NAGOYA UNIVERSITY

Lesson 04
Basic Rules of Matrix

4A
• Definitions of matrices
• Various types of matrices

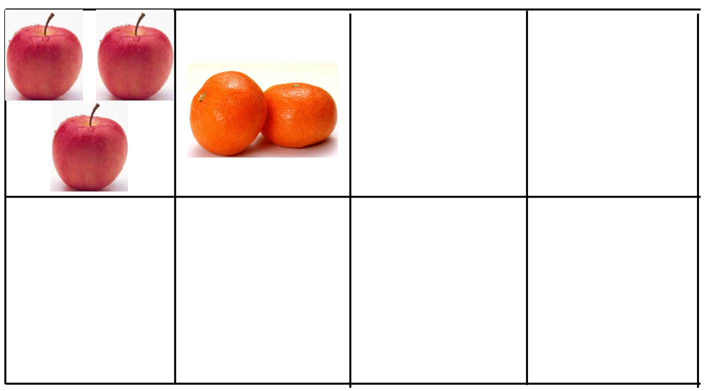
Definition of a Matrix

Definition

- A matrix is a rectangular array of numbers arranged in rows and columns.

<Example>

Arrangement box



Matrix

column

row

$$\begin{pmatrix} 3 & 2 & 2 & 1 \\ 2 & 5 & 3 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}$$

Types of Matrices

Square matrix

The number of rows and that of columns are equal.

$$\begin{pmatrix} 12 & 5 & 7 \\ 5 & 5 & 6 \\ 1 & 3 & 2 \end{pmatrix}$$

Diagonal matrix

A square matrix with all non-diagonal elements zero.

$$\begin{pmatrix} 12 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Row matrix

A matrix with one row

$$\begin{pmatrix} 12 & 5 & 7 \end{pmatrix}$$

Column matrix

A matrix with one column

$$\begin{pmatrix} 12 \\ 5 \\ 1 \end{pmatrix}$$

Types of Matrices (*Contd.*)

Transposed matrix A^T

A matrix $B(= A^T)$ which is obtained by interchanging the j-th row and the i-th column of A

$$b_{ij} = a_{ji}$$

$$A = \begin{pmatrix} 12 & 5 \\ 4 & 1 \\ 3 & 2 \end{pmatrix} \Rightarrow B = A^T = \begin{pmatrix} 12 & 4 & 3 \\ 5 & 1 & 2 \end{pmatrix}$$

Identity matrix

A diagonal matrix with all the diagonal elements are 1.
Notation: I

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Types of Matrices (Contd.)

Symmetric matrix

A matrix which is equal to its transpose.

$$a_{ij} = a_{ji}$$

$$\begin{pmatrix} 12 & 5 & 7 \\ 5 & 4 & 3 \\ 7 & 3 & 1 \end{pmatrix}$$

Skew-symmetric matrix

A matrix which is equal to the negative of its transpose

$$a_{ij} = -a_{ji}$$

$$\begin{pmatrix} 0 & 5 & -7 \\ -5 & 0 & -3 \\ 7 & 3 & 0 \end{pmatrix}$$

Equality of Matrices

Matrices of the same kind

Matrices A and B which have the same number of rows and the same number of columns

$$A = \begin{pmatrix} 12 & 4 & 3 \\ 5 & 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 3 & 3 \\ 5 & 10 & 9 \end{pmatrix}$$

Equal matrices

When the corresponding elements of the matrices of the same kind are the same, it is said they are equal.

$$A = B \iff a_{ij} = b_{ij}$$

Example

[Examples 1-1] Find the values of x, y, u and v

$$\begin{pmatrix} x+y & x-y \\ u-1 & 2v \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Ans. Since the corresponding elements are equal,

$$x+y=1 \qquad x-y=3 \qquad u-1=2 \qquad 2v=4$$

Solving these equations

$$x=2,\ y=-1 \qquad u=3 \qquad v=2$$

Exercise

[Ex.4-1] (1) Find the 2×3 matrix in which elements are given by

$$a_{ij} = 3i - 2j.$$

(2) Solve the following equality

$$\begin{pmatrix} 5 & x \\ 2x & x-y \end{pmatrix} = \begin{pmatrix} x+y & z \\ z^2 & -1 \end{pmatrix}$$

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex.4-1] (1) Find the 2×3 matrix in which elements are given by

$$a_{ij} = 3i - 2j.$$

(2) Solve the following equality $\begin{pmatrix} 5 & x \\ 2x & x-y \end{pmatrix} = \begin{pmatrix} x+y & z \\ z^2 & -1 \end{pmatrix}$

Ans.

(1) $a_{11} = 3\times1-2\times1 = 1 \quad a_{12} = 3\times1-2\times2 = -1 \quad a_{13} = 3\times1-2\times3 = -3$

$a_{21} = 3\times2-2\times1 = 4 \quad a_{22} = 3\times2-2\times2 = 2 \quad a_{23} = 3\times2-2\times3 = 0$

Therefore, $A = \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$

(2) $5 = x+y, \quad x = z, \quad 2x = z^2, \quad x-y = -1$

Therefore, $x = 2, \; y = 3, \; z = 2$

Lesson 04
Basic Rules of Matrix

4B

- Addition of matrices
- Subtraction of matrices
- Scalar multiplication

Benefits of Matrices

Matrix provides the following two merits:

1. **Compact notation** for describing sets of data and sets of equations

2. **Efficient methods for manipulating** sets of data and solving sets of equations.

Addition and Subtraction

Dimension

When a matrix has m rows and n columns, the matrix is said to be of dimension $m \times n$.

Addition and subtraction

Two matrices may be added or subtracted only if they have the same dimensions

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{pmatrix}$$

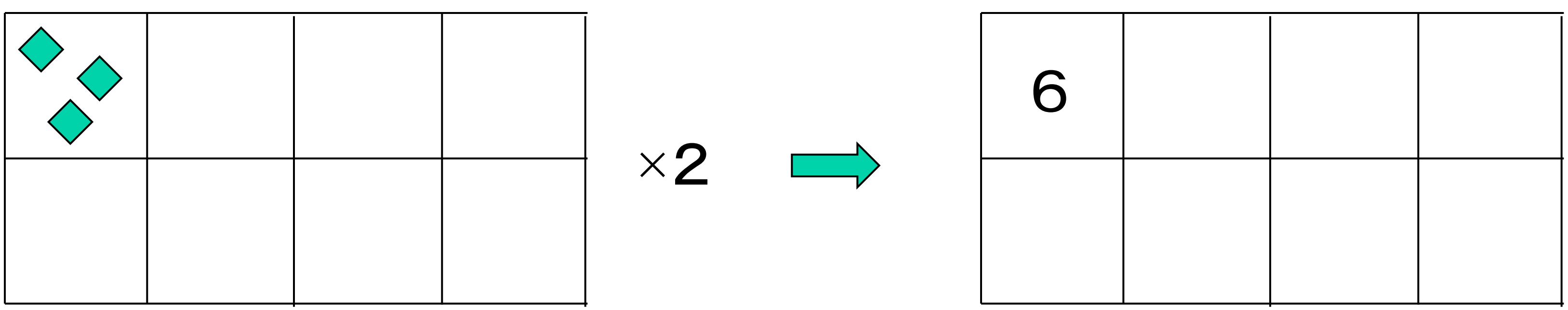
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} - \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} = \begin{pmatrix} a_{11}-b_{11} & a_{12}-b_{12} & a_{13}-b_{13} \\ a_{21}-b_{21} & a_{22}-b_{22} & a_{23}-b_{23} \end{pmatrix}$$

Multiply a Matrix by a Number

Multiple every element in the matrix by the same Number.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{then} \quad kA = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{pmatrix}$$

Arrangement box



Basic Rules of Calculation

If the matrices are those of the same kind, the following rules hold for addition, subtraction and scalar multiplication.

1. Commutative Law

$$A + B = B + A$$

2. Associative law

$$(A + B) + C = A + (B + C)$$

3. Distributive law

$$k(A + B) = kA + kB$$

4 About the scalar multiplication

$$(kl)A = k(lA)$$

Example

[Examples 4-2] When $A = \begin{pmatrix} 2 & -3 & 0 \\ 4 & 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

find the following matrix.

$$2(3A - 2B) - 3(A - B)$$

Ans.

$$2(3A - 2B) - 3(A - B) = 6A - 4B - 3A + 3B = 3A - B$$

$$= 3\begin{pmatrix} 2 & -3 & 0 \\ 4 & 1 & 5 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 4 \\ 2 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 6 & -9 & 0 \\ 12 & 3 & 15 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 4 \\ 2 & 3 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -11 & -4 \\ 10 & 0 & 17 \end{pmatrix}$$

Exercise

[Ex4-2] When $A = \begin{pmatrix} 5 & -1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$, find the matrix which satisfy the following equation .

$$2A + X = B$$

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex4-2] When $A = \begin{pmatrix} 5 & -1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$, find the matrix which satisfy the following equation .

$$2A + X = B$$

Ans.

$$X = B - 2A = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} - 2\begin{pmatrix} 5 & -1 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} - \begin{pmatrix} 10 & -2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} -8 & 3 \\ 1 & -7 \end{pmatrix}$$