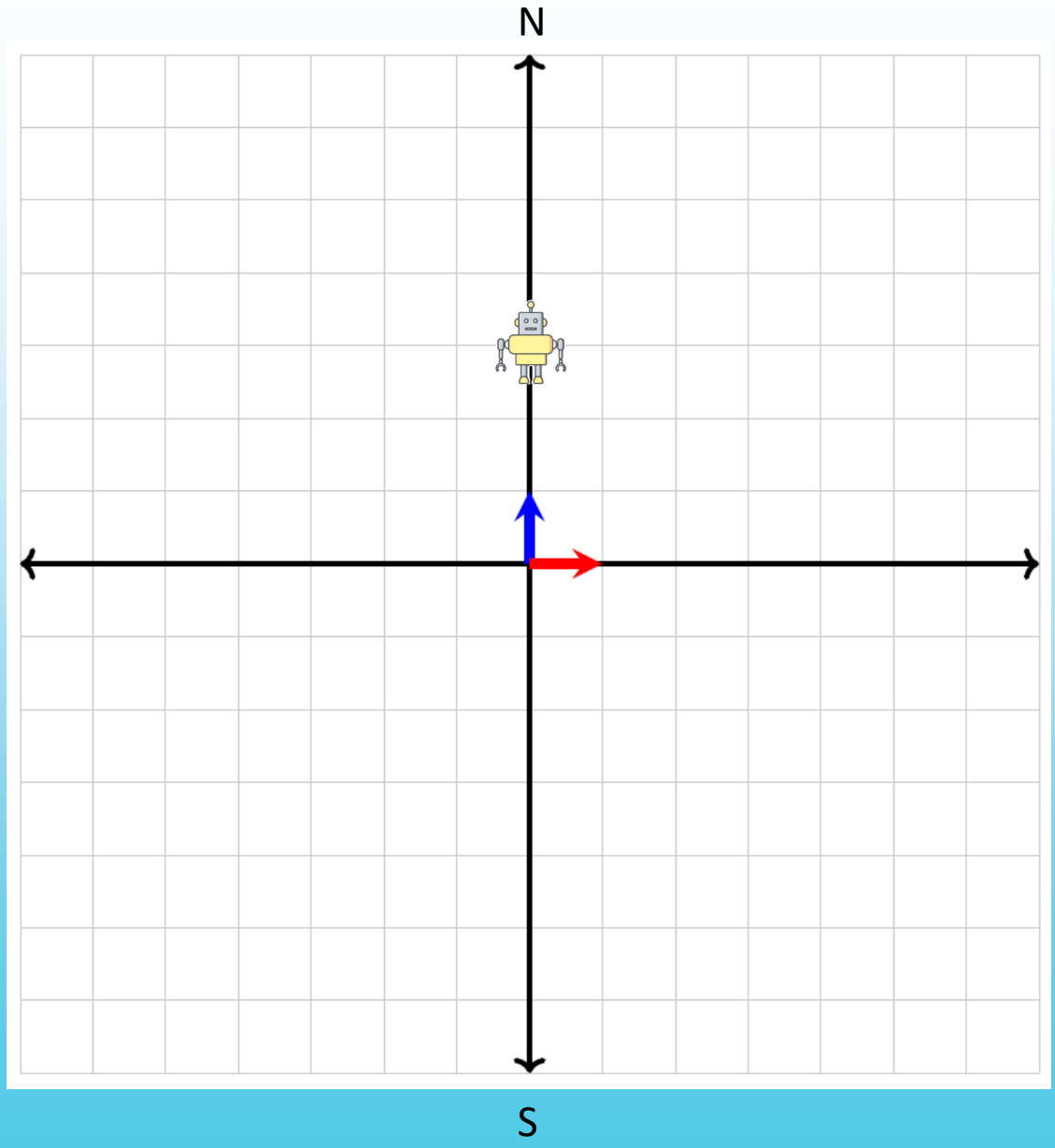

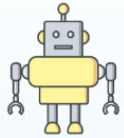

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Movement 2 (M2): Go 1 step north

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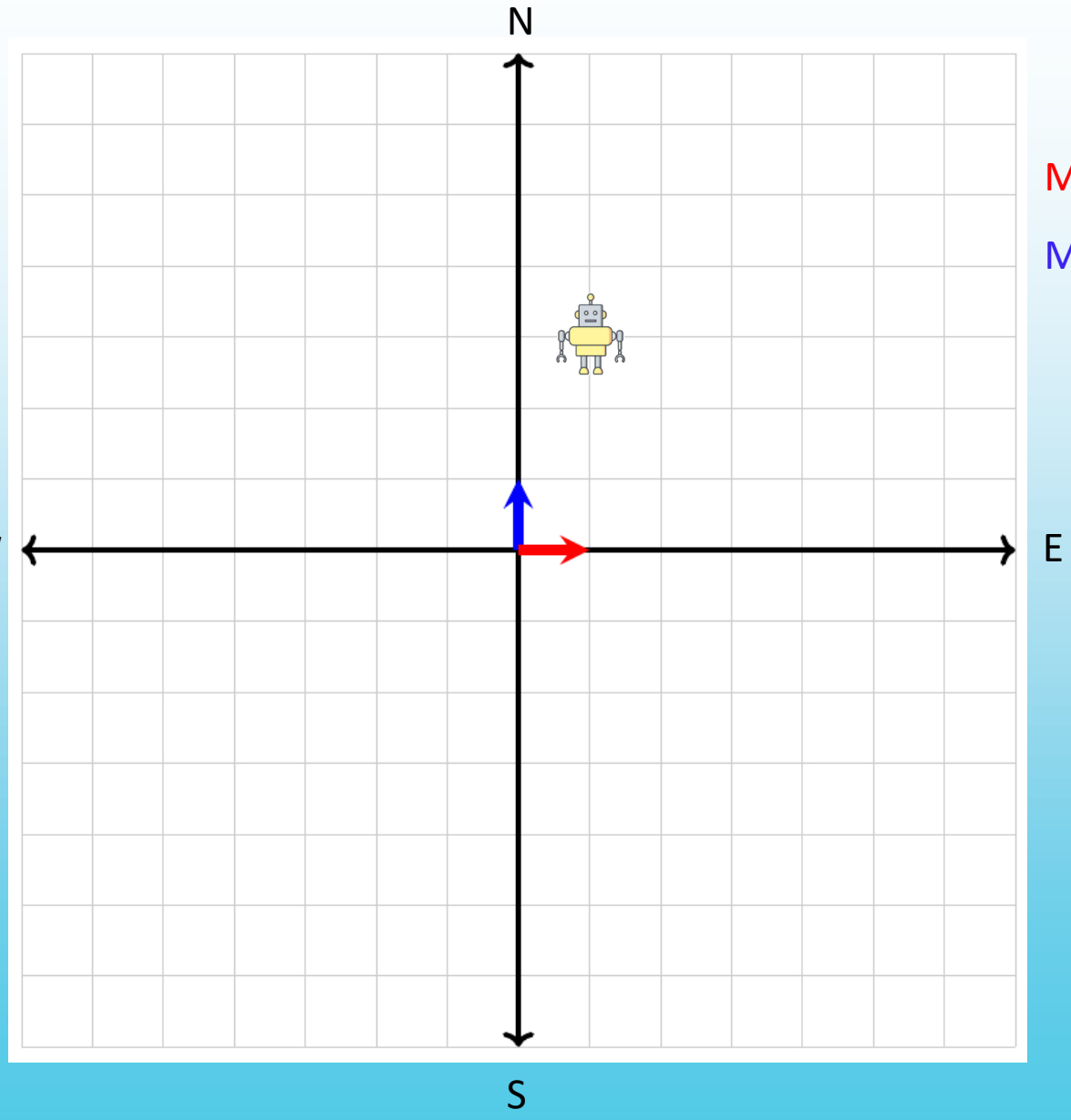

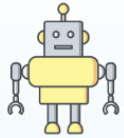

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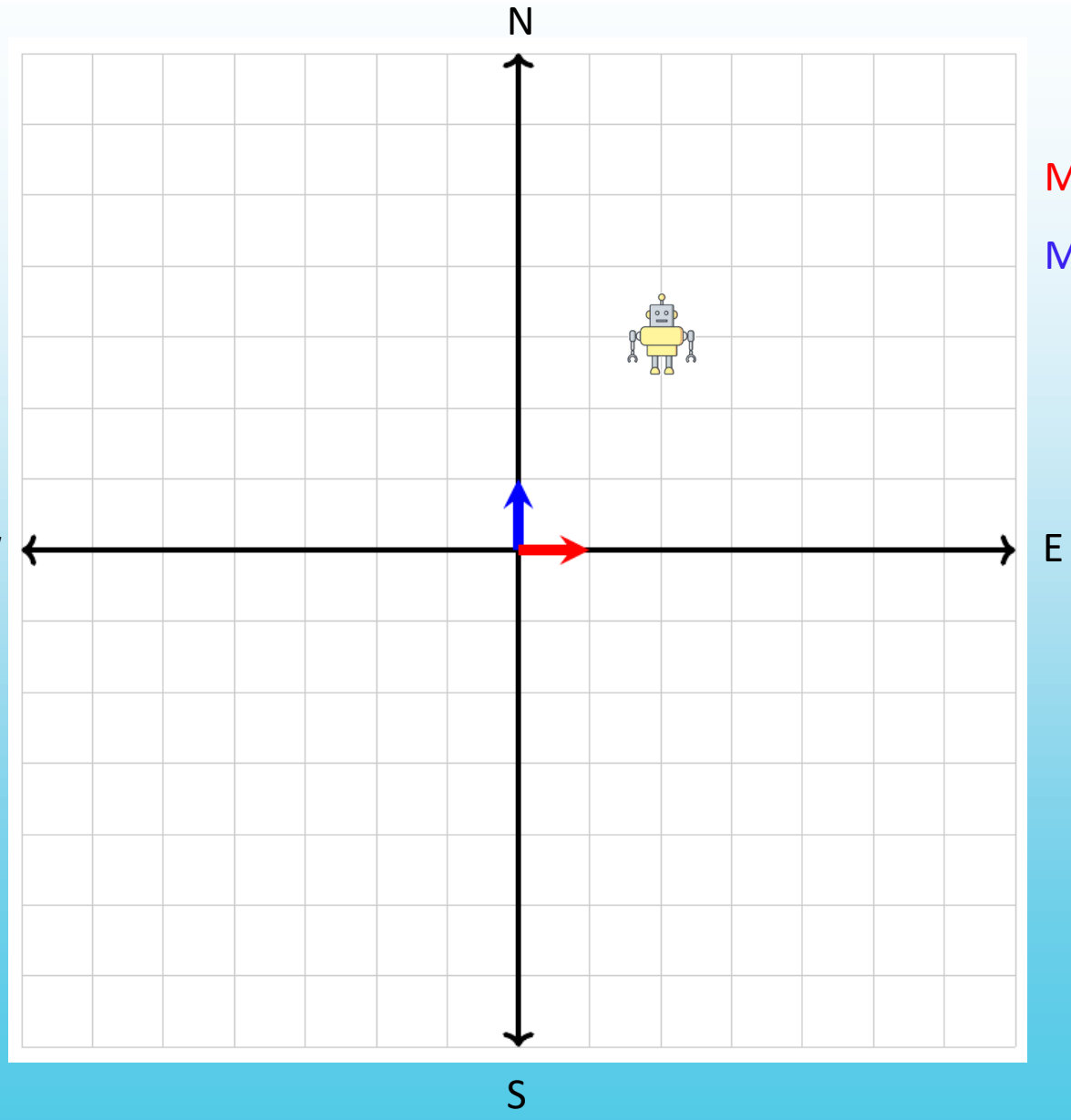

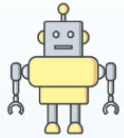

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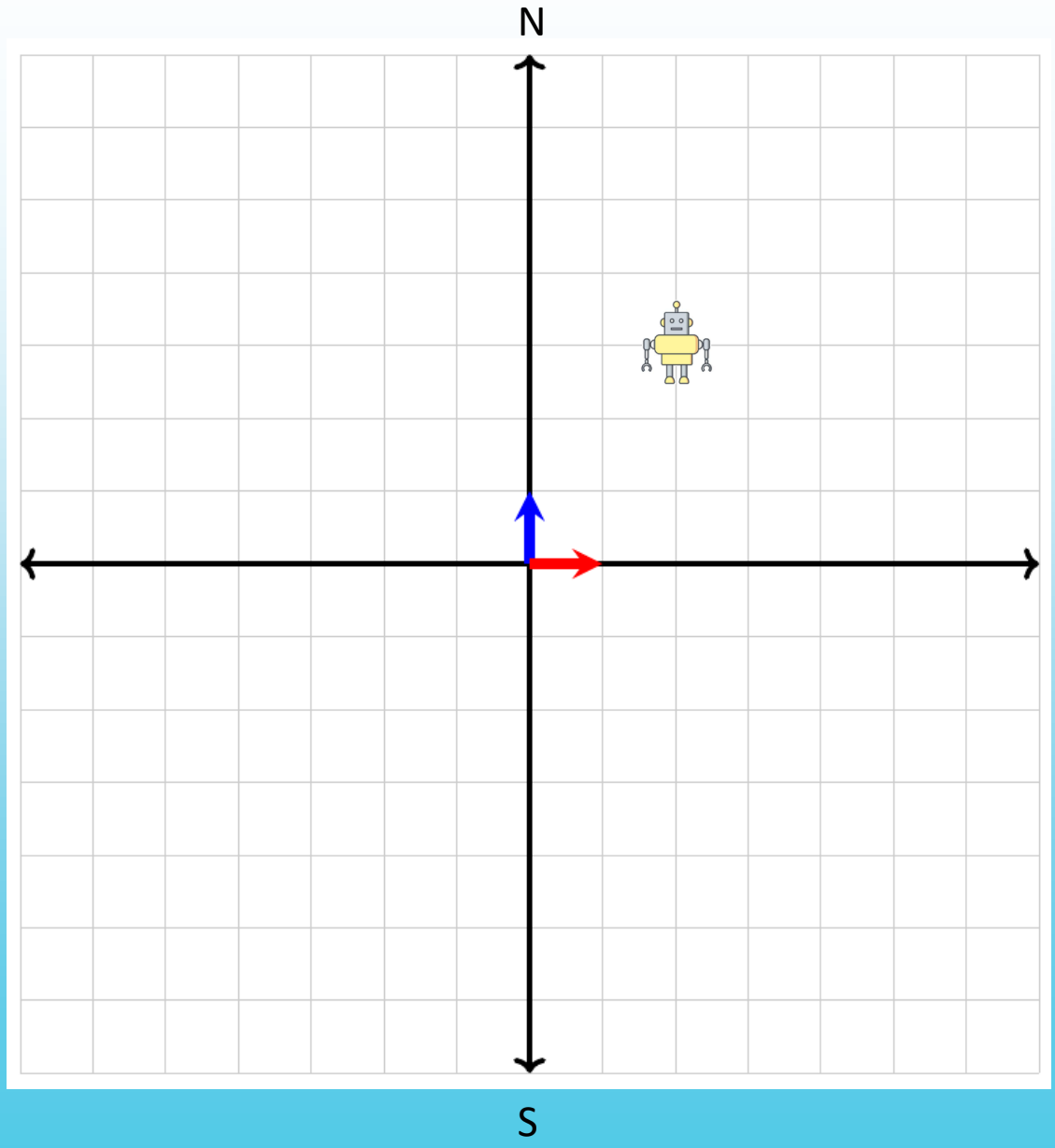

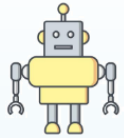

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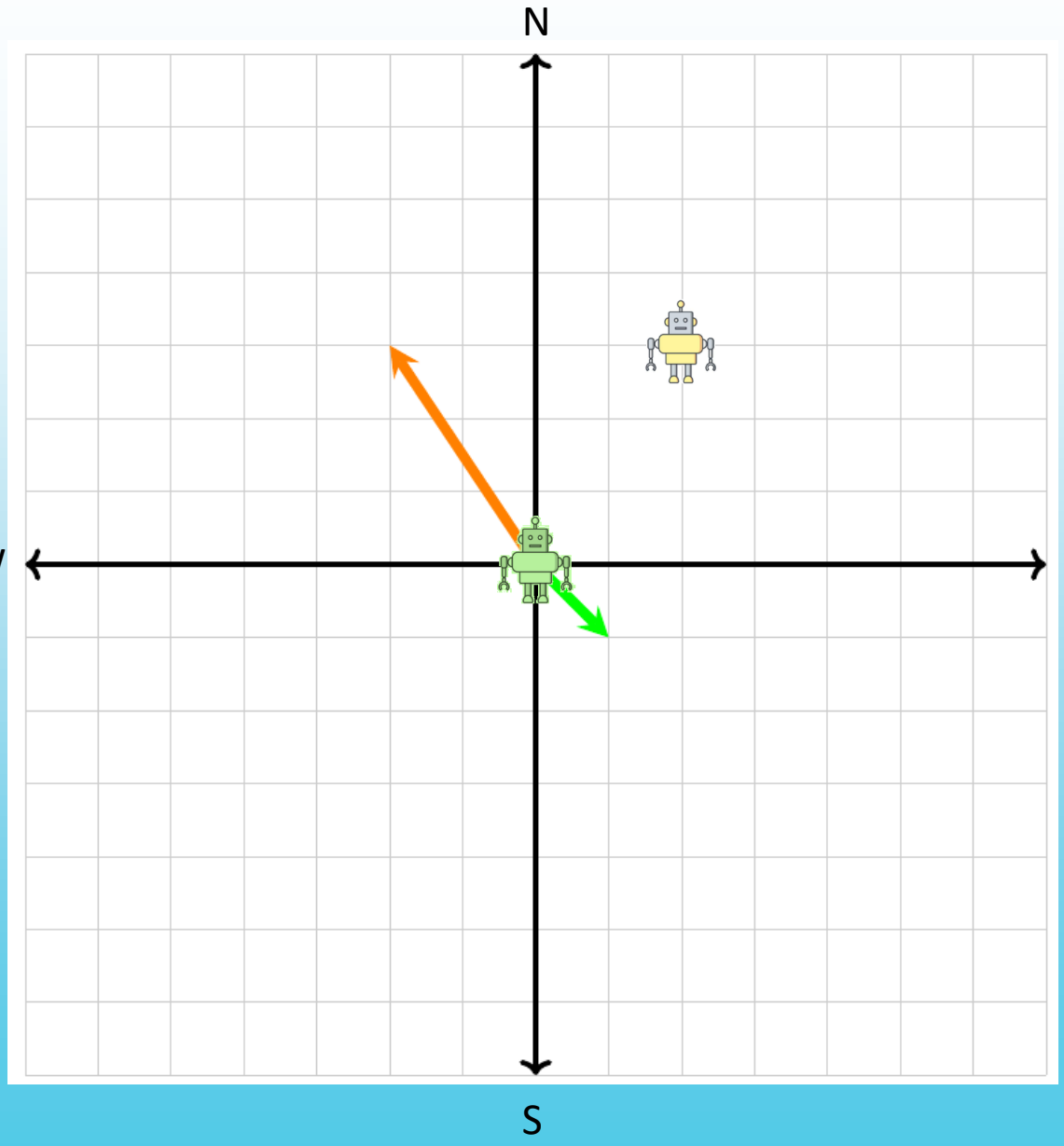
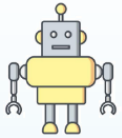
Movement 1 (M1): Go 1 step east

Movement 2 (M2): Go 1 step north

2 (M1) + 3 (M2)

Some linear algebra…





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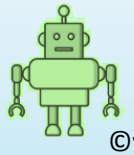

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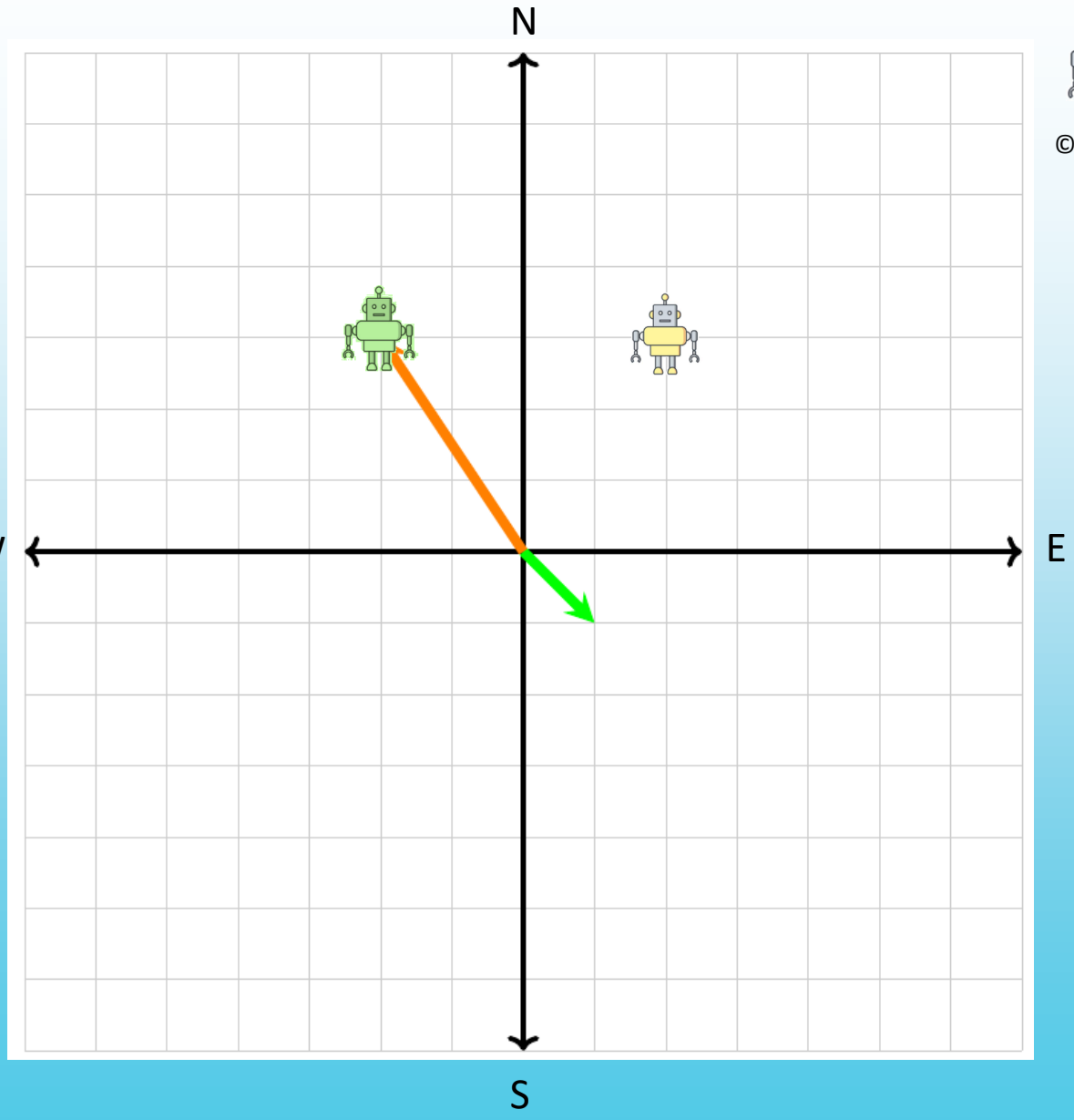
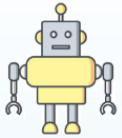
Movement 3 (M3): Go 1 step east & 1 step south

Movement 4 (M4): Go 2 steps west & 3 steps north

Some linear algebra…





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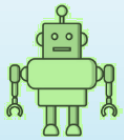
2 (M1) + 3 (M2)



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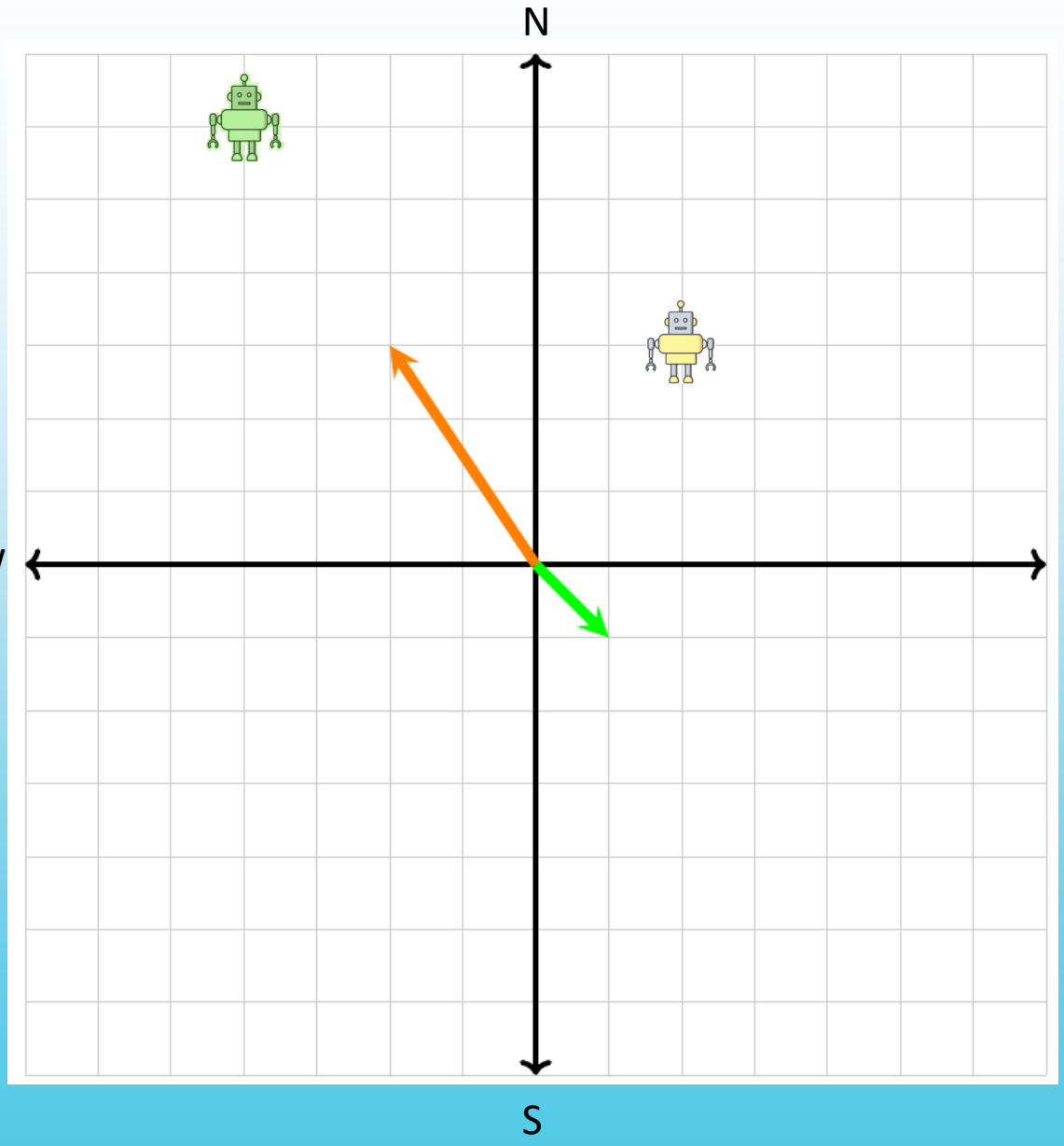
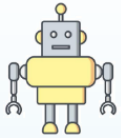
Some linear algebra…





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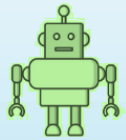
Movement 1 (M1): Go 1 step east

Movement 2 (M2): Go 1 step north

2 (M1) + 3 (M2)



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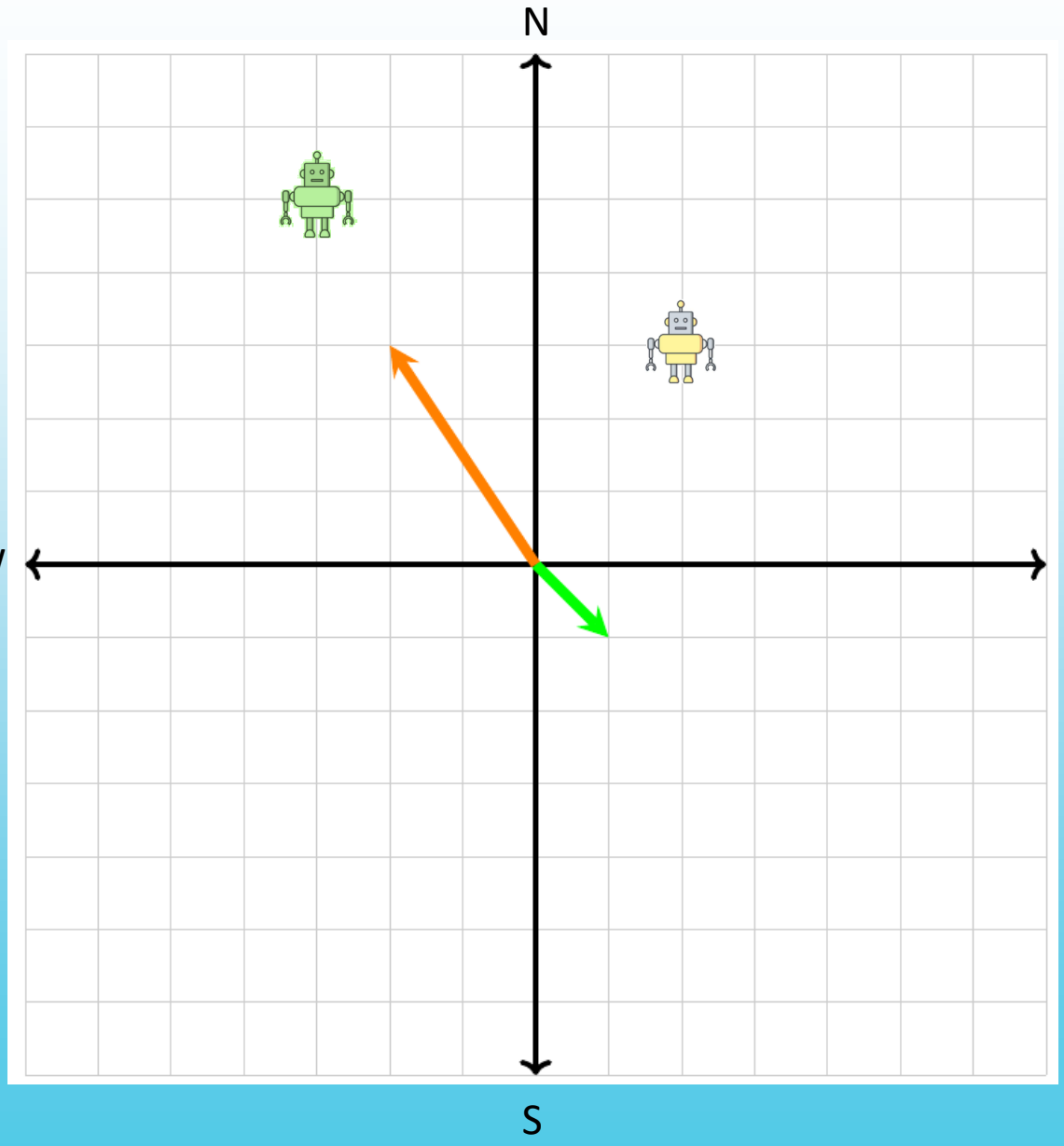
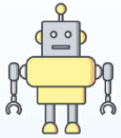
Movement 3 (M3): Go 1 step east & 1 step south

Movement 4 (M4): Go 2 steps west & 3 steps north

Some linear algebra…





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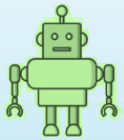

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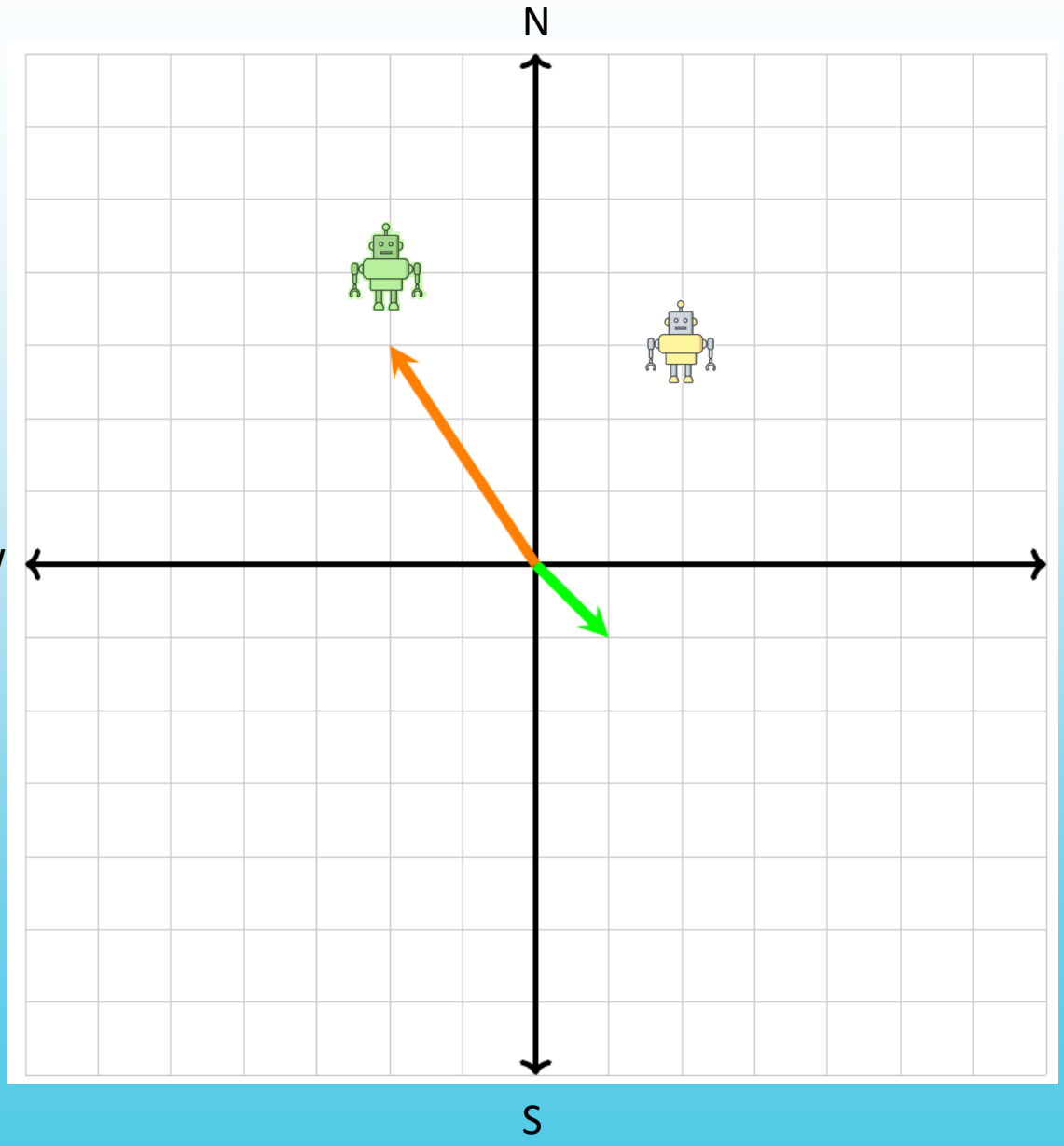

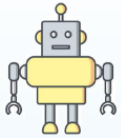

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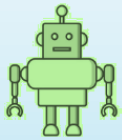
Movement 1 (M1): Go 1 step east

Movement 2 (M2): Go 1 step north

2 (M1) + 3 (M2)

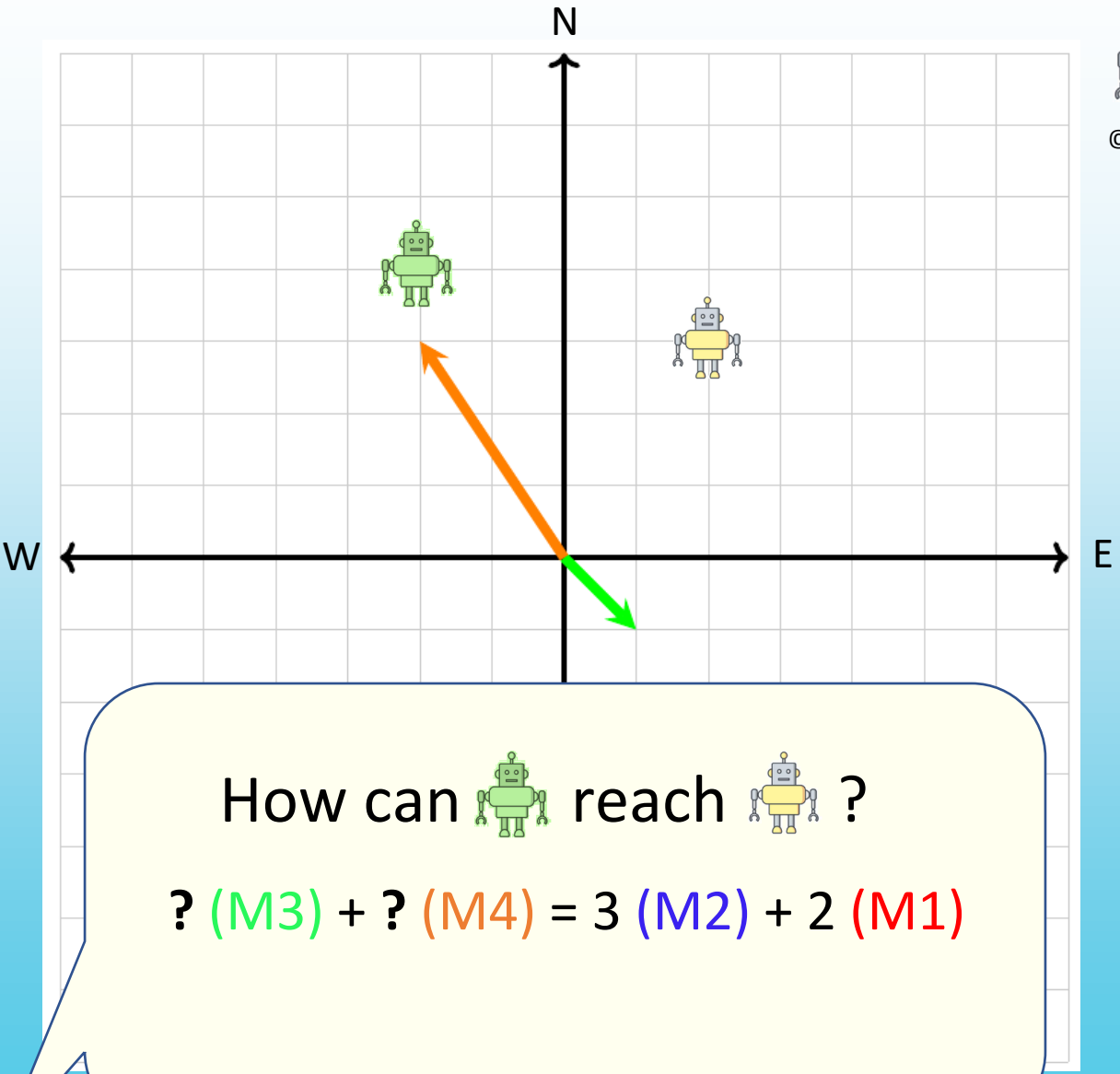


©vecteezy

Movement 3 (M3): Go 1 step east & 1 step south

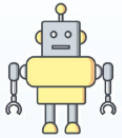
Movement 4 (M4): Go 2 steps west & 3 steps north

Some linear algebra…



How can reach ?

? (M3) + **?** (M4) = 3 (M2) + 2 (M1)

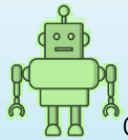


©vecteezy

Movement 1 (M1): Go 1 step east

Movement 2 (M2): Go 1 step north

2 (M1) + 3 (M2)



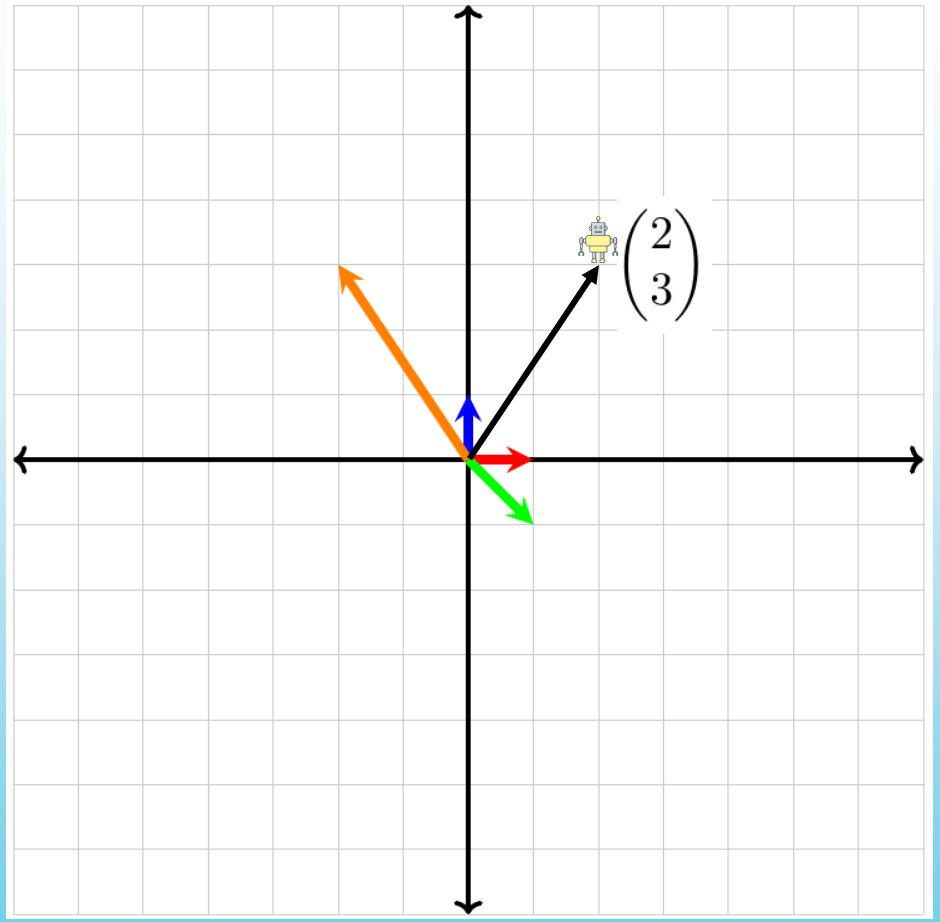
©vecteezy

Movement 3 (M3): Go 1 step east & 1 step south

Movement 4 (M4): Go 2 steps west & 3 steps north

2 (M4) + 2 (M3)

Some linear algebra…a bit more serious



Vector Notation:

$$M_1 = \begin{pmatrix}1\\0\end{pmatrix}, \; M_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

$$M_3 = \begin{pmatrix}1\\-1\end{pmatrix}, \; M_4 = \begin{pmatrix}-2\\3\end{pmatrix}$$

$$2M_1 + 3M_2 = \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

Find numbers a and b such that

$$aM_3 + bM_4 = a\begin{pmatrix}1\\-1\end{pmatrix} + b\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

Matrix * Vector = Vector

$$\underbrace{\begin{pmatrix}1 & -2\\-1 & 3\end{pmatrix}}_{A}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

$$A^{-1} = \begin{pmatrix}3 & 2\\1 & 1\end{pmatrix}$$

$$\begin{pmatrix}3 & 2\\1 & 1\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}2\cdot 3 + 3\cdot 2\\2+3\end{pmatrix} = \begin{pmatrix}12\\5\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$$

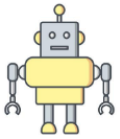
$$12M_3 + 5M_4 = 12\begin{pmatrix}1\\-1\end{pmatrix} + 5\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}12-10\\-12+15\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

Basis change

These two sets of „movements“ are examples of bases for the 2-dimensional space.

- Every point can be reached
- There is a unique way to reach a point

Basis 1 (Standard basis)



©vecteezy

$$M_1 = \begin{pmatrix}1\\0\end{pmatrix}, \; M_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

$$2M_1 + 3M_2 = \begin{pmatrix}2\\3\end{pmatrix}$$

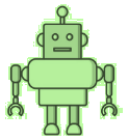
Change of basis

$$A = \begin{pmatrix}1 & -2\\-1 & 3\end{pmatrix}$$

$$A^{-1} = \begin{pmatrix}3 & 2\\1 & 1\end{pmatrix}$$

$$\begin{pmatrix}3 & 2\\1 & 1\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}12\\5\end{pmatrix}$$

Basis 2

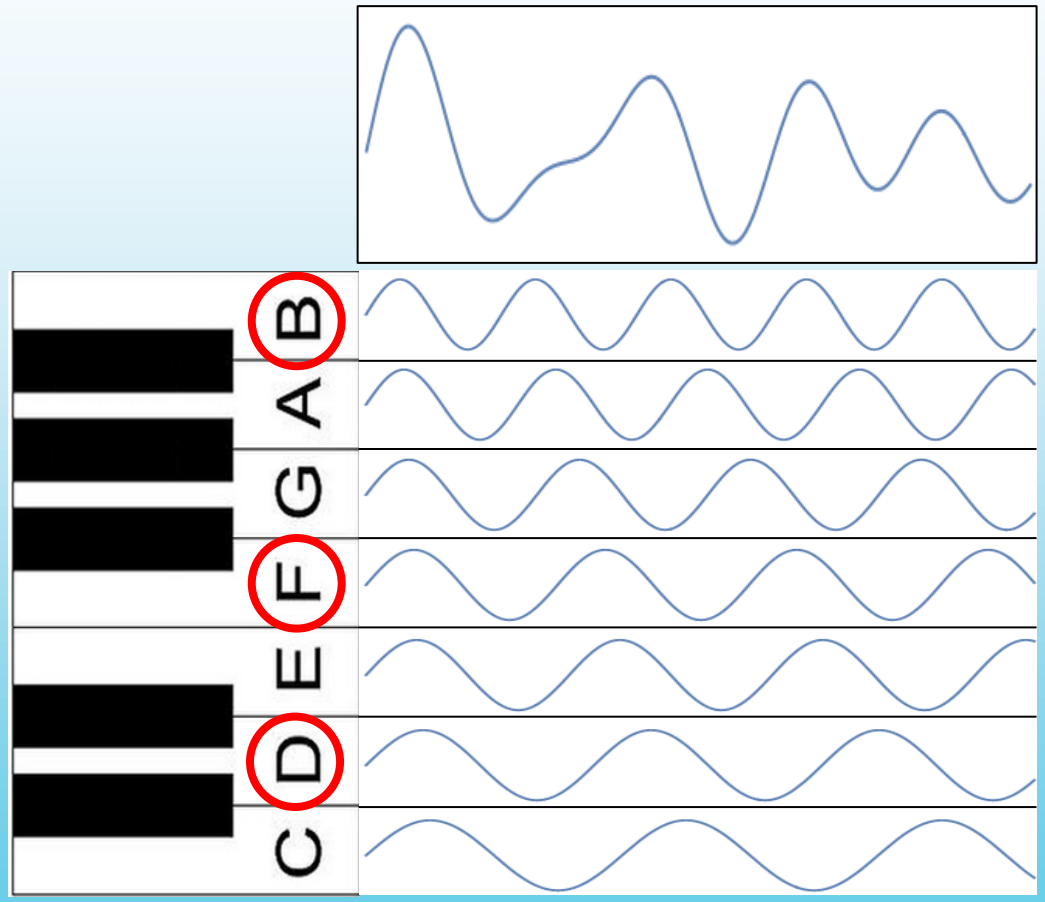


©vecteezy

$$M_3 = \begin{pmatrix}1\\-1\end{pmatrix}, \; M_4 = \begin{pmatrix}-2\\3\end{pmatrix}$$

$$12M_3 + 5M_4 = \begin{pmatrix}2\\3\end{pmatrix}$$

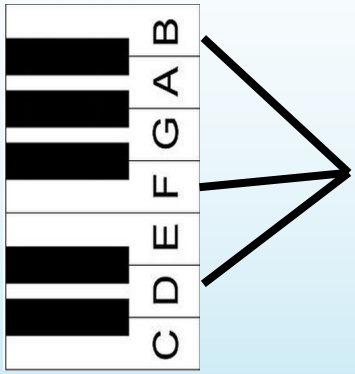
Back to SingStar

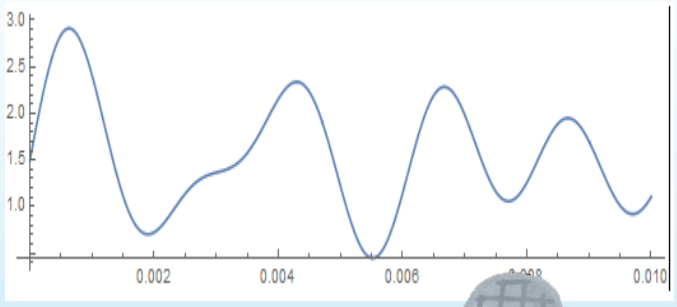


D + F + B

How to we get this result using math?

Back to SingStar: Recording with a microphone

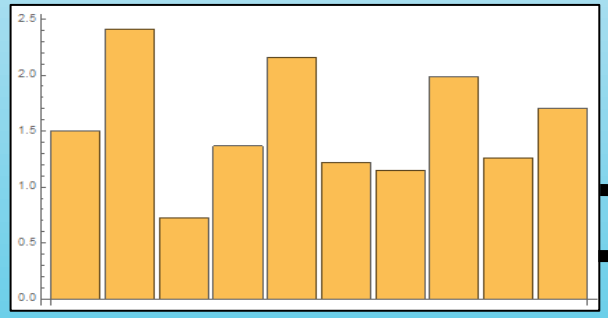




10ms

©いらすとや

Sampling rate 1KHz
(measures 10 values in 10ms)

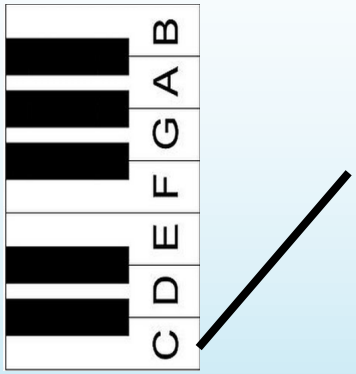


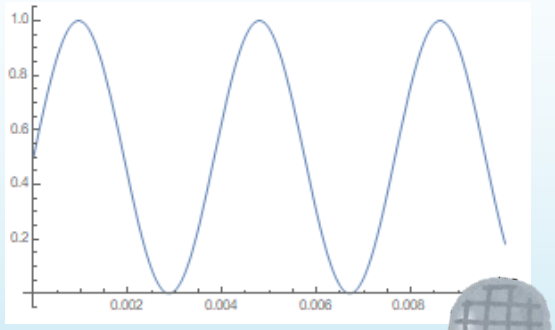
$$\begin{pmatrix} 1.5 \\ 2.41 \\ 0.725 \\ 1.37 \\ 2.16 \\ 1.22 \\ 1.15 \\ 1.98 \\ 1.26 \\ 1.7 \end{pmatrix}$$

Soundwave → Vector

Back to SingStar: Recording with a microphone

But we can do that for each Key seperately first!

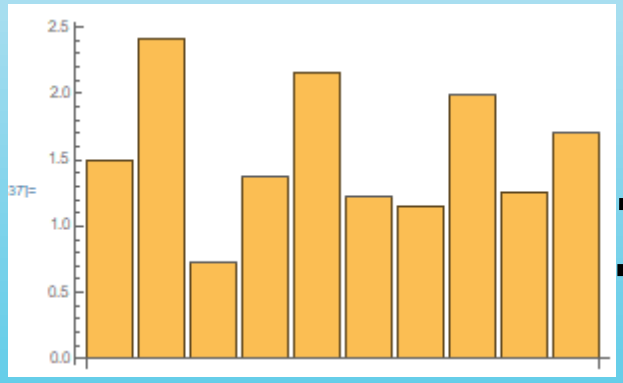




10ms

©いらすとや

Sampling rate 1KHz
(measures 10 values in 10ms)

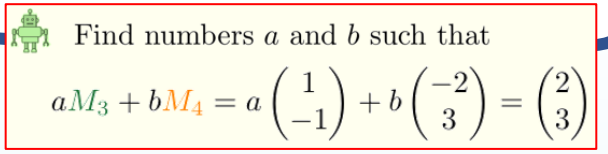


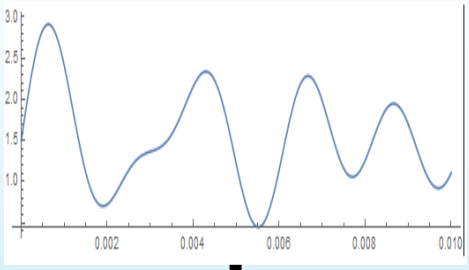
$$\begin{pmatrix} 0.5 \\ 0.999 \\ 0.431 \\ 0.0107 \\ 0.636 \\ 0.97 \\ 0.299 \\ 0.0574 \\ 0.763 \\ 0.906 \end{pmatrix}$$

Soundwave → Vector

SingStar: Just Linear Algebra

We obtain the same question as before

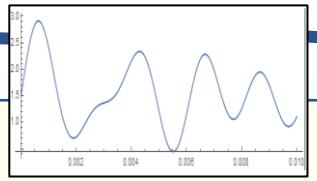




$$\begin{pmatrix}1.5\\2.41\\0.725\\1.37\\2.16\\1.22\\1.15\\1.98\\1.26\\1.7\end{pmatrix} = C\begin{pmatrix}0.5\\0.999\\0.431\\0.0107\\0.636\\0.97\\0.299\\0.0574\\0.763\\0.906\end{pmatrix} + D\begin{pmatrix}0.5\\0.982\\0.243\\0.155\\0.941\\0.609\\0.000632\\0.657\\0.915\\0.121\end{pmatrix} + \ldots$$

Find numbers C,D,… such that this equations holds

SingStar: Just Linear Algebra

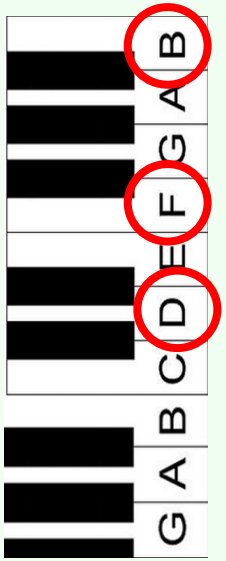


Solution

$$\begin{pmatrix} 1.5 \\ 2.41 \\ 0.725 \\ 1.37 \\ 2.16 \\ 1.22 \\ 1.15 \\ 1.98 \\ 1.26 \\ 1.7 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.52 & 0.68 & 0.82 & 0.91 & 0.94 & 0.98 & 1. & 1. & 0.99 & 0.97 \\ 0.46 & 0.16 & 0.01 & 0.026 & 0.081 & 0.24 & 0.43 & 0.53 & 0.68 & 0.82 \\ 0.57 & 0.95 & 0.94 & 0.65 & 0.46 & 0.16 & 0.011 & 0.0014 & 0.078 & 0.25 \\ 0.41 & 0.00099 & 0.3 & 0.8 & 0.96 & 0.94 & 0.64 & 0.45 & 0.16 & 0.0089 \\ 0.61 & 0.98 & 0.36 & 0.00025 & 0.1 & 0.61 & 0.97 & 1. & 0.79 & 0.42 \\ 0.37 & 0.11 & 0.91 & 0.78 & 0.42 & 0.00063 & 0.3 & 0.58 & 0.95 & 0.94 \\ 0.65 & 0.74 & 0.0017 & 0.68 & 0.97 & 0.66 & 0.057 & 0.0077 & 0.38 & 0.88 \\ 0.33 & 0.44 & 0.86 & 0.017 & 0.13 & 0.92 & 0.76 & 0.4 & 0.00099 & 0.32 \\ 0.69 & 0.38 & 0.44 & 0.89 & 0.38 & 0.12 & 0.91 & 0.99 & 0.44 & 0.00025 \end{pmatrix} \begin{pmatrix} 1. \\ -2.45 \times 10^{-14} \\ 1.07 \times 10^{-13} \\ 1. \\ 7.59 \times 10^{-13} \\ 1. \\ 1.27 \times 10^{-12} \\ -1.07 \times 10^{-12} \\ 2.76 \times 10^{-13} \\ -4.59 \times 10^{-14} \end{pmatrix}$$

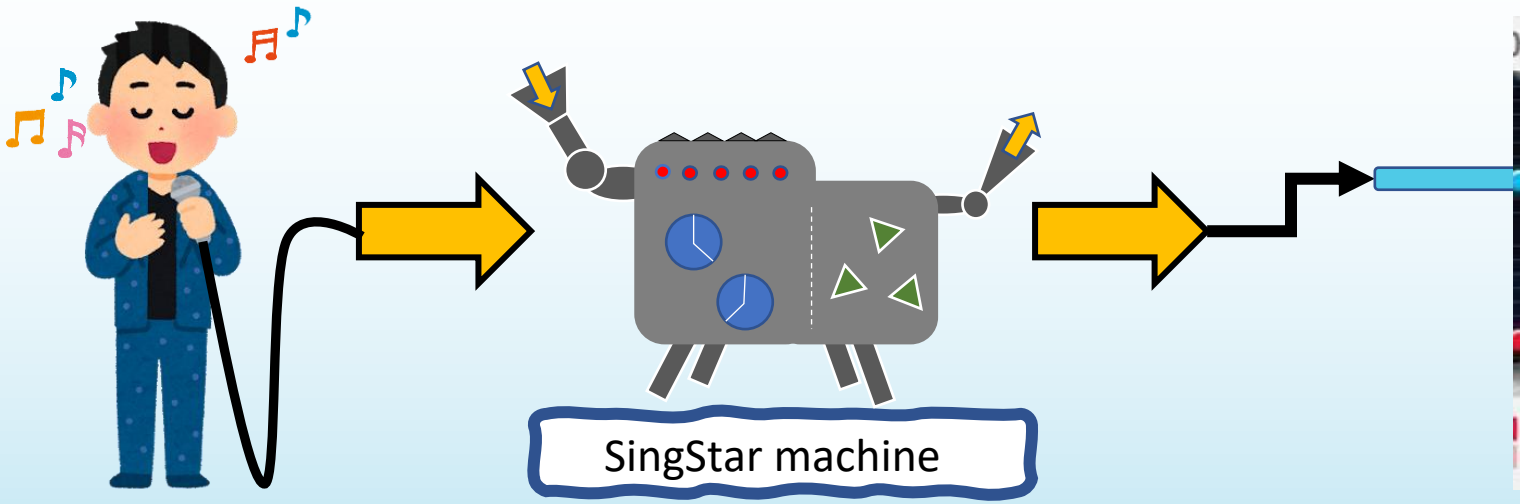
Interpretation of the solution:

$$\begin{pmatrix} 1. \\ -2.45 \times 10^{-14} \\ 1.07 \times 10^{-13} \\ 1. \\ 7.59 \times 10^{-13} \\ 1. \\ 1.27 \times 10^{-12} \\ -1.07 \times 10^{-12} \\ 2.76 \times 10^{-13} \\ -4.59 \times 10^{-14} \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



The keys D,F and B were played with the same volume (all values are the same)

SingStar: How does it work?



Soundwave

Vector
(of pressure data)

$$\begin{pmatrix} 1.5 \\ 2.41 \\ 0.725 \\ 1.37 \\ 2.16 \\ 1.22 \\ 1.15 \\ 1.98 \\ 1.26 \\ 1.7 \end{pmatrix}$$

Matrix multiplication
(basis change)

Discrete Fourier transform

Vector
(of amplitudes, i.e.
Intensity in each frequency)

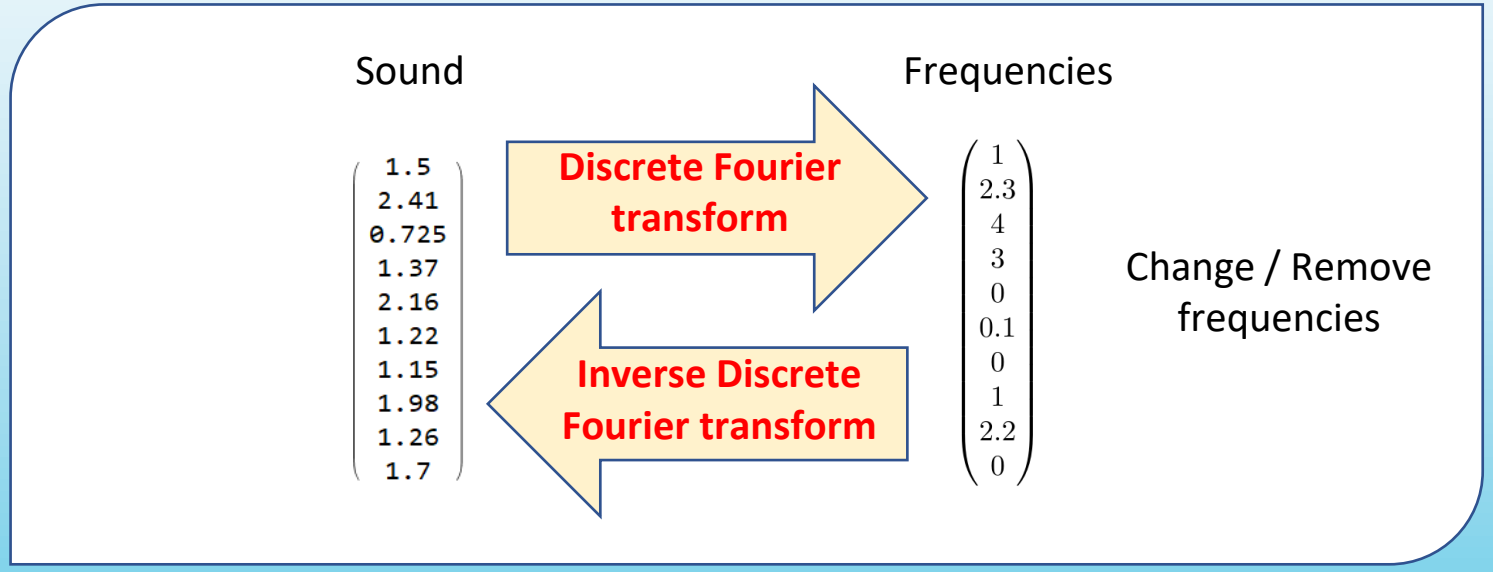
$$\begin{pmatrix} 1 \\ 2.3 \\ 4 \\ 3 \\ 0 \\ 0.1 \\ 0 \\ 1 \\ 2.2 \\ 0 \end{pmatrix}$$

The highest values gives the current pitch

Fourier transform

The (discrete) Fourier transform has various applications

- Digital filter

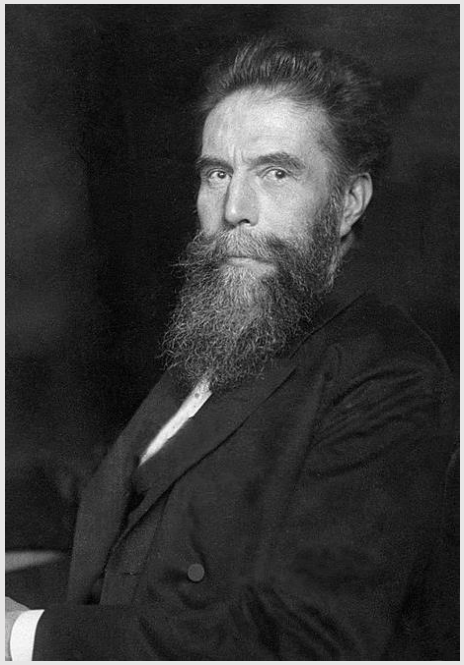


- Image processing
- Data compressions (JPEG)
- Appears in various areas of mathematics and physics

A little bit history

Today: Friday 8th November, 2019

124 years ago: Friday 8th November, 1895



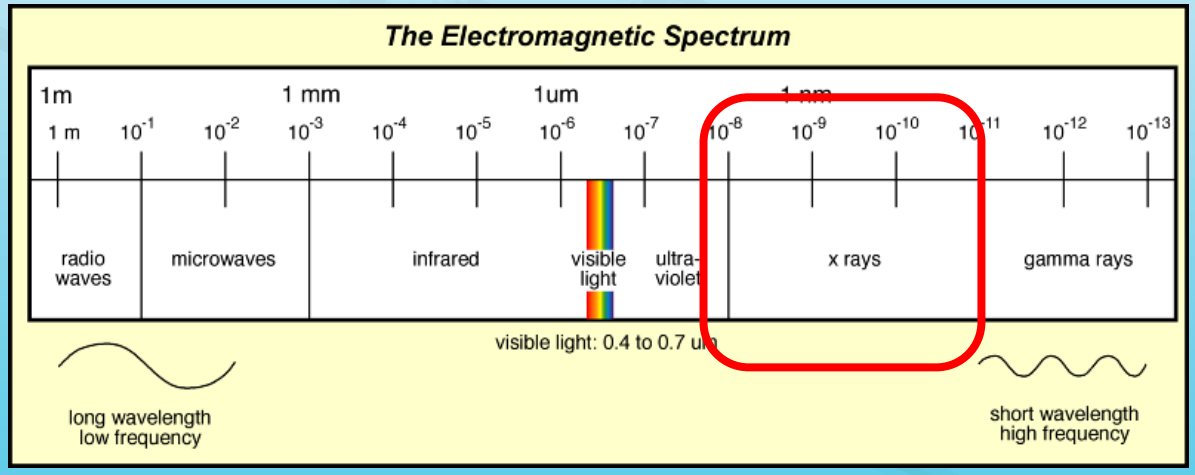
Wilhelm Conrad Röntgen

1845 - 1923

https://ja.wikipedia.org/wiki/%E3%83%B4%E3%82%A3%E3%83%AB%E3%83%98%E3%83%AB%E3%83%A0%E3%83%BB%E3%83%AC%E3%83%B3%E3%83%88%E3%82%B2%E3%83%B3

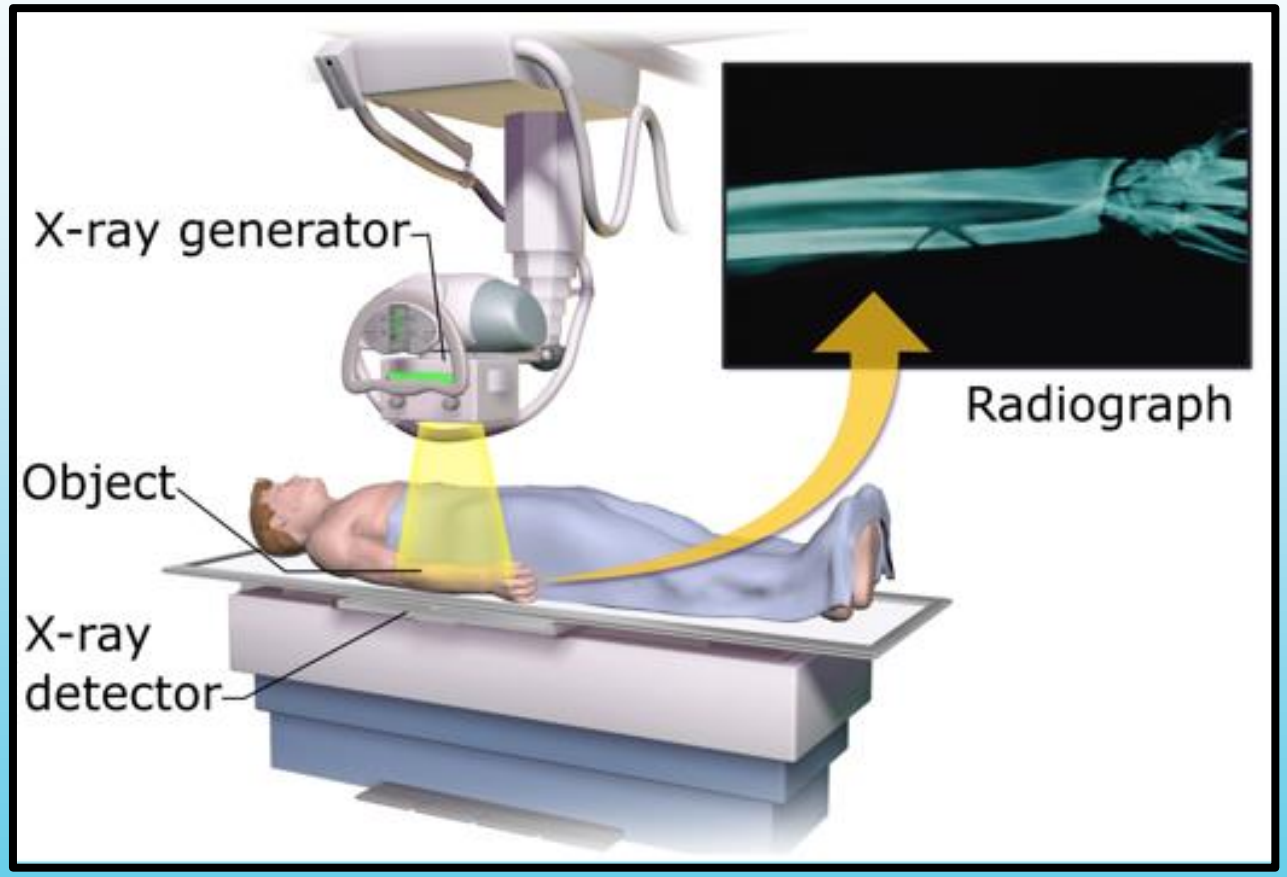
Today 124 years ago W. C. Roentgen discovered X-Rays (Röntgenstrahlen, レントゲン線)

For this discovery he obtained the first Nobel Prize in Physics (1901)

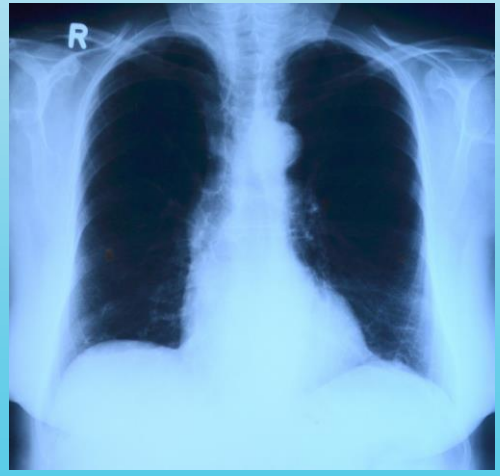


https://ja.wikipedia.org/wiki/%E3%83%95%E3%82%A1%E3%82%A4%E3%83%AB:Wilhelm_R%C3%B6ntgen_signature.svg

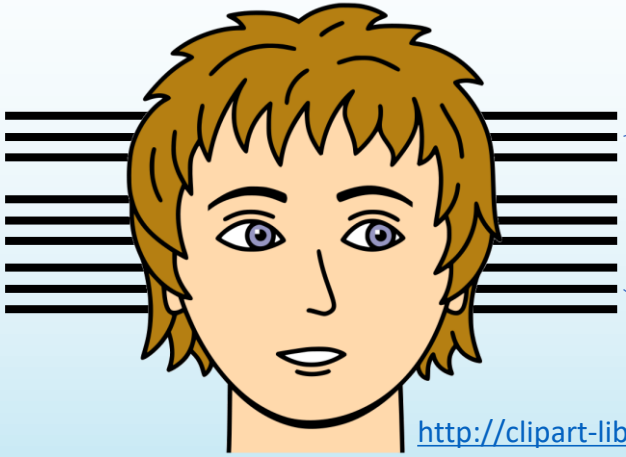
X-rays



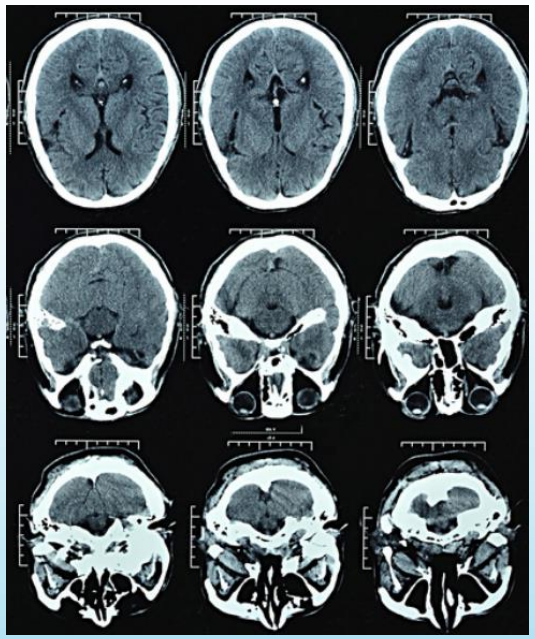
https://commons.wikimedia.org/wiki/File:X-Ray.png



Different point of view



http://clipart-library.com/



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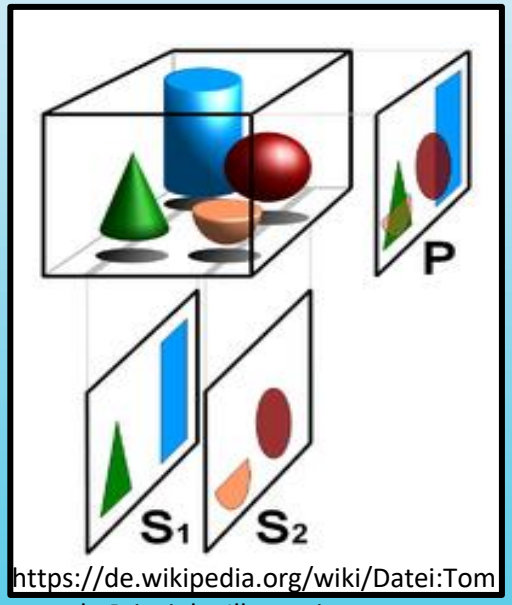
http://www.okakenko.jp/column/prevention/%E7%A6%81%E7%85%99%E5%A4%96%E6%9D%A5%E3%81%AB%E3%81%A4%E3%81%84%E3%81%A6/

Computed tomography scan (CT Scan)

https://www.gurunanakinstitute.com/



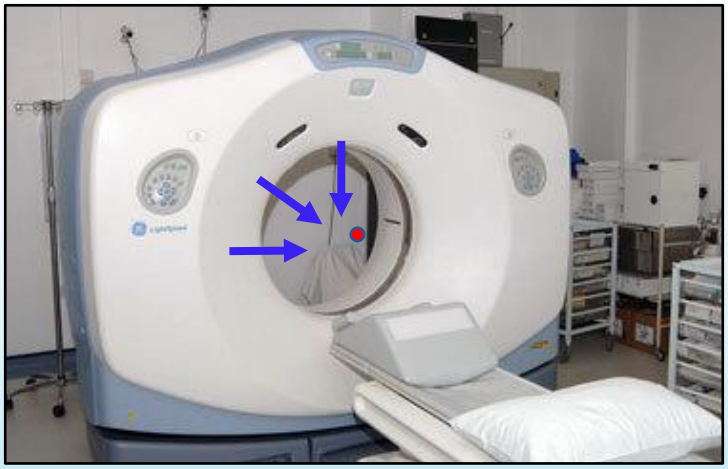
https://de.wikipedia.org/wiki/Datei:TomographyPrinciple_Illustration.png

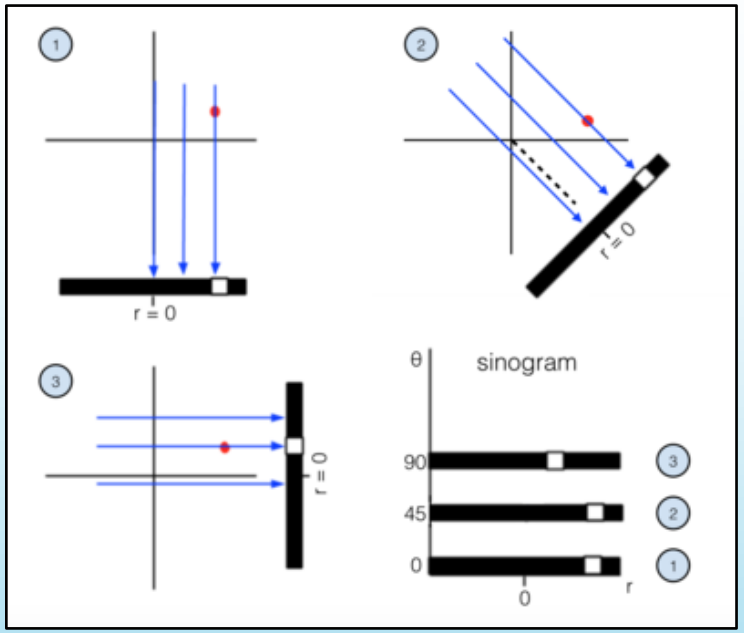
Basic principle of **tomography**
(tomos = slice/section)

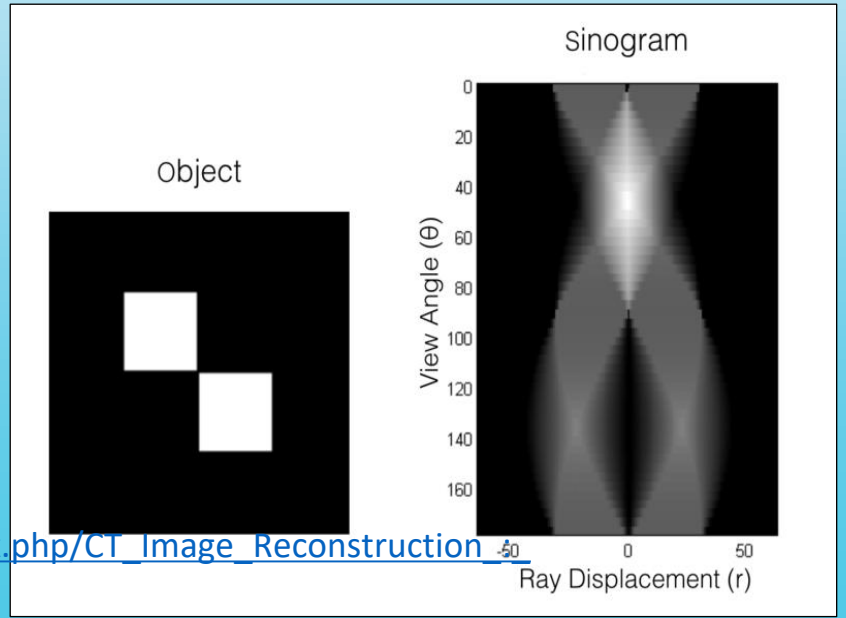
Video removed due to copyright restrictions

CT Scanner
(without cover)

X-Ray from different angles: Sinogram



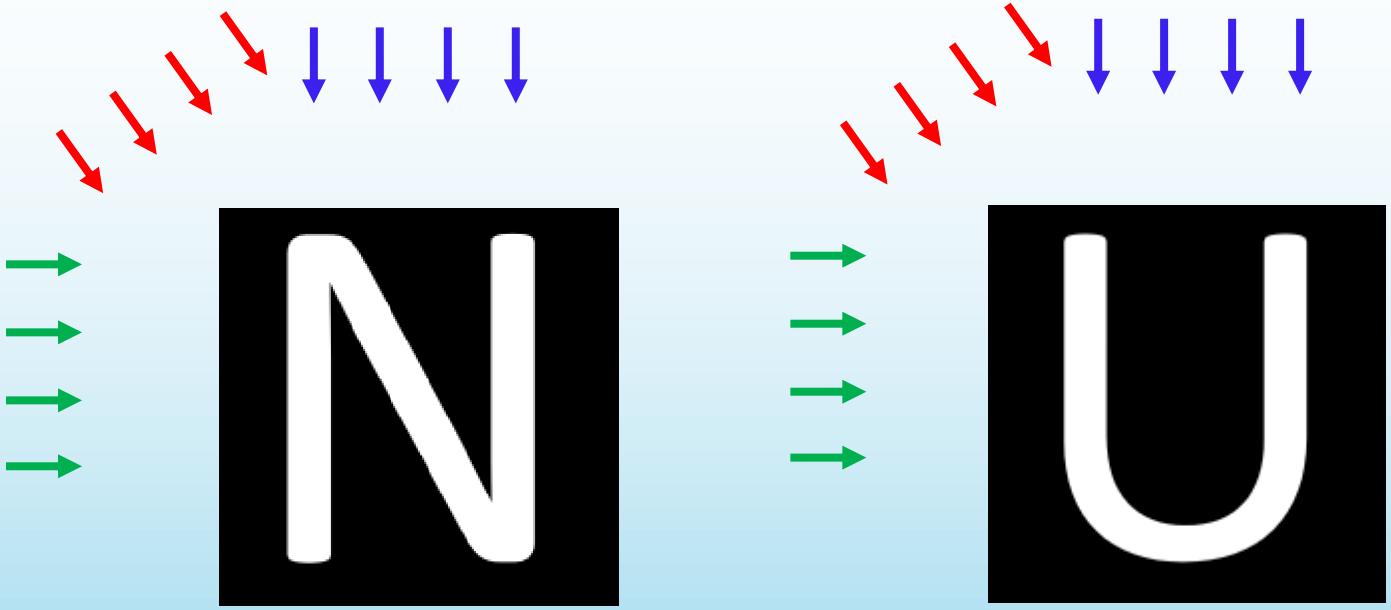




http://199.116.233.101/index.php/CT_Image_Reconstruction_-_Third_Edition

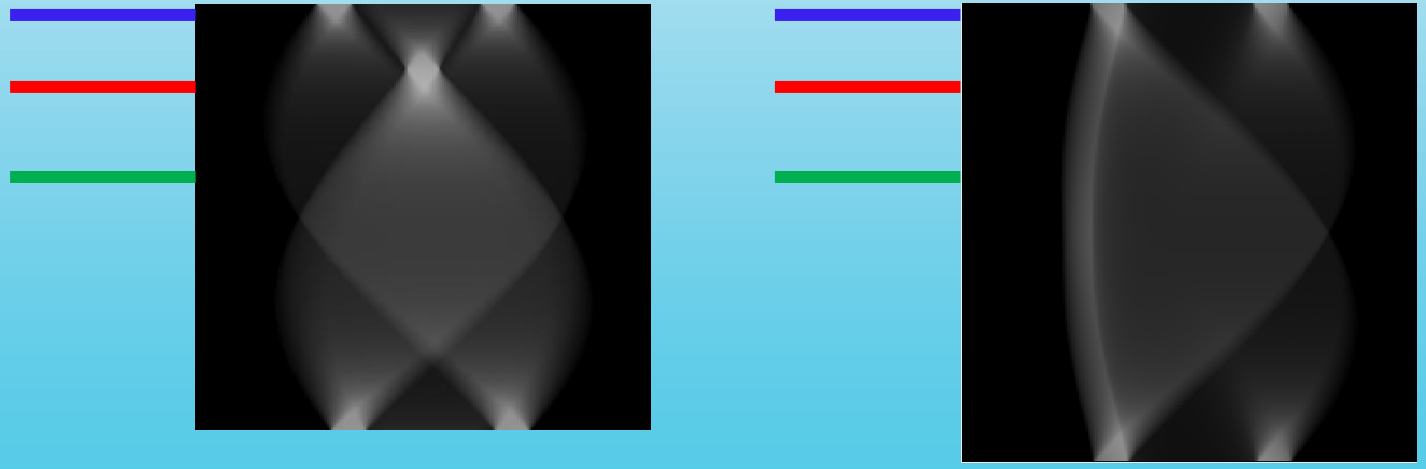
X-Ray from different angles: Sinogram

Original object

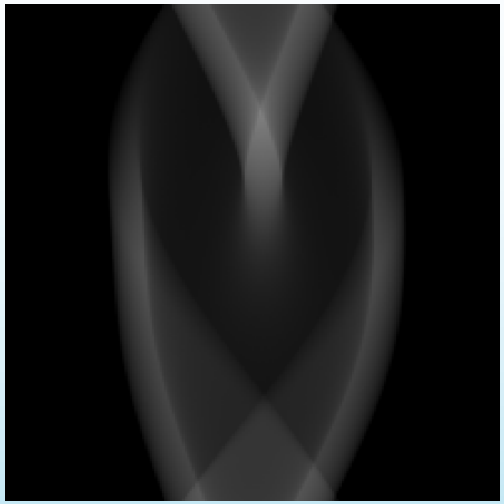


Sinogram (taking X-ray from different angels)

angle (0 – 180)



Sinogram: Can you invert it?



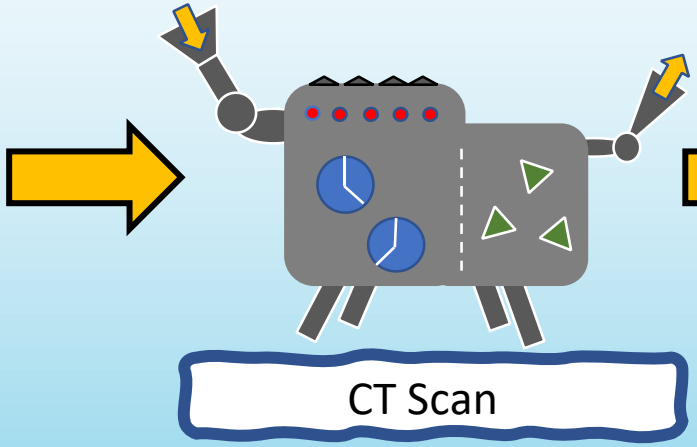
Is this the Sinogram of S or G?

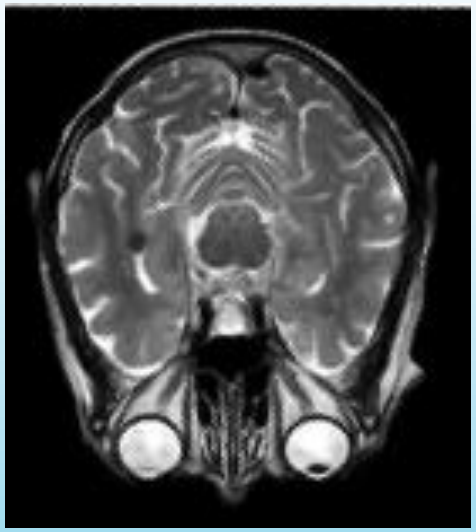
Yes

No

Sinogram: Can you invert it?

CT Scan – How does it work?

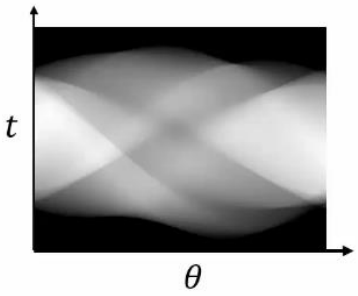


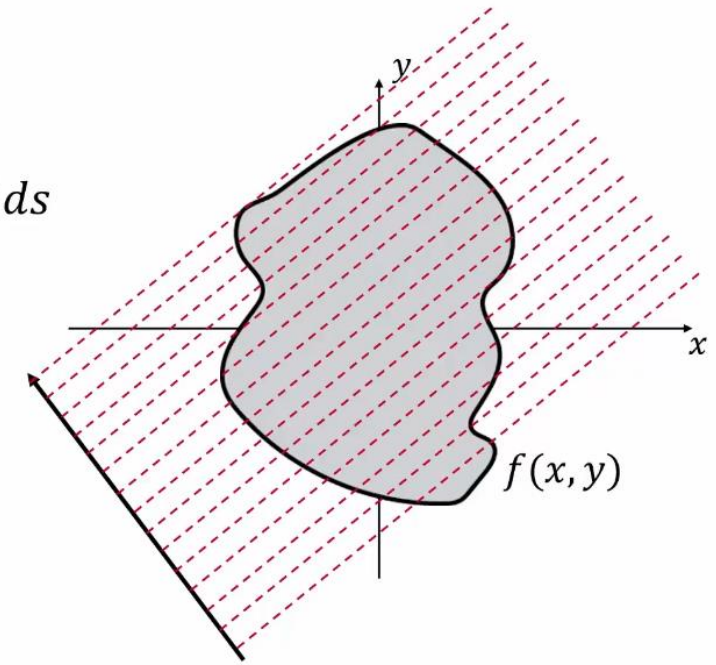


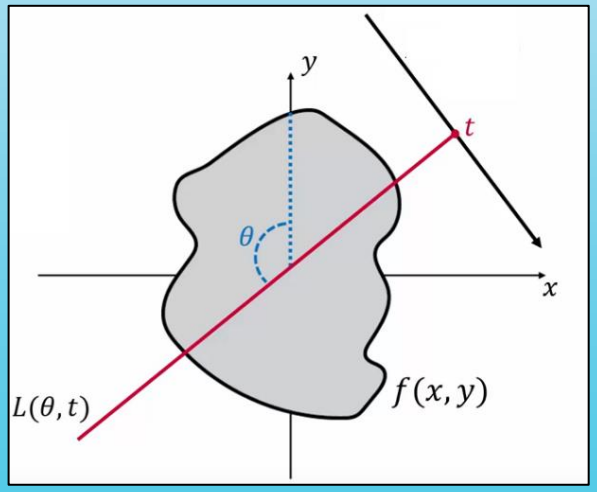
Radon transform

Radon transform (sorry a little bit math)

$$\mathcal{R}f(\theta, t) = \int_{L(\theta,t)} f(x, y)ds$$





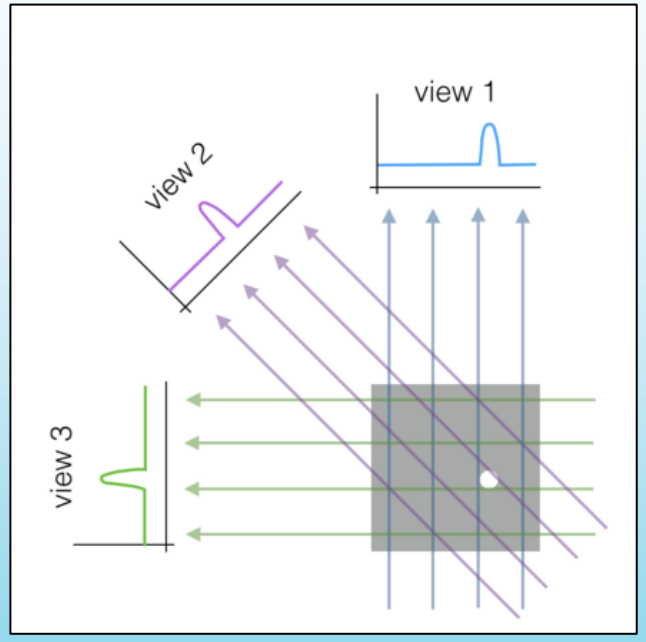


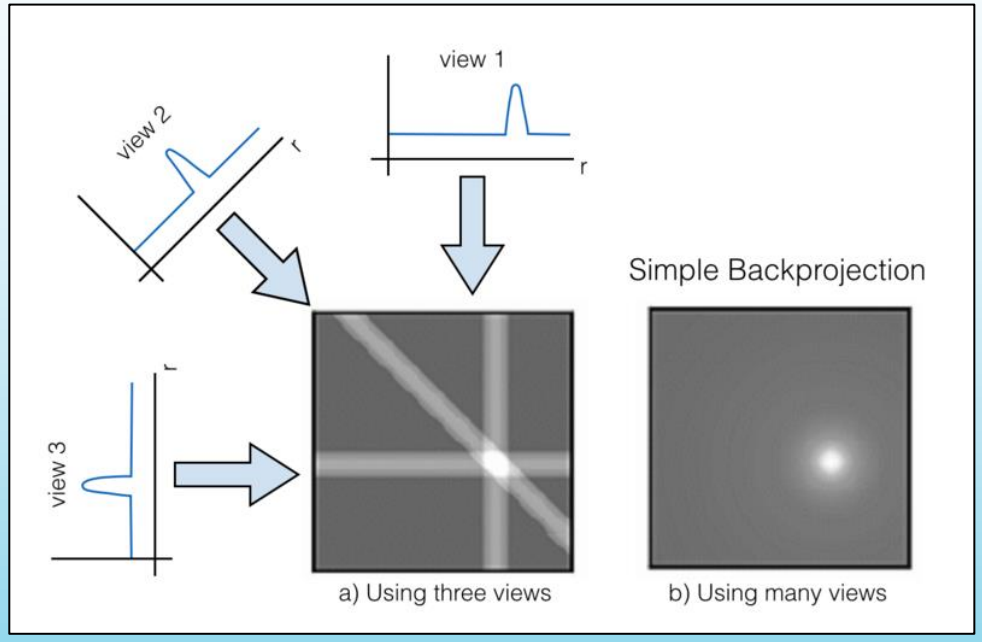
https://www.youtube.com/watch?v=YIvTpW3IevI

This transform can be inverted by using the Fourier transform and the „Projection slice theorem“.

Backprojection: From Sinogram to the original object

- One easy way: Simple backprojection



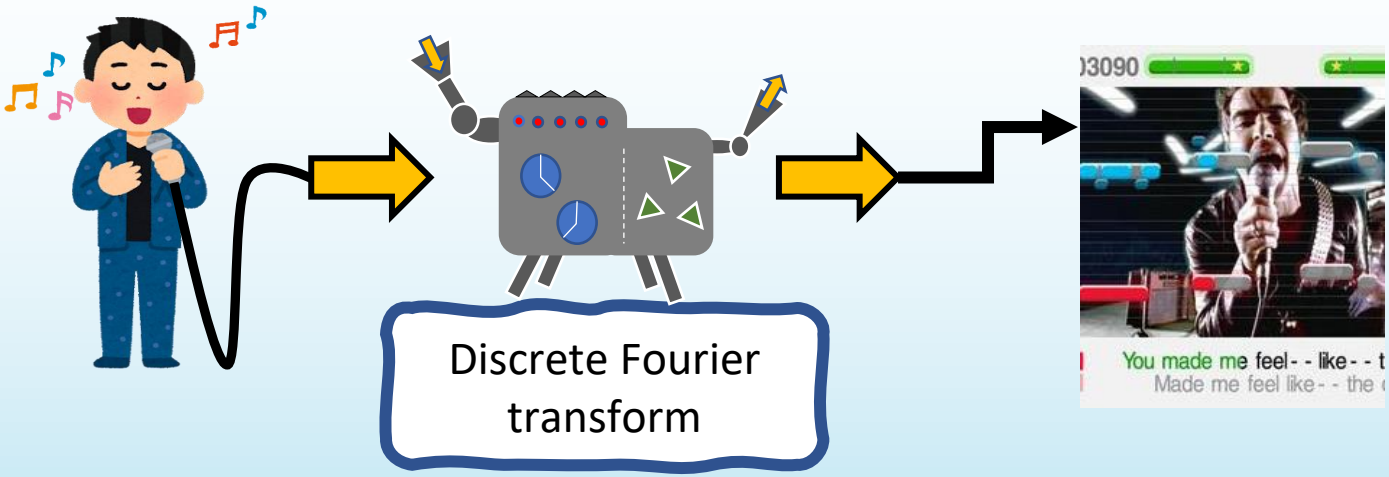


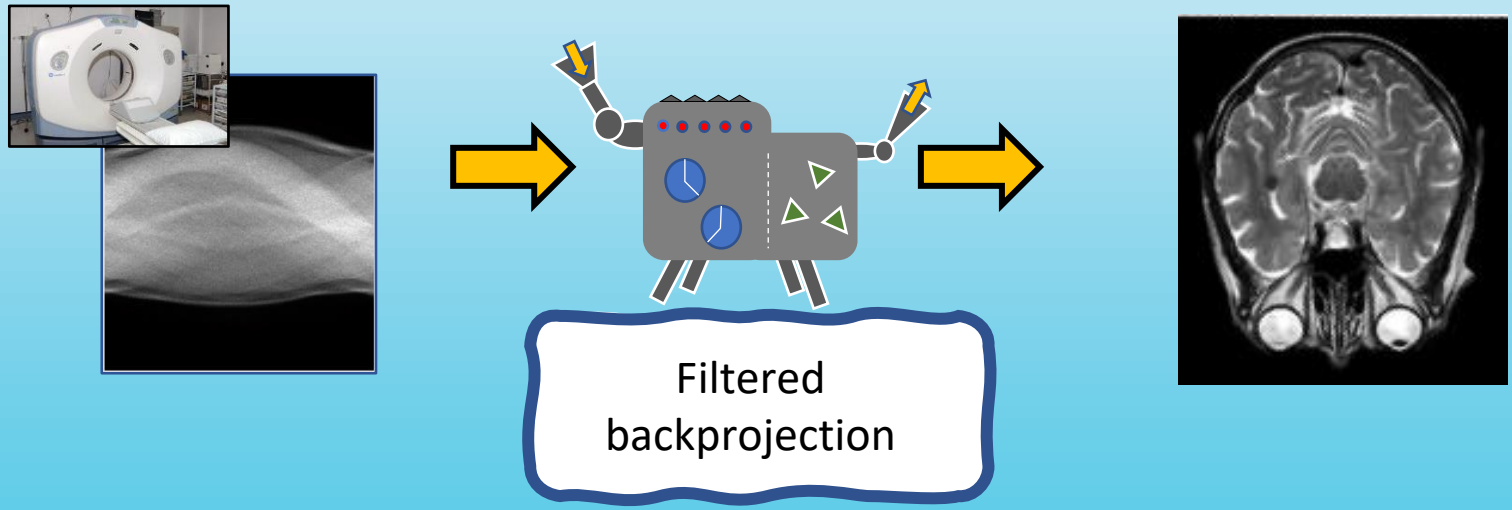
http://199.116.233.101/index.php/CT_Image_Reconstruction_:_Third_Edition

Results are blurry!

In reality **filtered backprojection** is used
(uses discrete Fourier transform)

Summary: Math gives magic machines





Some problems are too hard..

In reality, we also have problems where it is really hard/impossible to create a magic machine.

Example: Ramen classification

Input
Picture of Ramen

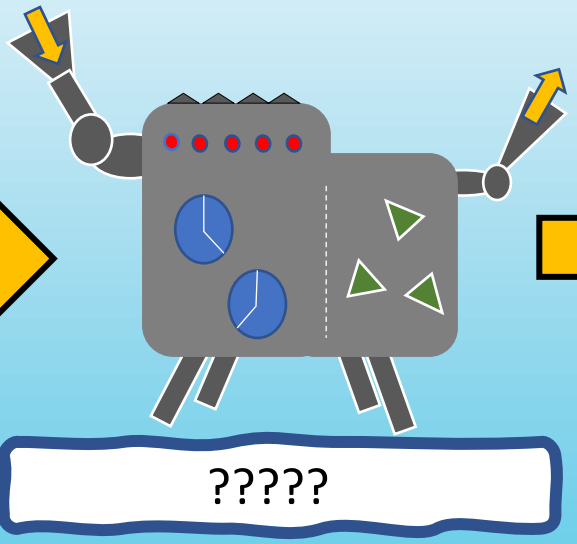

https://taiken.co/single/taiwan-ramen-not-from-taiwan/


https://www.nipponizakaya.co.uk/product/tonkotsu/


https://tripjepang.co.id/trik-makan-ramen-ala-orang-jepang.html


https://skyticket.jp/guide/180956/2

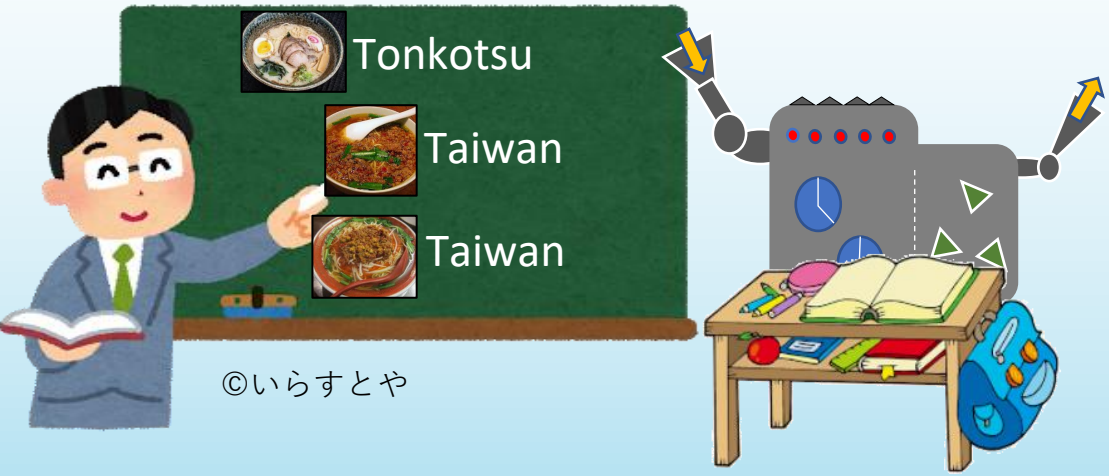

https://www.japandigest.de/reisen/essen/essen/ramen-landkarte/

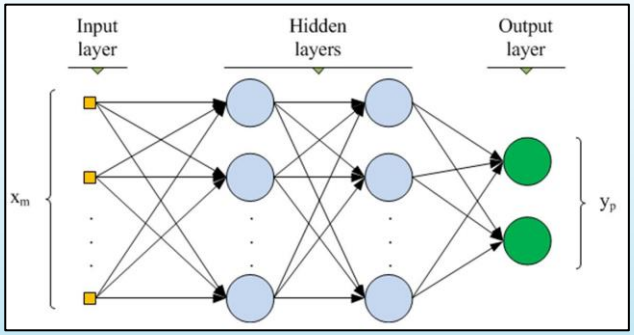


Output
What type of Ramen is it?
Taiwan Ramen?
Tonkotsu Ramen?
...

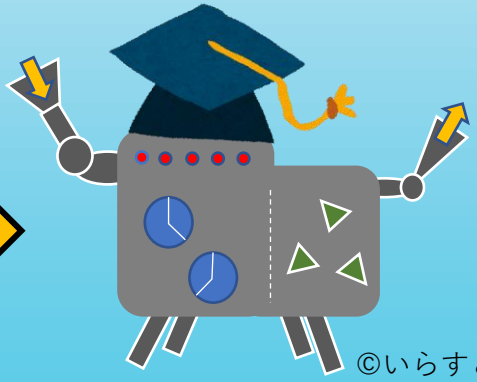
Machine learning

We can simulate “brains” and teach them!





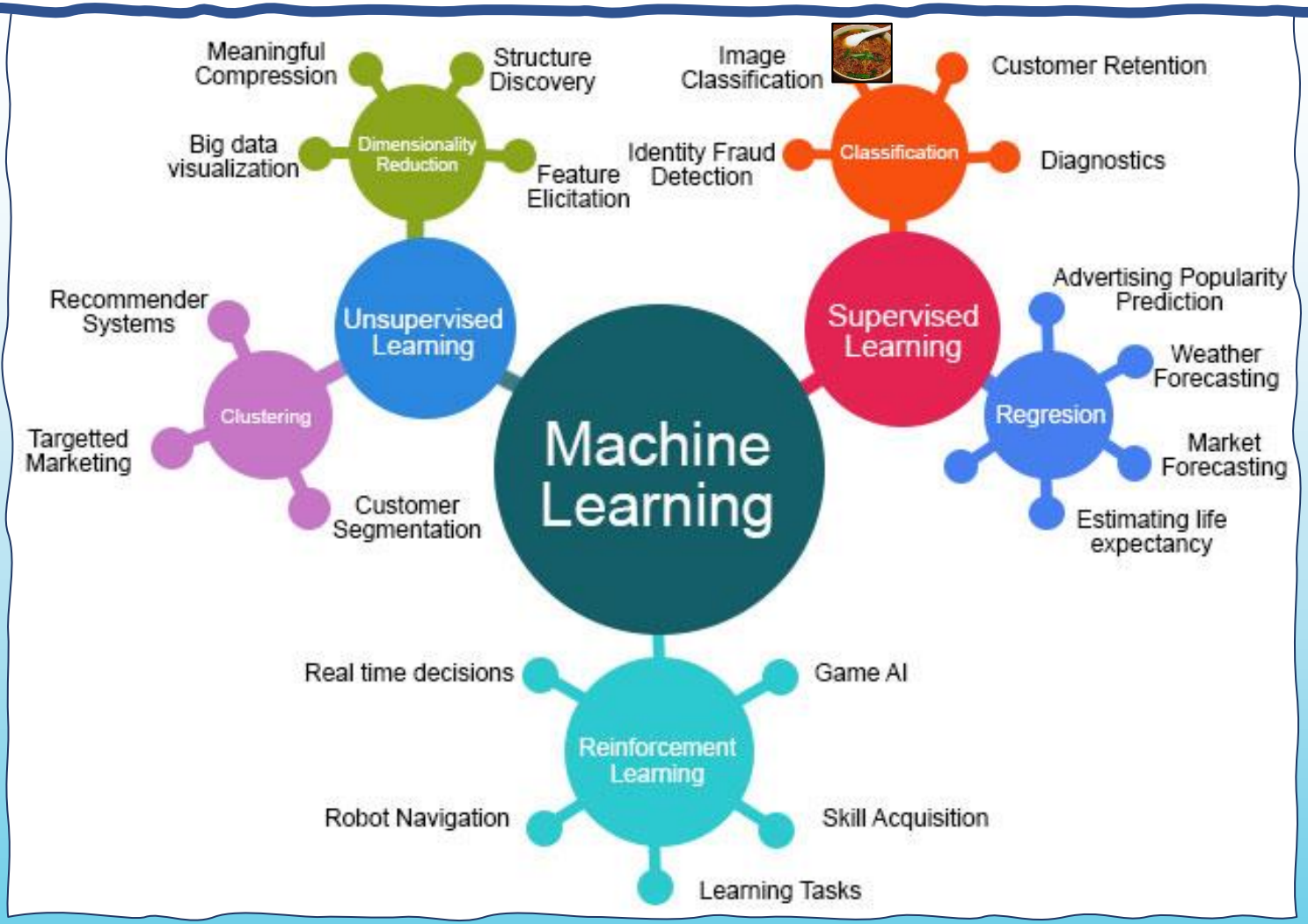
Neural network



Taiwan Ramen!
... with 95% certainty!

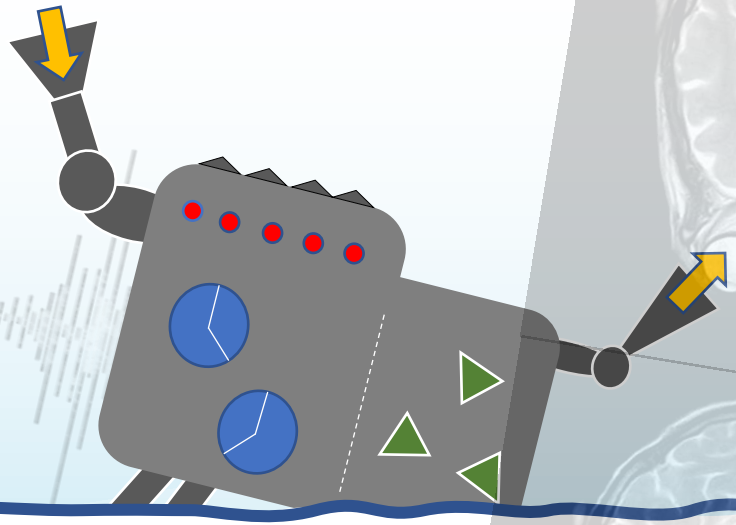
Ramen classifier

Machine learning



https://www.kdnuggets.com/2019/04/poll-data-science-machine-learning-methods-algorithms-use-2018-2019.html

In Fall 2020 I am planning to offer a „Math for machine learning“ course in the G30 Program.



Thank you very much for your attention!