



Lesson 06 Inverse Matrix and Simultaneous Equations

6A

Inverse Matrix

Inverse Matrix

Inverse of a number

Original number $a \longrightarrow$ The inverse number $x = \frac{1}{a} = a^{-1}$ xa = 1

Inverse of a matrix

Original square matrix A

If X satisfies
$$AX = XA = I$$
, then X is an inverse of matrix
Notation : A^{-1}

How to Find the Inverse Matrix (2x2-matrix)

For 2×2 matrix

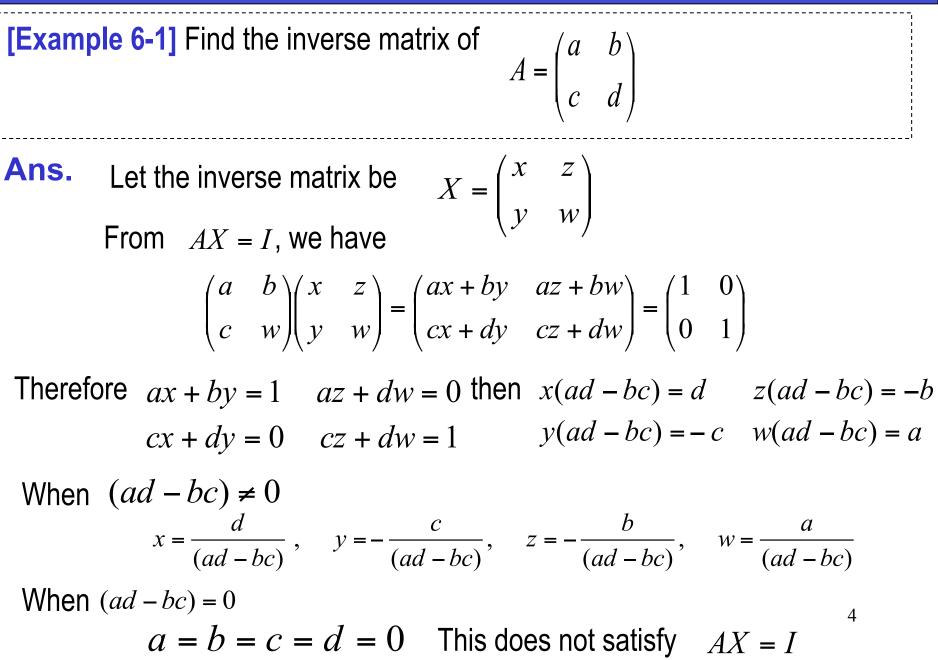
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

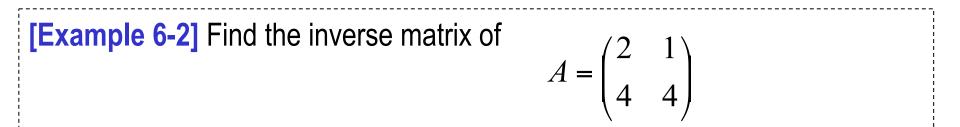
The inverse matrix is given as follows

When
$$|A| = ad - bc \neq 0$$

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$
When $|A| = ad - bc = 0$
$$A^{-1}$$
 does not exist

Proof : Refer to the next example





Ans. The determinant of matrix A

$$|A| = \begin{vmatrix} 2 & 1 \\ 4 & 4 \end{vmatrix} = 2 \cdot 4 - 1 \cdot 4 = 4$$

The inverse matrix

$$A^{-1} = \frac{1}{4} \begin{vmatrix} 4 & -1 \\ -4 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -1/4 \\ -1 & 1/2 \end{vmatrix}$$

[Ex.6-1] Find the inverse of the following matrices.

(1)
$$A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$$
 (2) $A = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex.6-1] Find the inverse of the following matrices. (1) $A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ (2) $A = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$ Ans. (1) The determinant is $|A| = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 3 \times 3 - 4 \times 2 = 1 \neq 0$ Therefore $A^{-1} = \frac{1}{1} \begin{vmatrix} 3 & -4 \\ -2 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -4 \\ -2 & 3 \end{vmatrix}$ The determinant is $|A| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 2 \times 6 - 4 \times 3 = 0$ (2)

Therefore, the inverse does not exist





Lesson 06 Inverse Matrix and Simultaneous Equations

6B

- Simultaneous equation
- •

Simultaneous Equations

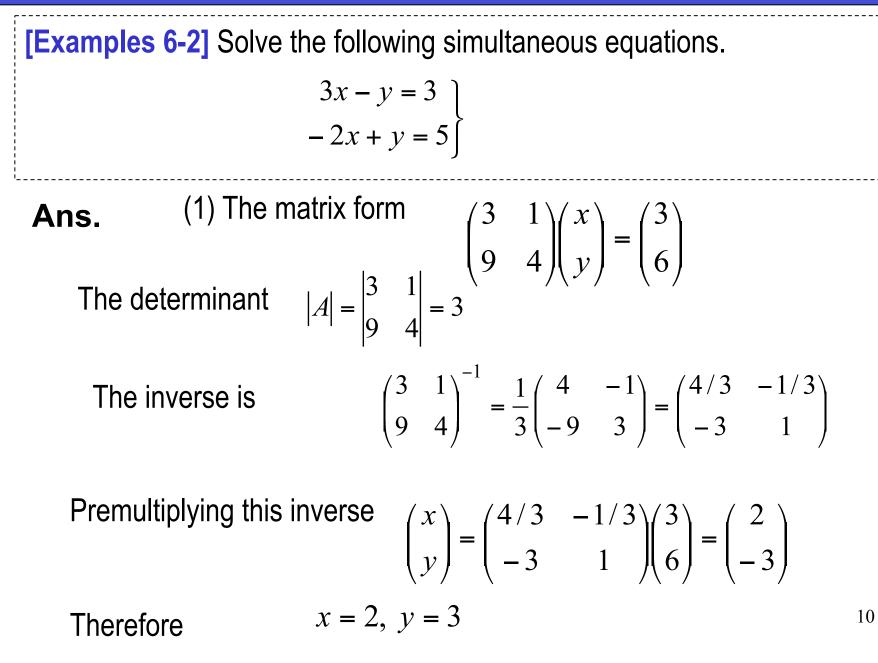
Simultaneous equations

$$ax + by = p \\ cx + dy = q$$

$$\Rightarrow \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$
sing notations
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad P = \begin{pmatrix} p \\ q \end{pmatrix}, \text{ we have}$$

$$AX = P$$

If $ad - bc \neq 0$, we have $A^{-1}AX = A^{-1}P$ $\therefore X = A^{-1}P$ If ad - bc = 0, there exist infinite number of solutions or there is no solution (refer to the example)



[Examples 6-3] Solve the following simultaneous equations. (1) 2x - y = -5 -6x + 3y = 15}
(2) 2x + y = 24x + 2y = 3

Ans. (1) The determinant of coefficients

$$|A| = \begin{vmatrix} 2 & -1 \\ -6 & 3 \end{vmatrix} = 0$$

Dividing the second equation by (-3), 2x - y = -5This is the same as the first equation.

There are infinite number of solution (x, y) which satisfy 2x - y = -5

(2) The determinant of coefficients

$$\left|A\right| = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0$$

4x + 2y = 4

This contradicts to the second equation.

Multiplying the first equation by 2, we have

So, there is no solution

[Ex. 6-2] Solve the following simultaneous equation.

$$3x - y = 3$$
$$-2x + y = 5$$

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex. 6-2] Solve the following simultaneous equation.

$$3x - y = 3$$
$$-2x + 3y = 5$$

Ans. The matrix form is $\begin{pmatrix} 3 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

The determinant of coefficients

$$|A| = \begin{vmatrix} 3 & -1 \\ -2 & 3 \end{vmatrix} = 7 \neq 0$$

The inverse is $\begin{pmatrix} 3 & -1 \\ -2 & 3 \end{pmatrix}^{-1} = \frac{1}{7} \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3/7 & 1/7 \\ 2/7 & 3/7 \end{pmatrix}$

By premultiplying this inverse matrix, we have

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3/7 & 1/7 \\ 2/7 & 3/7 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$
¹³

[Ex. 6-3] Solve the following simultaneous equation. (1) 2x - y = -5 -6x + 3y = 15 (2) 2x + y = 24x + 2y = 3

Pause the video and solve the problem by yourself.

[Ex. 6-3] Solve the following simultaneous equation.

(1)
$$\begin{array}{c} 2x - y = -5 \\ -6x + 3y = 15 \end{array}$$
 (2) $\begin{array}{c} 2x + y = 2 \\ 4x + 2y = 3 \end{array}$

Ans. (1) The determinant of coefficients

$$|A| = \begin{vmatrix} 2 & -1 \\ -6 & 3 \end{vmatrix} = 0$$

Dividing the second equation by (-3), 2x - y = -5This is the same as the first equation.

There are infinite number of solution (x, y) which satisfy 2x - y = -5

(2) The determinant of coefficients

$$\begin{vmatrix} A \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0$$

4x + 2y = 4

This contradicts to the second equation.

Multiplying the first equation by 2, we have

So, there is no solution