CHAPTER 11 : ROLLING

Application of Newton's laws both for linear and rotational motion

1. Rolling
   a. Pure roll (smooth rolling)
   b. Kinetic energy of rolling
   c. Rolling down a ramp

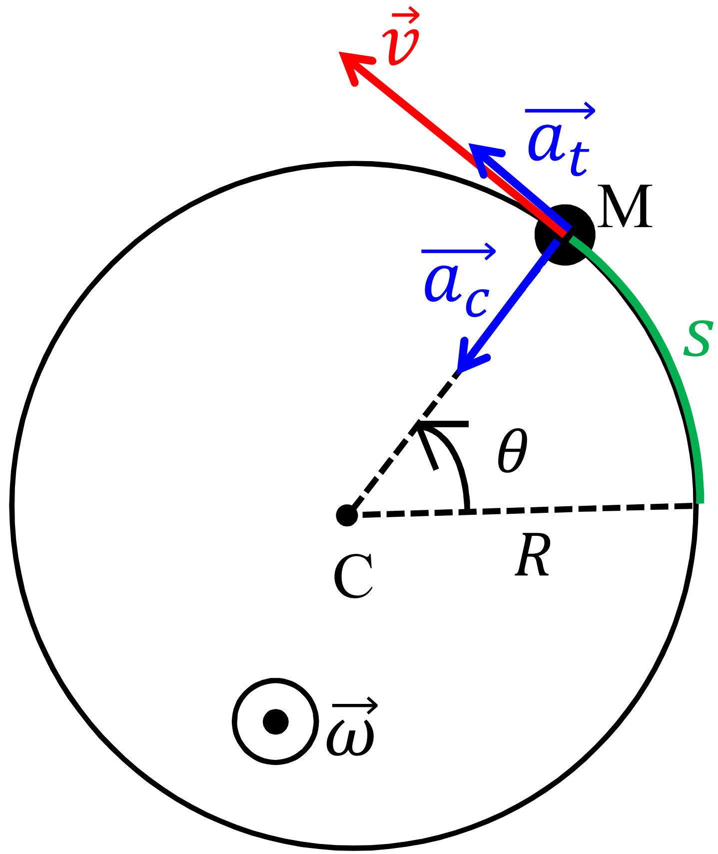
CIRCULAR MOTION : ${}^{dR}/_{dt} = 0$

$$s = R\theta$$

$$v = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

$$a_t = \frac{dv}{dt} = R\frac{d^2\theta}{dt^2} = R\frac{d\omega}{dt} = R\alpha$$

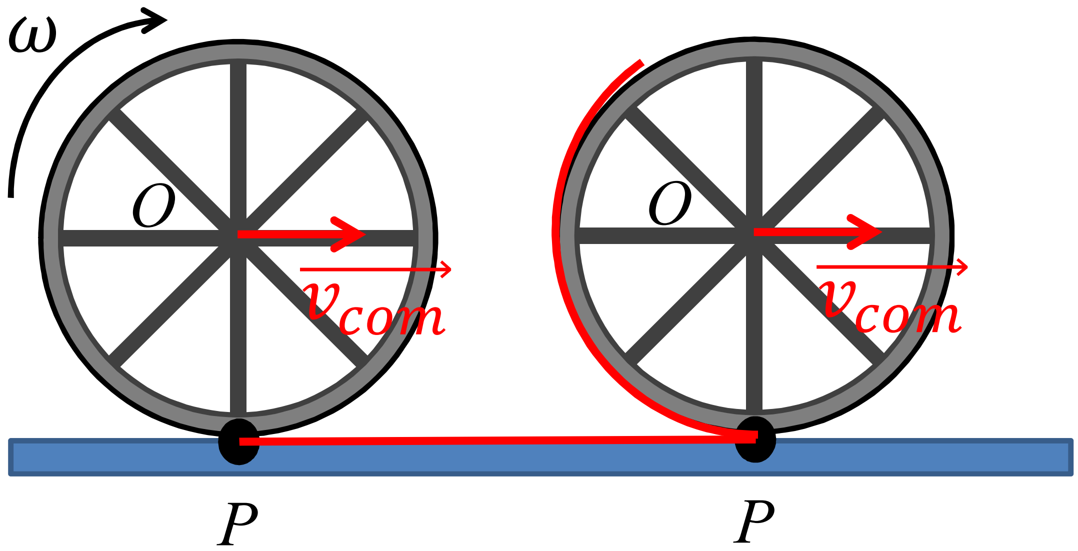
$$a_c = \frac{v^2}{R} = R\omega^2$$



ω is represented as a vector along the rotation axis

Its orientation is given by the right-hand rule of vector cross products

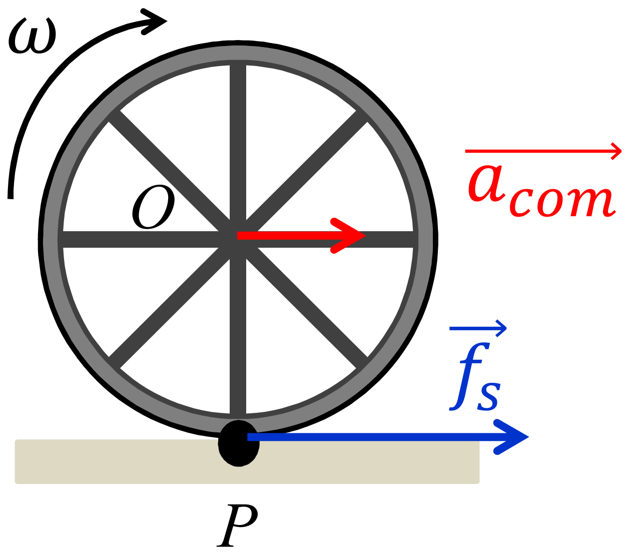
FRICTION AND ROLLING (HORIZONTAL MOTION) CASE OF UNIFORM MOTION



If a wheel rolls at constant speed, it has no tendency to slide at the point of contact P, and thus no frictional force acts there.

$$\overrightarrow{F_{net}} = m\overrightarrow{a_{com}} = 0$$

FRICTION AND ROLLING (HORIZONTAL MOTION)
CASE OF ACCELERATED MOTION



A wheel is being made to <u>rotate faster</u>. $\overrightarrow{a_{com}} \neq 0$. The faster rotation tends to make the bottom of the wheel slide to the left at point P.

A frictional force at P, directed to the right, opposes this tendency to slide. (Newton 3rd law)

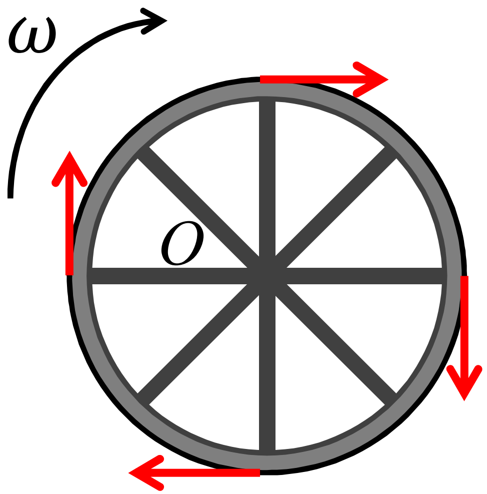
No sliding ➔ **Static** friction!

If the wheel were made to <u>rotate slower</u>:

The directions of the center-of-mass acceleration and frictional force at point P would now be to the left.

ROLLING AND RELATIVITY

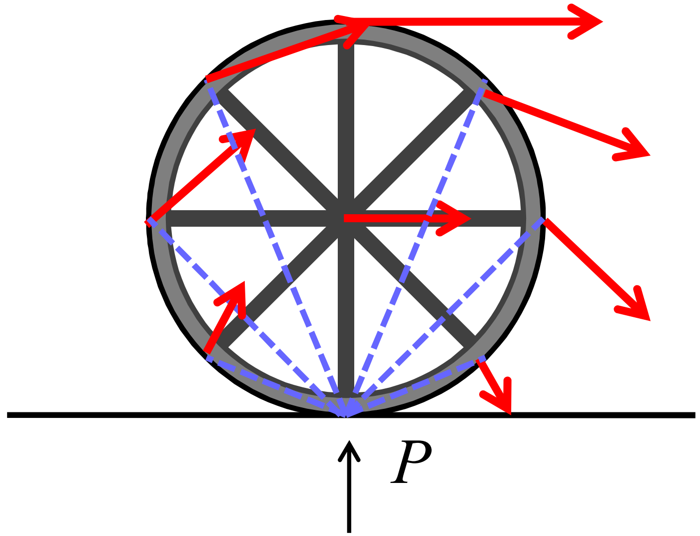
Velocities relative to center O



Each point on rim moving at speed $|\vec{v}|$ relative to center O

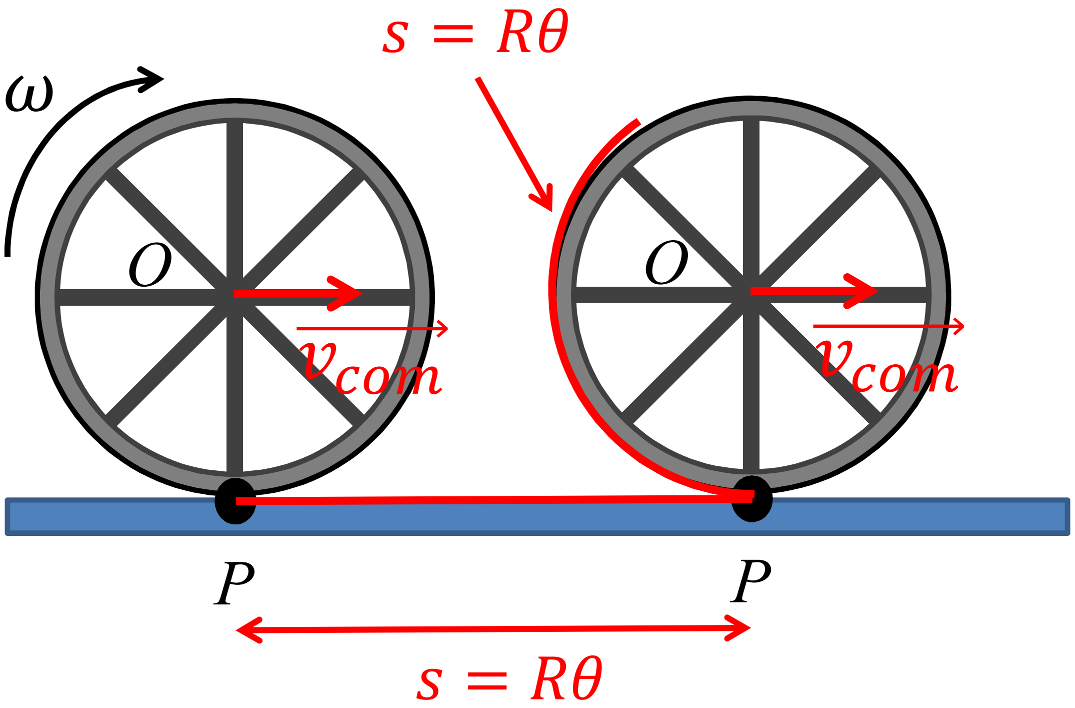
$$\boxed{|\vec{v}| = R\omega}$$

Velocities relative to road



Zero velocity at contact point P

PURE ROLL: ROTATION AND TRANSLATION COMBINED



$s = R\theta$

Also applies to pulleys

$$v_P = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega_O = v_{com} = R\omega_P$$

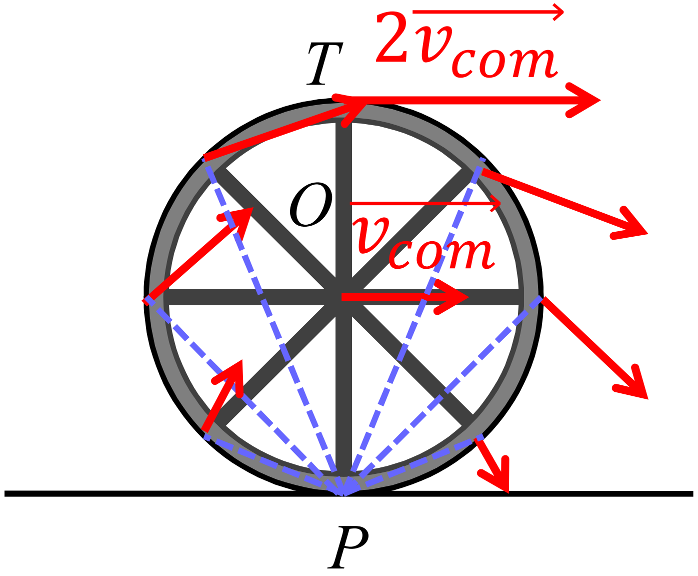
ω_O of wheel rotation
$= \omega_P$ of rotation about P

The com O and contact point P experience the same **linear motion**

ROLLING – WHEEL OF RADIUS R

Pure roll (no skidding, no sliding)

O: center of mass of wheel



Rotation about contact point P

$s = R\theta$

$$v = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

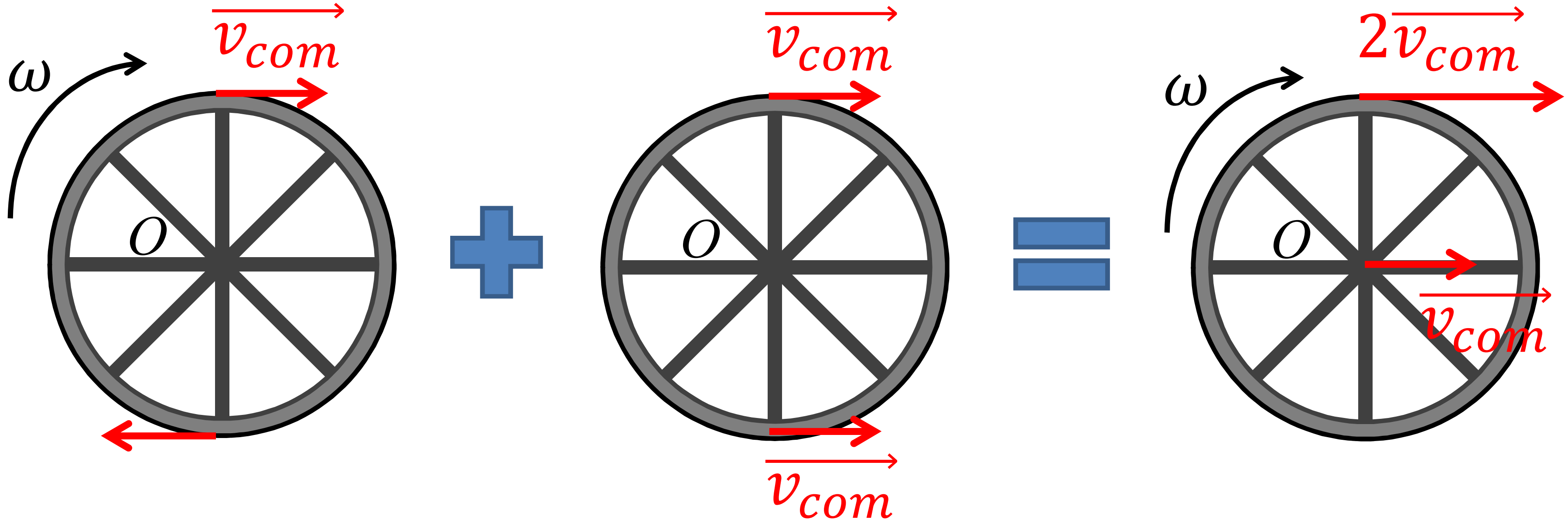
Instantaneous speed of O: $v_O = R\omega_P$

Instantaneous speed of T: $v_T = 2R\omega_P$

Rotation about P: V=0 at P!

$$\boxed{v_{com} = R\omega}$$

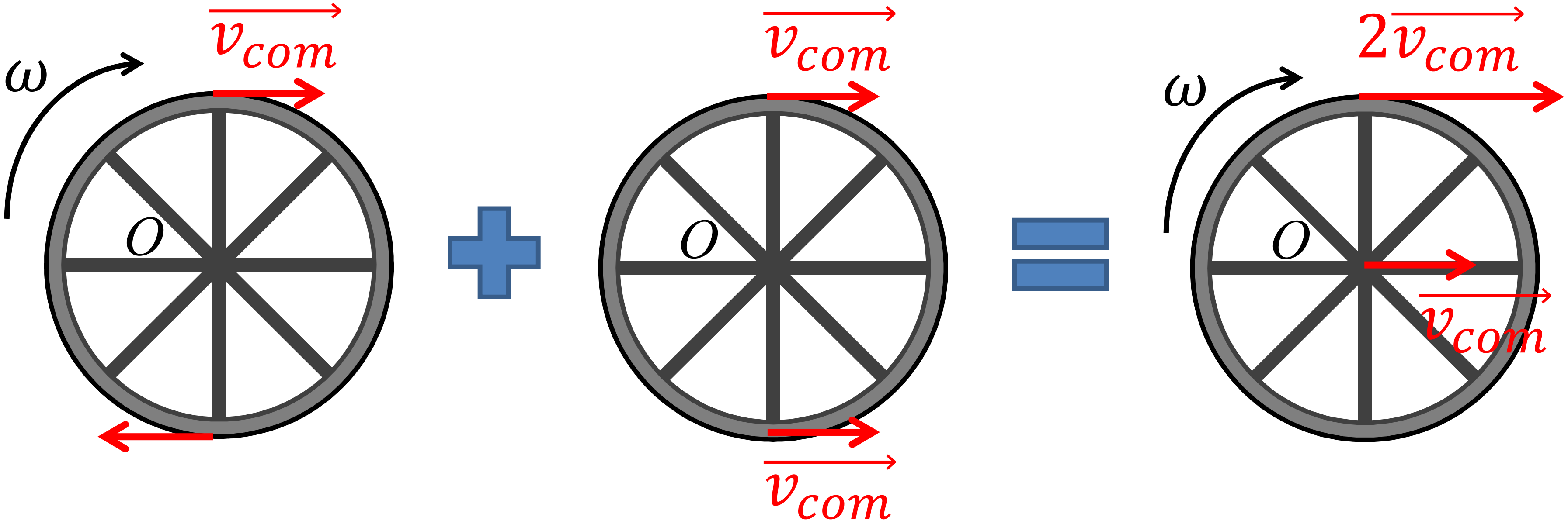
ROLLING AS PURE TRANSLATION + PURE ROTATION



Pure rotation + Pure translation = Rolling motion

Complicated motion can be decomposed into elementary simple ones

THE KINETIC ENERGY OF ROLLING



$K_{rot} = \frac{1}{2} I_{com}\omega^2$ $\qquad$ $K_{tran} = \frac{1}{2} M {v_{com}}^2$

$$\boxed{K = K_{rot} + K_{tran} = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M {v_{com}}^2}$$

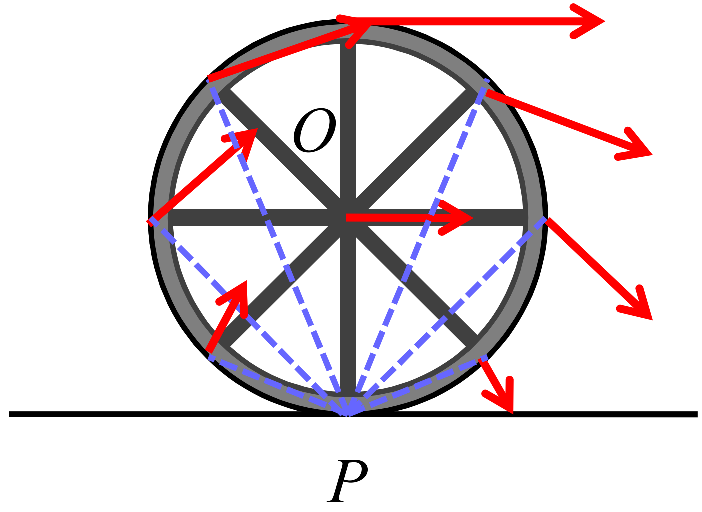
A rolling object has two types of kinetic energy:

1. **a rotational kinetic energy due to its rotation about its center of mass (=½ $I_{com}\omega^2$),**
2. **a translational kinetic energy due to translation of its center of mass (=½ Mv^2_{com})**

THE KINETIC ENERGY OF ROLLING - ANOTHER VIEW

If we view the rolling as pure rotation about an axis through P, then:

$$K = \frac{1}{2} I_P \omega^2$$



(ω is the angular speed of the wheel and I_P is the rotational inertia of the wheel about the axis through P).

Using the parallel-axis theorem ($I_P = I_{com} + Mh^2$): $I_P = I_{com} + MR^2$

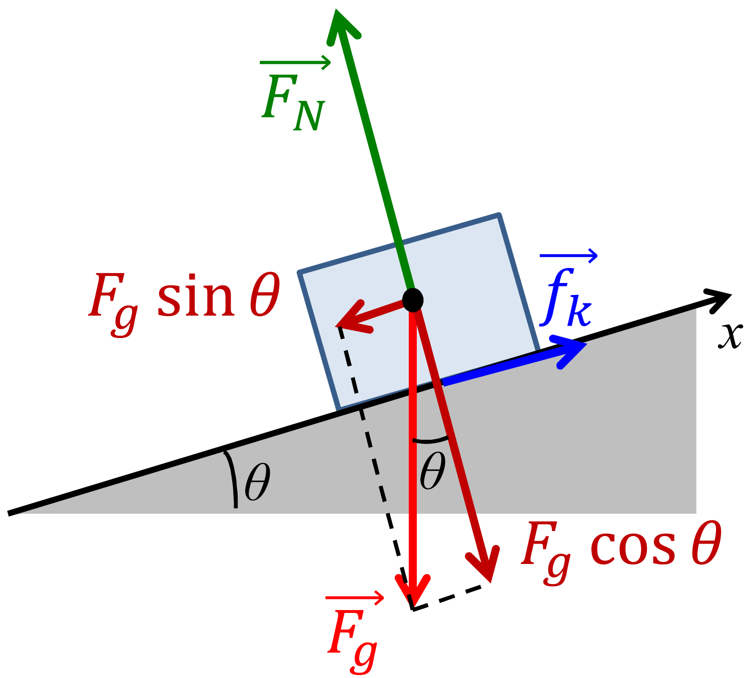
(M is the mass of the wheel, I_{com} is its rotational inertia about an axis through its com, and R is the wheel's radius, at a perpendicular distance h).

Using the relation $v_{com} = \omega R$, we get: $K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M {v_{com}}^2$

ROLLING DOWN A RAMP – FINDING a_{com}

- What about a block (not rolling) down a ramp?
- Depend on m? size?

BLOCK DOWN A RAMP – FINDING a_{com}



Motion: only translation

Linear form (translation of com)

$$f_k - Mg\sin\theta = Ma_{com,x}$$

$$f_k = \mu_k F_N = \mu_k Mg\cos\theta$$

$$a_{com,x} = -g(\sin\theta - \mu_k\cos\theta)$$

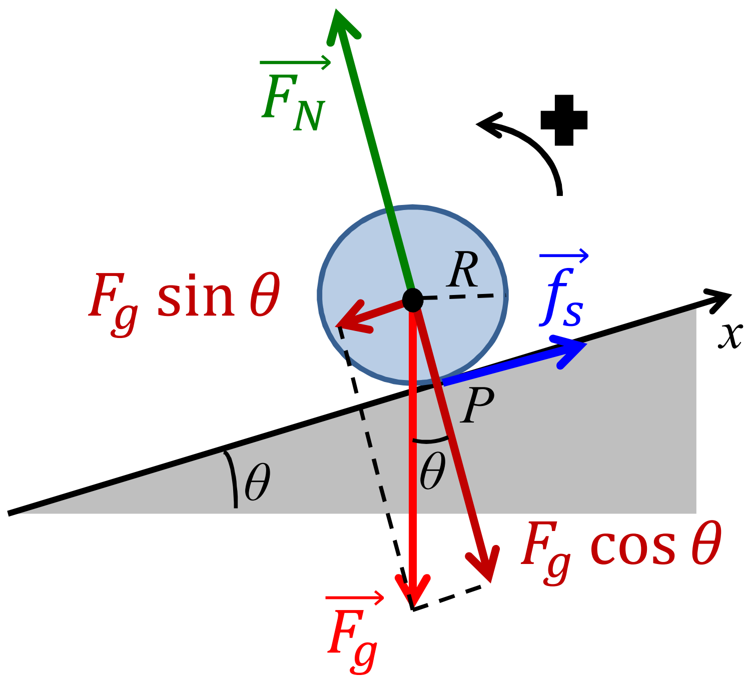
Frictional force is empirical (not fundamental)

Without friction (fundamental theory only):

$$a_{com,x} = -g\sin\theta$$

Acceleration independent of *M* and size

ROLLING DOWN A RAMP – FINDING a_{com}



General motion (translation+rotation)
➔ use Newton's 2nd law in both linear and angular forms

Linear form (translation of com)

$$f_s - Mg\sin\theta = Ma_{com,x} \quad (1)$$

Angular form (about com)

$$\tau_{net} = I\alpha \Rightarrow Rf_s = I_{com}\alpha \; (2)$$

Relation linear – rotational motion

$v_{com} = -R\omega \qquad a_{com} = -R\alpha \quad (3)$

3 unknowns: a_{com}, f_s, α
➔ need a 3rd equation

(2) and (3)

$$f_s = -I_{com}\, {a_{com}}/{R^2}$$

Then, with (1):

$$a_{com} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

ROLLING DOWN A RAMP – FINDING a_{com}

$$a_{com} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

Type	I_{com}	$a_{com,x}$	Rank
Solid cylinder	$\frac{1}{2}MR^2$	$\frac{2}{3}g\sin\theta$	2
Hollow cylinder	MR^2	$\frac{1}{2}g\sin\theta$	3
Solid sphere	$\frac{2}{5}MR^2$	$\frac{5}{7}g\sin\theta$	1

All accelerations are independent of *M* and *R*!

Ranking: rotational inertia at work.

FORMULA SUMMARY

Pure roll: pure rotation + pure translation

$$s = R\theta \qquad v_{com} = R\omega$$

$$K = K_{rot} + K_{tran} = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}{}^2$$