NAGOYA UNIVERSITY

Nagoya University G30
Preliminary Lecture Series

Course III : Linear Algebra

Contents

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Lesson 01
Basic Rules of Vectors

1A

- Definitions of vectors
- Basic rules
- Components of vectors

Scalars and Vectors

Scalar

- A scalar : a quantity described by a magnitude.
- Notation : normal italic type alphabet, Greek letters, etc.

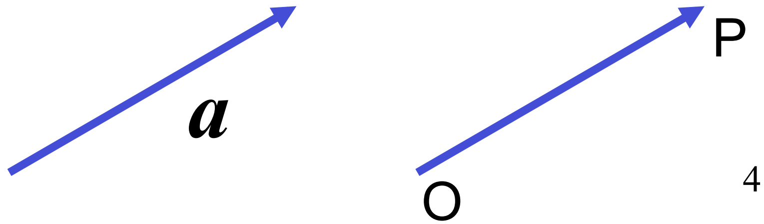
  [Ex.] area A, temperature t, speed v, angle θ

Vector

- A vector : a quantity described by magnitude and direction.

  [Ex.] force, velocity

- A vector is commonly illustrated by "an arrow".
- Typical notation : $\boldsymbol{a}$, $\vec{a}$, $\overrightarrow{\mathrm{OP}}$
- The magnitude of a vector is denoted by $|\boldsymbol{a}|$ or a .

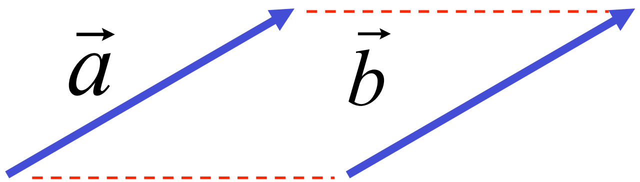


Basic Properties of Vectors

1. Equality $\vec{a} = \vec{b}$

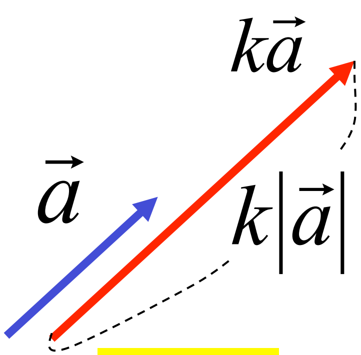
Same magnitude and direction

⇨ they are equal.

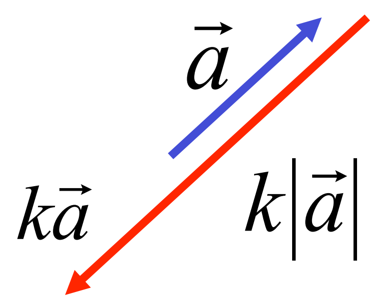


2. Scalar Multiplication $k\vec{a}$

k : a scalar



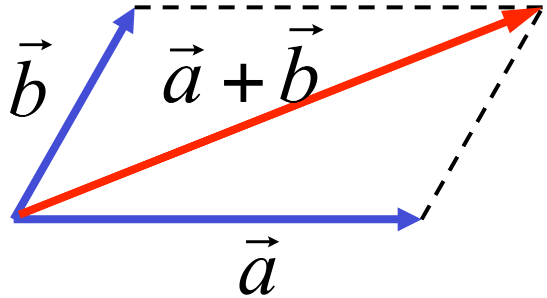
$k > 0$



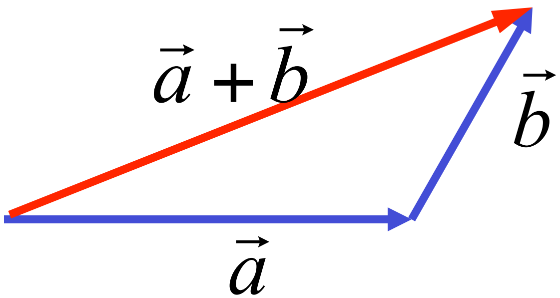
$k < 0$

3. Addition

Sum = the diagonal of the parallelogram



Sum = the closing third side.

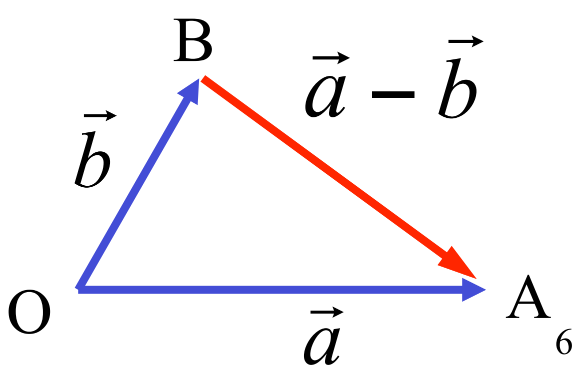


4. Subtraction

$$\vec{b} + \overrightarrow{BA} = \vec{a}$$

Therefore

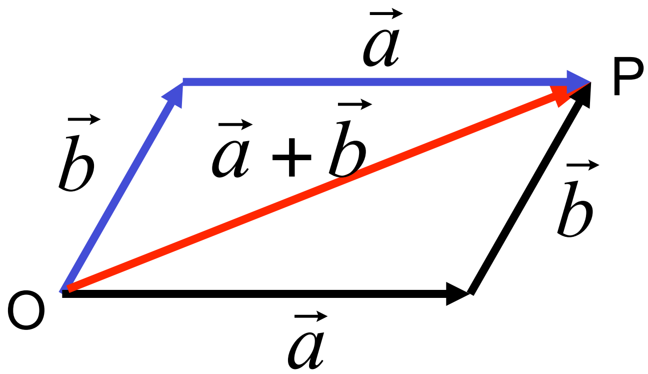
$$\overrightarrow{BA} = \vec{a} - \vec{b}$$



Basic Laws of Vectors

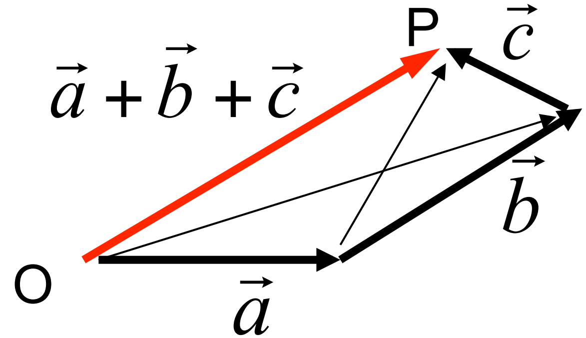
1. Commutative law

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$



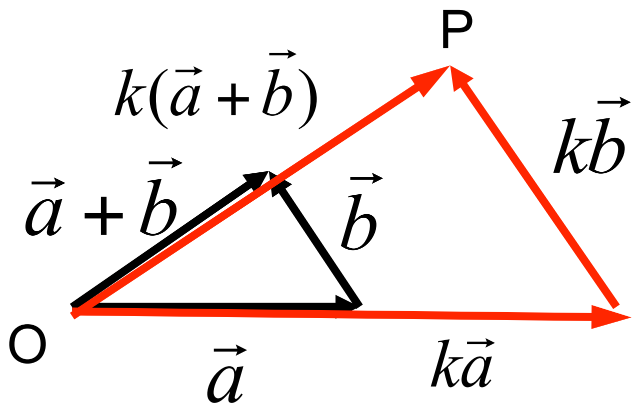
2. Associative law

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$



3. Distributive law

$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$$



Example

[Examples 1-1] Let $\vec{p} = 3\vec{a} + 2\vec{b}$ and $\vec{q} = -2\vec{a} + \vec{b}$. Answer the following questions. (1) Find $\vec{x}$ which satisfies the equation . $3(\vec{x} - \vec{q}) = 2\vec{p} + \vec{x}$

(2) Find $\vec{x}$ and $\vec{y}$ which satisfy

$$\left.\begin{aligned} 2\vec{x} - 3\vec{y} &= \vec{p} \quad \text{(i)} \\ \vec{x} + \vec{y} &= \vec{q} \quad \text{(ii)} \end{aligned}\right\}$$

Ans.

(1) Substituting $\vec{p}$ and $\vec{q}$, we have

$$3\vec{x} - 3\left(-2\vec{a} + \vec{b}\right) = 2\left(3\vec{a} + 2\vec{b}\right) + \vec{x} \qquad \therefore \quad \vec{x} = \frac{7}{2}\vec{b}$$

(2) From (i)－(ii)×2, we have $-5\vec{y} = \vec{p} - 2\vec{q}$

$$\therefore \quad \vec{y} = -\frac{1}{5}\vec{p} + \frac{2}{5}\vec{q} = -\frac{1}{5}(3\vec{a} + 2\vec{b}) + \frac{2}{5}(-2\vec{a} + \vec{b}) = -\frac{7}{5}\vec{a}$$

From (ii), we have

$$\vec{x} = \vec{q} - \vec{y} = (-2\vec{a} + \vec{b}) - (-\frac{7}{5}\vec{a}) = -\frac{3}{5}\vec{a} + \vec{b}$$

Exercise

[Ex.1-1] Find $\vec{x}$ and $\vec{y}$ which satisfy the following equation

$$\left.\begin{aligned} 3\vec{x} + 2\vec{y} &= \vec{a} \qquad \text{(i)} \\ 4\vec{x} - 3\vec{y} &= \vec{b} \qquad \text{(ii)} \end{aligned}\right\}$$

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex.1-1] Find $\vec{x}$ and $\vec{y}$ which satisfy the following equation

$$\left.\begin{aligned} 3\vec{x} + 2\vec{y} &= \vec{a} \\ 4\vec{x} - 3\vec{y} &= \vec{b} \end{aligned}\right\}$$

Ans.

From Eq.(i) $\times 3$, we have $9\vec{x} + 6\vec{y} = 3\vec{a}$

From Eq.(ii) $\times 2$, we have $8\vec{x} - 6\vec{y} = 2\vec{b}$

Adding, we have $17\vec{x} = 3\vec{a} + 2\vec{b}$ $\quad \therefore \ \vec{x} = \dfrac{3}{17}\vec{a} + \dfrac{2}{17}\vec{b}$

$$\therefore \ \vec{y} = -\frac{3}{2}\vec{x} + \frac{1}{2}\vec{a} = -\frac{3}{2}\left(\frac{3}{17}\vec{a} + \frac{2}{17}\vec{b}\right) + \frac{1}{2}\vec{a} = \frac{4}{17}\vec{a} - \frac{3}{17}\vec{b}$$

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Lesson 01
Basic Rules of Vectors

1B

- Components of Vectors

Components of a Vector

Unit Vector

Vector whose length is 1

$$\vec{e} = \frac{\vec{a}}{|\vec{a}|} \qquad |\vec{e}| = 1$$

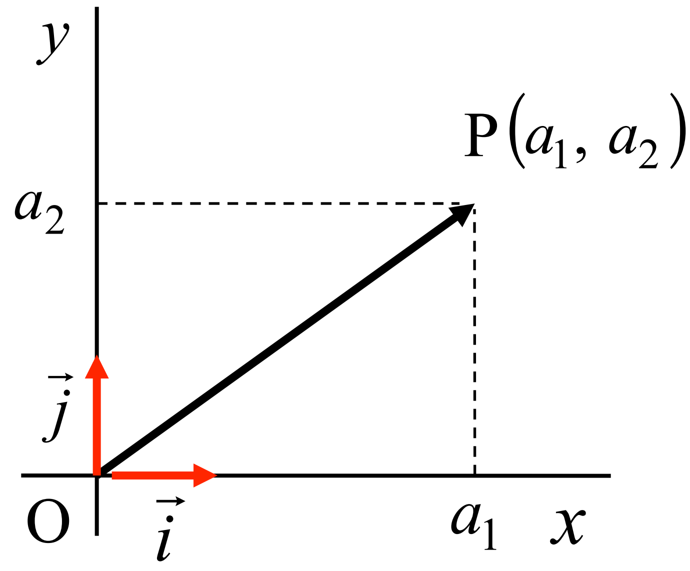
Basic Unit Vector

$$\vec{i} = (1,0) \quad \text{and} \quad \vec{j} = (0, 1)$$

Components of Vector

$$\vec{a} = a_1\vec{i} + a_2\vec{j} = (a_1, a_2)$$

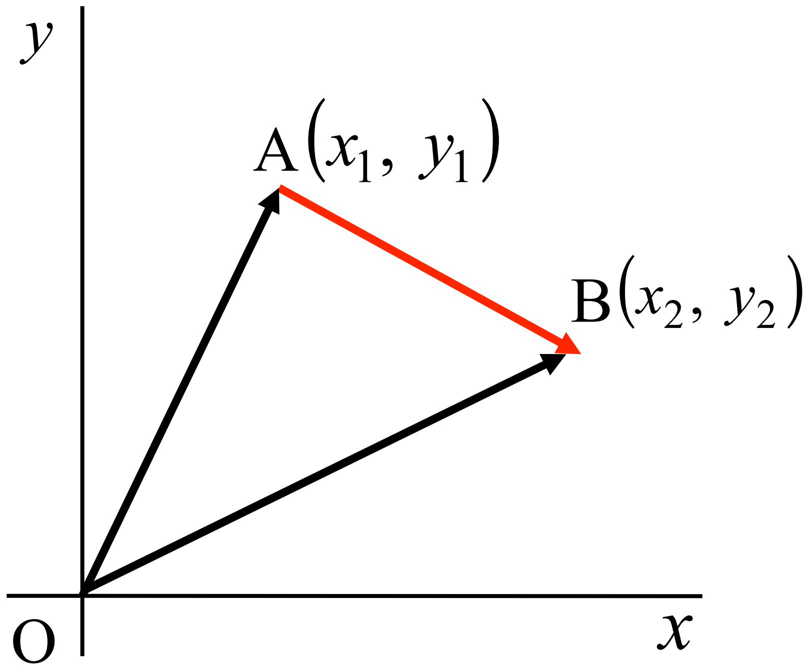
Component



Vector Connecting Two Points

Vector Connecting A and B

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= (x_2, y_2) - (x_1, y_1)$$
$$= (x_2 - x_1, y_2 - y_1)$$

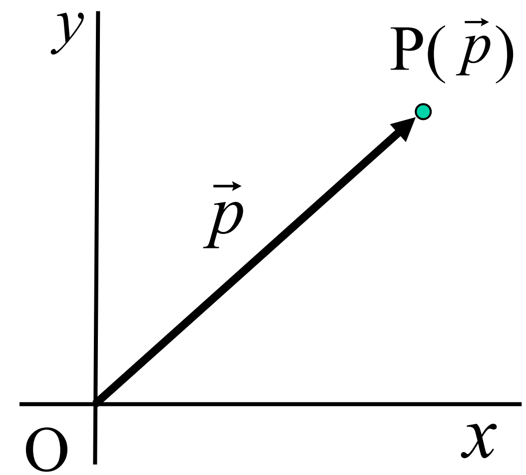


Length AB

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

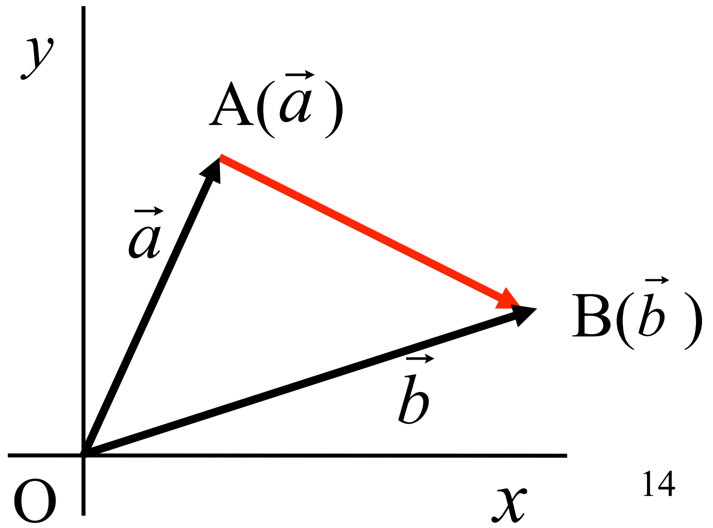
Position Vector

If we select the initial point of the vector at the origin, a point is designated by a vector.



Vector Connecting A and B

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$



Example

[Examples 1-2] Find the position vector of point C which divide the line connecting A($\vec{a}$) and B($\vec{b}$) internally in the ratio $m:n$

Ans.

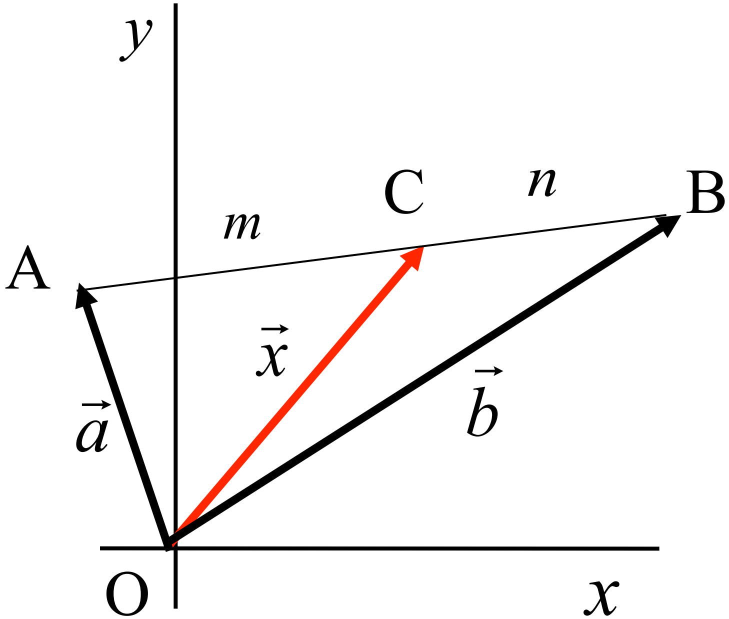
$\overrightarrow{AC}$ and $\overrightarrow{CB}$ have the same direction.

Magnitudes $\left|\overrightarrow{AC}\right| : \left|\overrightarrow{CB}\right| = m:n$

Therefore

$$n(\vec{x}-\vec{a}) = m(\vec{b}-\vec{x})$$

$$\therefore \quad \vec{x} = \frac{n\vec{a}+m\vec{b}}{(m+n)}$$



Exercise

[Ex1-2] Find the position vector $\vec{g}$ of the center of gravity of the Δ ABC. The position vectors of A, B, and C are $\vec{a}$, $\vec{b}$ and $\vec{c}$.
[Note] The center of gravity is given by the point which divide the line AM by the ratio 2:1 where M is the center of side BC.

Ans.

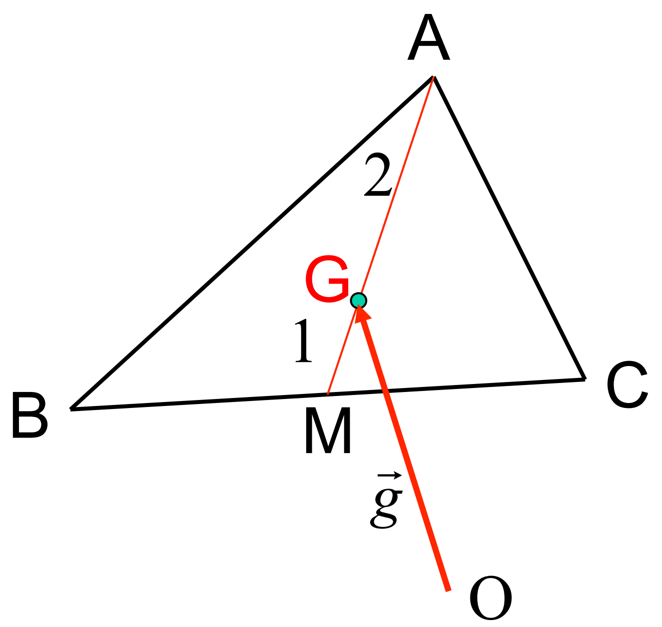
Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex1-2] Find the position vector $\vec{g}$ of the center of gravity of the Δ ABC. The position vectors of A, B, and C are $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Ans.

The center of side BC is $\vec{m} = \dfrac{\vec{b}+\vec{c}}{2}$



Since the center of gravity G divide the line AM internally in the ratio 2:1, we have

$$\vec{x} = \frac{\vec{a}+2\vec{m}}{2+1} = \frac{\vec{a}+2\left(\dfrac{\vec{b}+\vec{c}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$