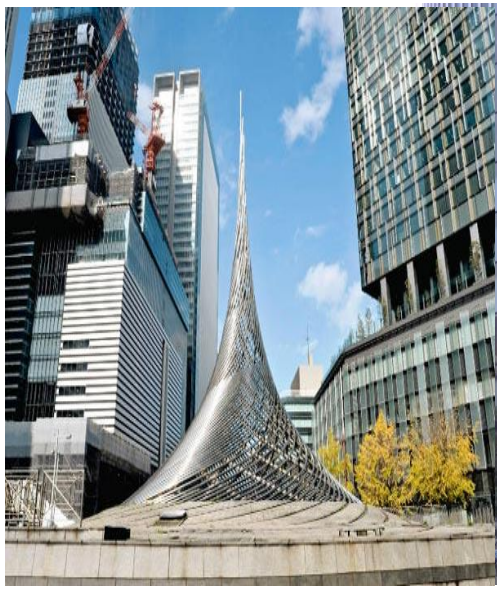
Symmetry and Singularity

https://www.feel-kobe.jp/facilities/detail.php?code=0000000036 2018/06/12



https://jouhou.nagoya/hishou-move/ 2018/06/12

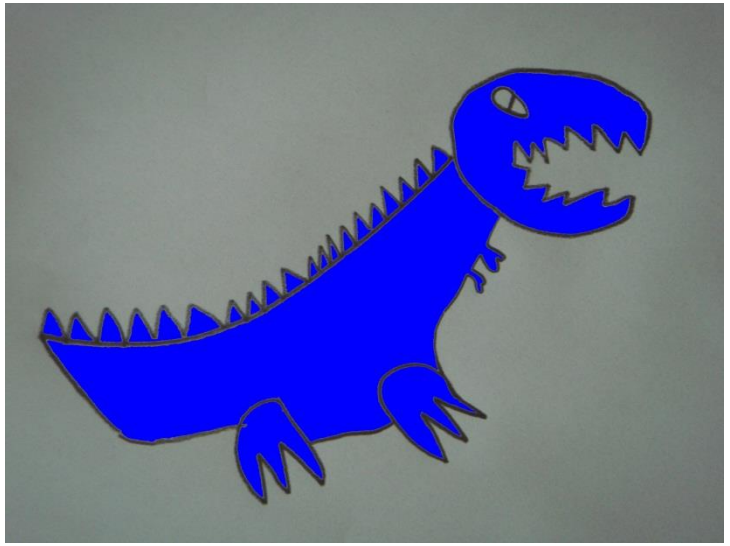
Yukari ITO

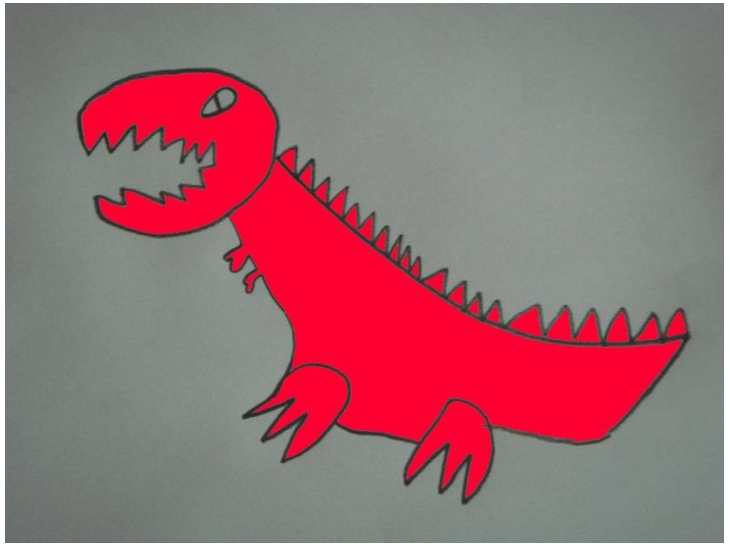
Graduate School of Mathematics, Nagoya University

2018.6.5 at Studium Generale

What is Symmetry in Mathematics?

- Symmetry with a person in the mirror.
- Forget everything except the shape!





Geometric symmetry

- Definition of Symmetry.

【Symmetry】

Symmetry can be observed after we

Rotate around a point,

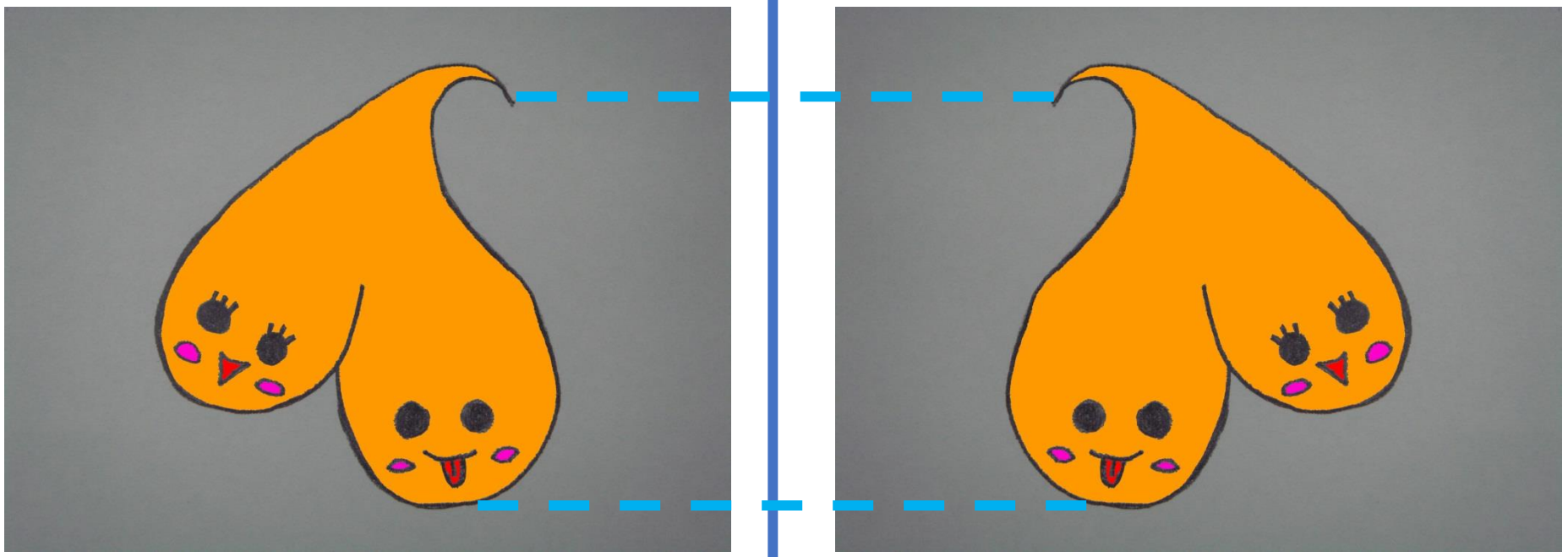
See the image in the mirror, etc.,

the shape is the same as the original one.

Group can explain this situation well mathematically.

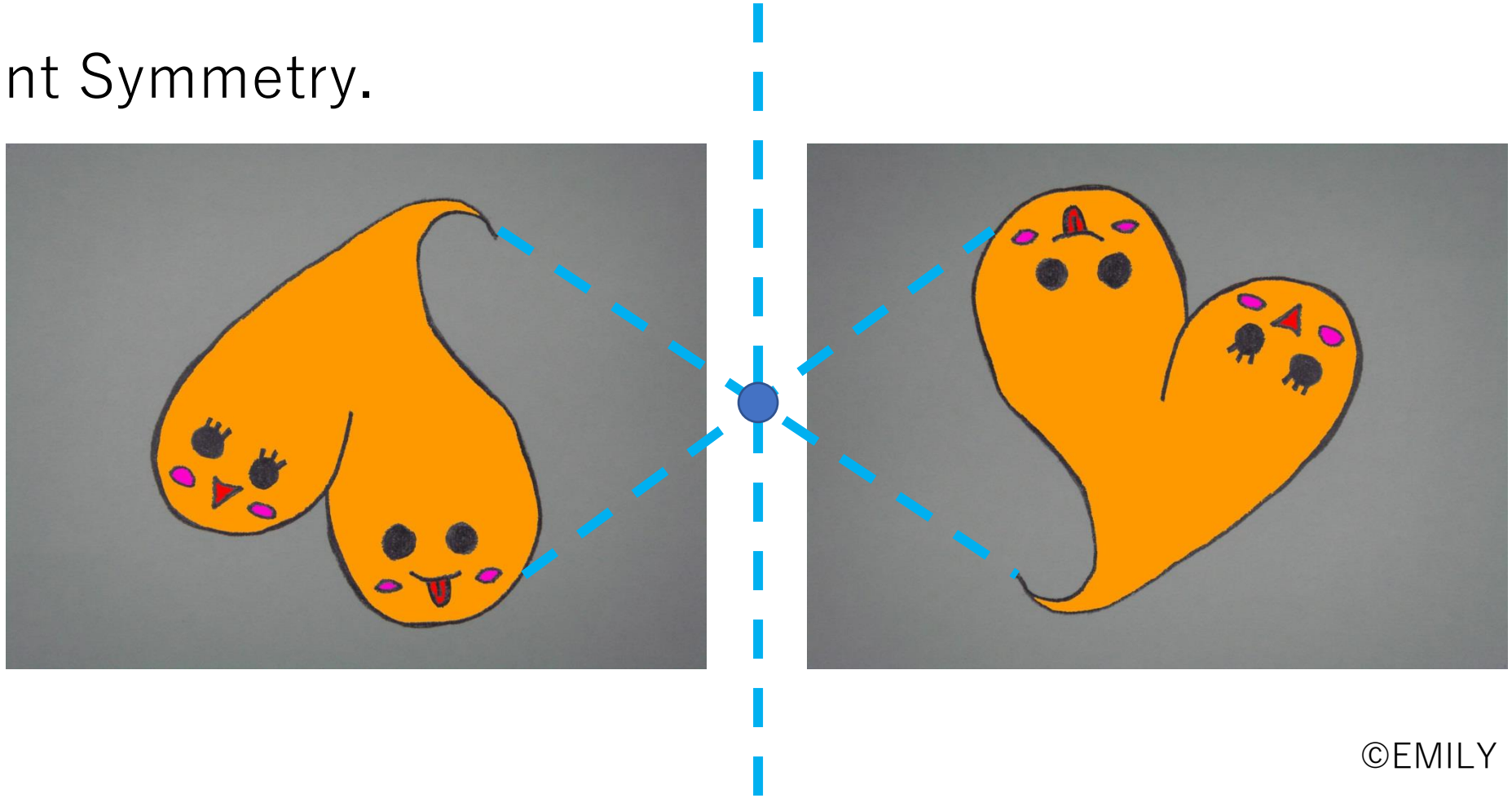
Geometric Symmetry

- Line Symmetry (regerection symmetry)



Geometric Symmetry

- Point Symmetry.



Definition of a Group

【Group（群）】

Let G be a set with operator ＊、we call the set G a **Group** if it satisfies the following conditions:

0．If x, y is in G, then x＊y is also in G.

1．(x＊y)＊z=x＊(y＊z)　**Associative Law（結合法則）**.

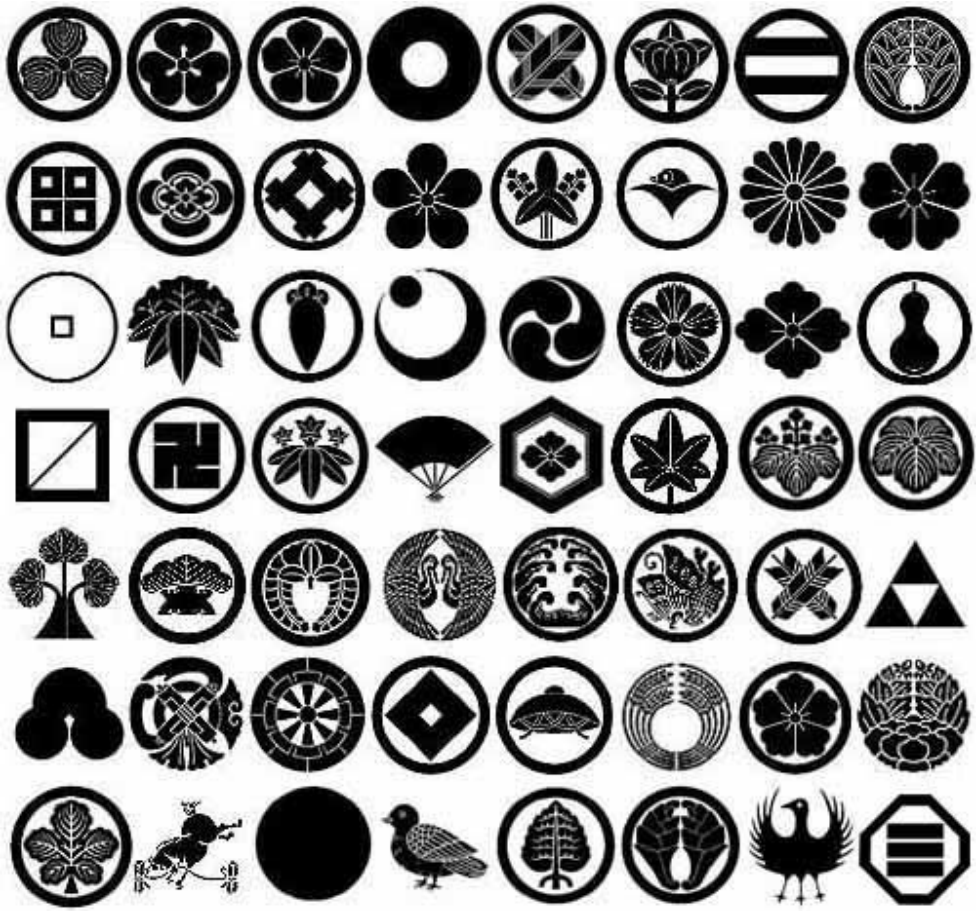
2．There exists **identity element** e in G such that

x＊e=e＊x=x .

3．There exists the **inverse element** of x such that

x＊y=y＊x=e.

Let’s find symmetries (Family crest)



http://www.harimaya.com/o_kamon1/yurai/a_yurai/yurai.html 2018/06/12

How to move the shape → a group.

- The meaning of the definition of a group in terms of movement of a family crest.

1. Associative law: Rule for computation.
2. Identity element（no move, do Nothing!）
3. Inverse element＝opposite movement.

Example 1

- Rotation・・・**Cyclic group (巡回群)**

  Rotate suitably!

Mitsudomoe(三つ巴)

Move it for 120 degrees, then it will coincide with the original shape.

This group is called cyclic group of order three.

https://kamon.myoji-yurai.net/kamonDetail.htm?kamonName=%E5%B7%A6%E4%B8%89%E3%81%A4%E5%B7%B4
2018/06/12

Exmaple 2

- Rotation and turning over・・・**Dihedral group（二面体群）**

  Rotate and turn over!

Kikyou(桔梗)

Rotate for 72 degrees or turn it over,

Then it will coincide with the original one

This group is called Dihedral group of order five.

https://kamon.myoji-yurai.net/kamonDetail.htm?kamonName=%E5%B7%A6%E4%B8%89%E3%81%A4%E5%B7%B4
2018/06/12

What kind of symmetry?

- There are several symmetries!

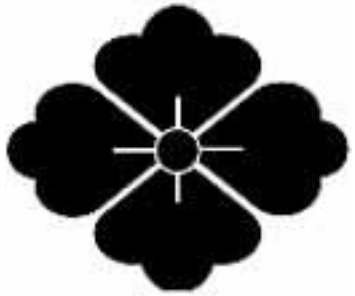
Kiku (菊)

https://www.pinterest.jp/pin/645070346596867118/?lp=true 2018/06/12

Kiri (桐)

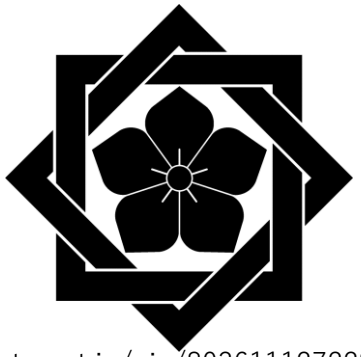
http://kamoeya.hamazo.tv/e2941161.html 2018/06/12

Hanabishi (花菱)



https://www.waichiba.com/item/itemgenre/kamonlist/k0301.html 2018/06/12

Chigai-masu ni kikyo　(違い枡に桔梗)



https://www.pinterest.jp/pin/803611127228706010/?lp=true 2018/06/12

Design and Group

- These two Family crests has difference design.
- Mathematically, they have the same symmetry and the corresponding groups are the same!

Kiri (桐)

http://kamoeya.hamazo.tv/e2941161.html
2018/06/12

Tachibana (橘)

https://sengoku-g.net/crests/view/51
2018/06/12

Two dimensional crystal group

- **Rotation, Turning over and Parallel displacement !**

We will obtain
two dimensional crystal groups!
The number of the crystal groups are only 17!　(1891,Fedlov)

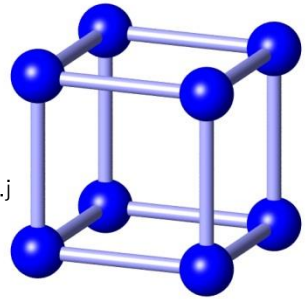
1．Tiles in Alhambra Palace in Spain.
2．Designs for wall paper, Fabric prints. Etc.

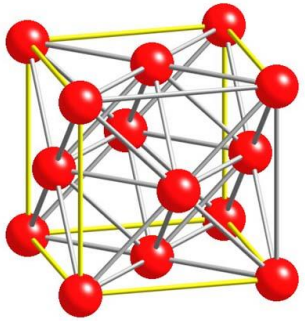
reference:『この定理が美しい』（数学書房）の「対称性の美」

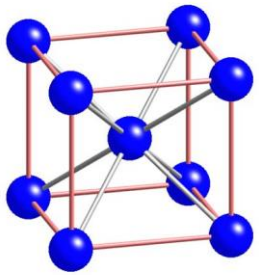
Three dimensional crystal group

- The number of the three crystal groups are **217**
- Most of them are exist in the nature!

http://www.cmpt.phys.tohoku.ac.jp/~otsuki/sc.html 2018/06/12







- The number of the Four dimensional crystal groups is **4783**

Symmetry in the nature!

- Group theory is used in Physics and Chemistry.
- It is useful to know the

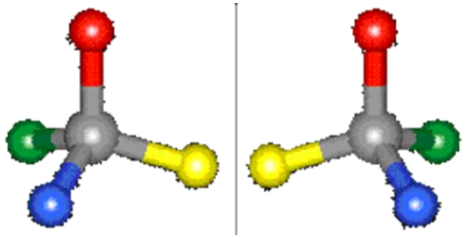
Structure of Molecule

Make new medicine, etc.

Symmetry in the nature

- 2001 Novel Prize in Chemistry

Professor Ryoji Noyori found a way to make one of the mirror symmetries.



http://www.org-chem.org/yuuki/chirality/L-R.html 2018/06/12

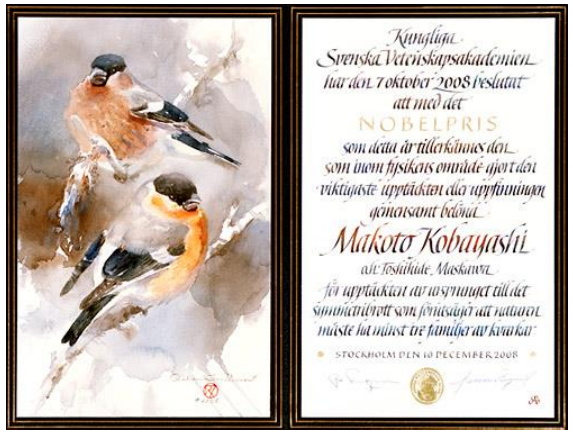
http://www.nndb.com/people/851/000100551/ 2018/06/12

http://www.nndb.com/people/851/000100551/ 2018/06/12

名古屋大学野依記念物質科学研究館2F

ケミストリーギャラリー　平日10:00～16:00

- 2008 Novel Prize in Physics
- Professors Makoto Kobayashi and Hidetoshi Maskawa find anti-symmetry of the Universe.



http://blog.livedoor.jp/hanano47/archives/51510692.html
2018/06/12

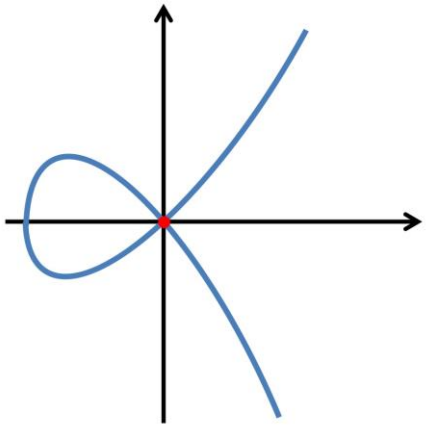
https://www.nobelprize.org/nobel_prizes/physics/laureates/2008/kobayashi-facts.html 2018/06/12

https://www.nobelprize.org/nobel_prizes/physics/laureates/2008/kobayashi-facts.html 2018/06/12

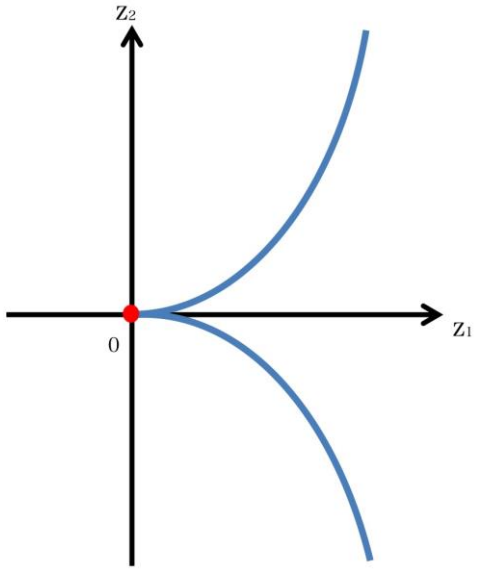
http://blog.livedoor.jp/hanano47/archives/51510692.html 2018/06/12

ノーベル物理学賞・化学賞展示室（理学部Ａ館1階）
月～木　10:00～12:00、13:00～16:00

Singularity



http://store.iroya-inc.co.jp/products/?id=1117 2018/06/12



Group action make a singularity!

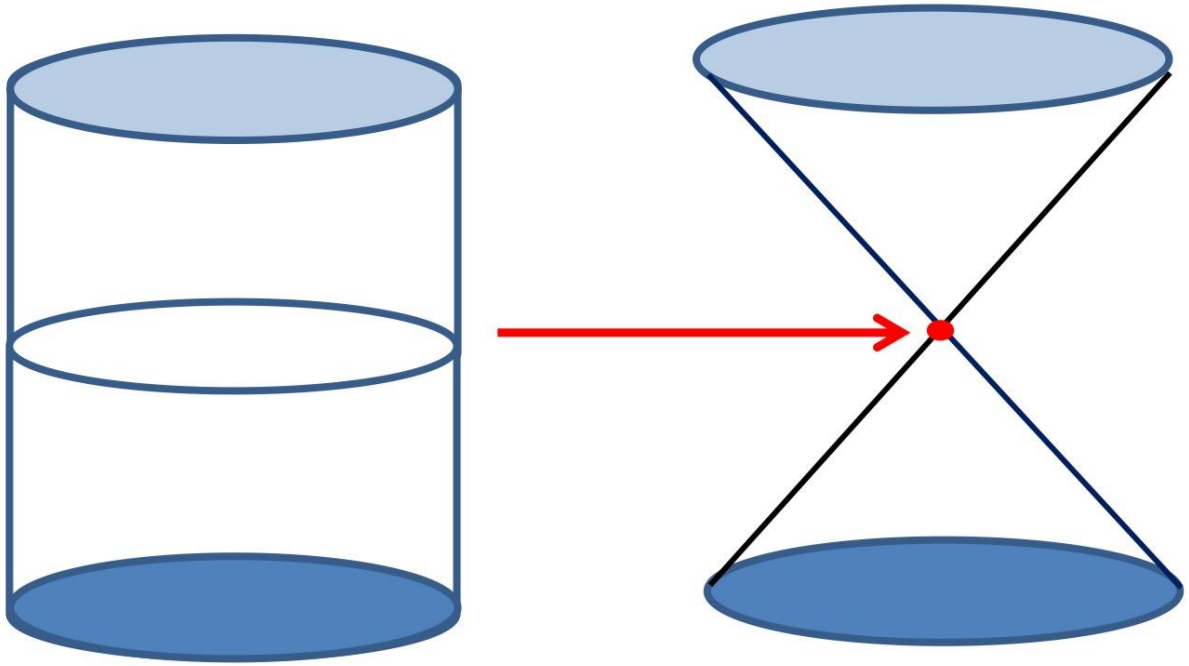
- Consider invariant under the group action :
- When G is cyclic group of order two, then the group action on two dimensional space can be written by a matrix $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

- The invariant monomials are x^2, y^2, xy . If we put them as $x^2 = X, y^2 = Y, xy = Z$, *then* $XY = Z^2$. More naturally,

$$X^2 + Y^2 + Z^2 = 0$$

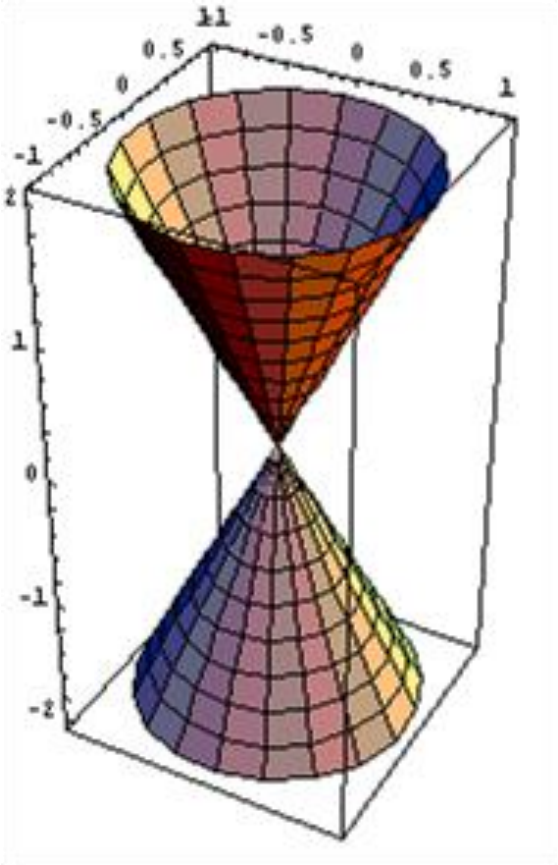
The singualrity by the group action

$$X^2 + Y^2 + Z^2 = 0$$



Real picture!

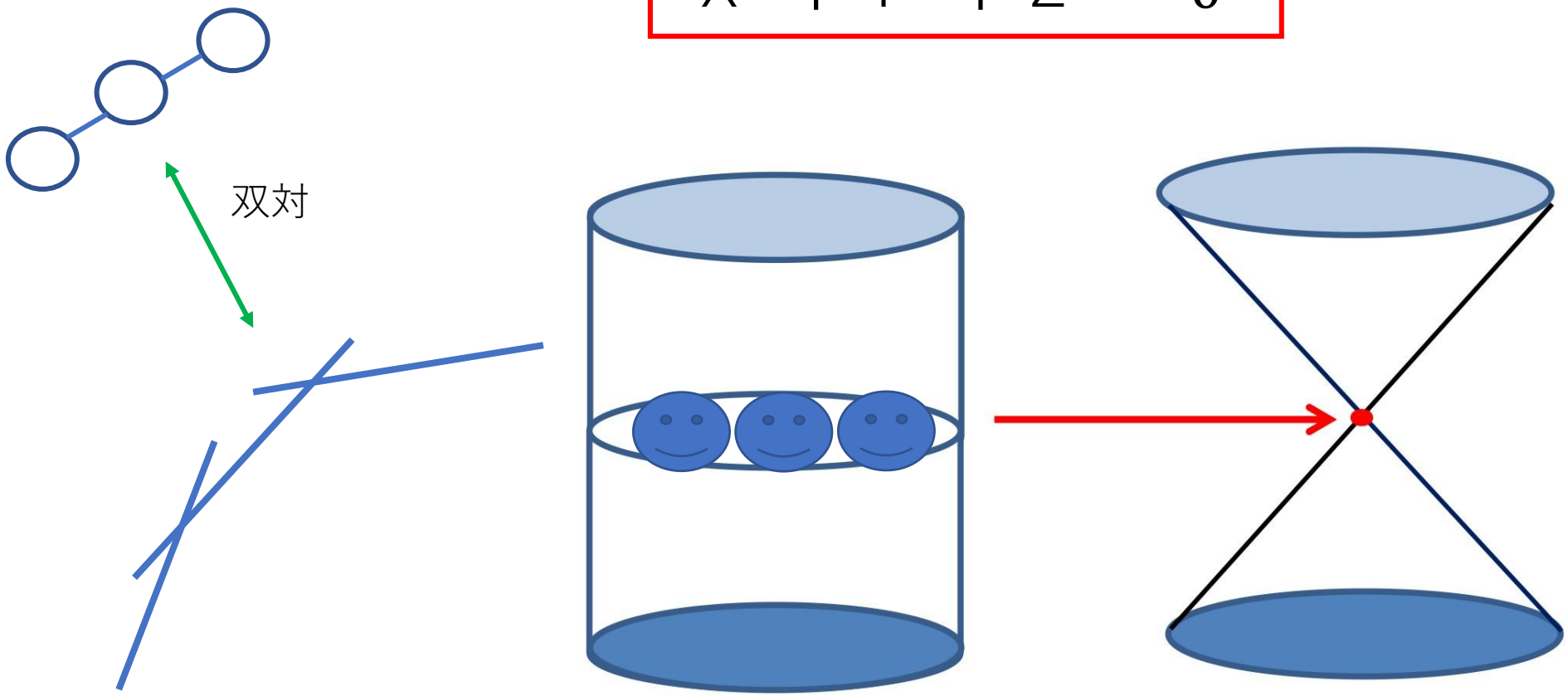
https://jouhou.nagoya/hishou-move/
2018/06/12



https://www.feel-kobe.jp/facilities/detail.php?code=0000000036
2018/06/12

Cyclic group of order 4

$$X^2 + Y^2 + Z^4 = 0$$



McKay correspondence in dimension two

- If G is a cyclic group of oder n+1
- The defining equation is

$$X^2 + Y^2 + Z^{n+1} = 0$$

- They will have n curves in the resolution of singularities!

It is also the same as the number of the group non-trivial irreducible representation!

Higher dimensional McKay correspondece

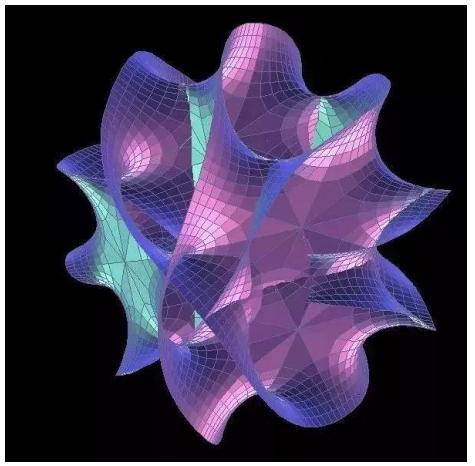
- We can also extend these to higher dimension,
- But there are many difficulties!

- On the other hand, there some relations with results in Physics!

Singularities in Physics

- **Black Holes**
- **Big Bang**
- **Super String theory**:

The Universe(10) = space time(4) + Calabi-Yau (6)



- **Multiverse**: