Course I

NAGOYA UNIVERSITY

Lesson 5
Trigonometric Functions (II)

5A

- **Radian – Another Unit of Angle**
- **Graphs of Trigonometric Functions**

Radian

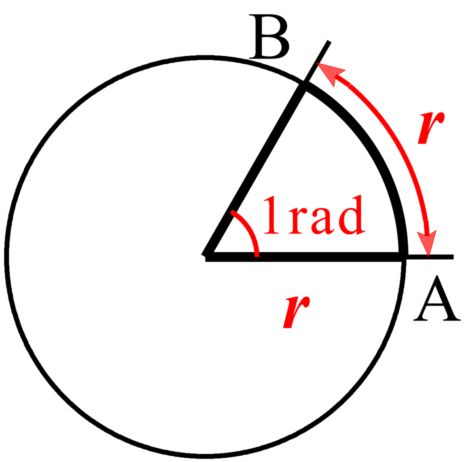
Degree (°)

- Angle of 1/360 of one circle
- 360 is a familiar number in astronomy. (One year = 365day ≈360)

Radian (non-dimention)

- Angle is described by the ratio of the arc to the radius.

$$360° \Leftrightarrow \quad 2\pi = \frac{2\pi r}{r} \quad \begin{matrix} \leftarrow \text{Arc} \\ \leftarrow \text{Radius} \end{matrix}$$



- A pure number (no unit) but symbol "rad" is used.
- 1rad=57.29••• (Memorize 360°=2π rad.)

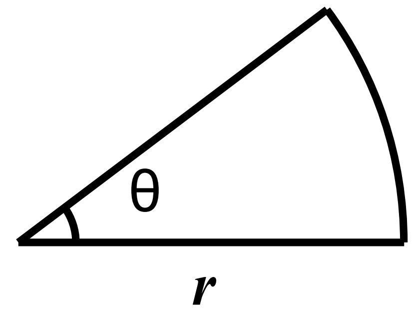
Merits of Radian

Example 1: ***Expression becomes simple***.

Area of a sector with angle θ and radius *r*

θ[degree] : $\pi r^2 \dfrac{\theta}{360}$

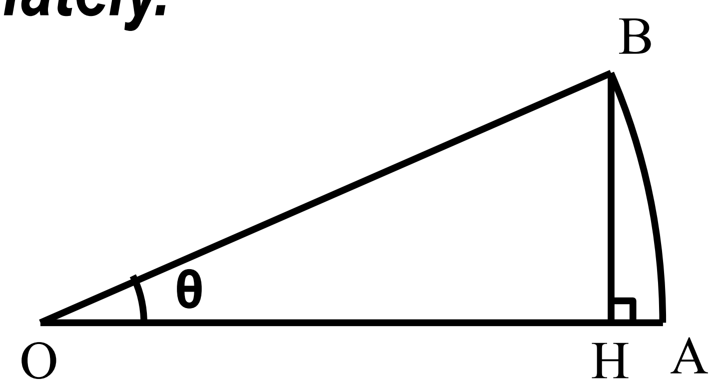
θ[rad] : $r^2 \dfrac{\theta}{2} \quad \left(= \pi r^2 \dfrac{\theta}{2\pi} \right)$



Example 2: ***Values of trigonometric functions of a small angle can be obtained approximately.***

When angle θ is small

$$\sin\theta = \frac{\mathrm{BH}}{\mathrm{OB}} \approx \frac{\mathrm{arcAB}}{\mathrm{OB}} = \theta$$

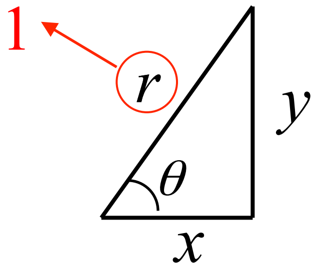


EX.

$$\sin\frac{\pi}{180} \approx \frac{\pi}{180} \approx \frac{3.1416}{180} = 0.01745$$

(From Table, 1deg ≈ 0.0175)

Graphs of the Sine/Cosine Functions

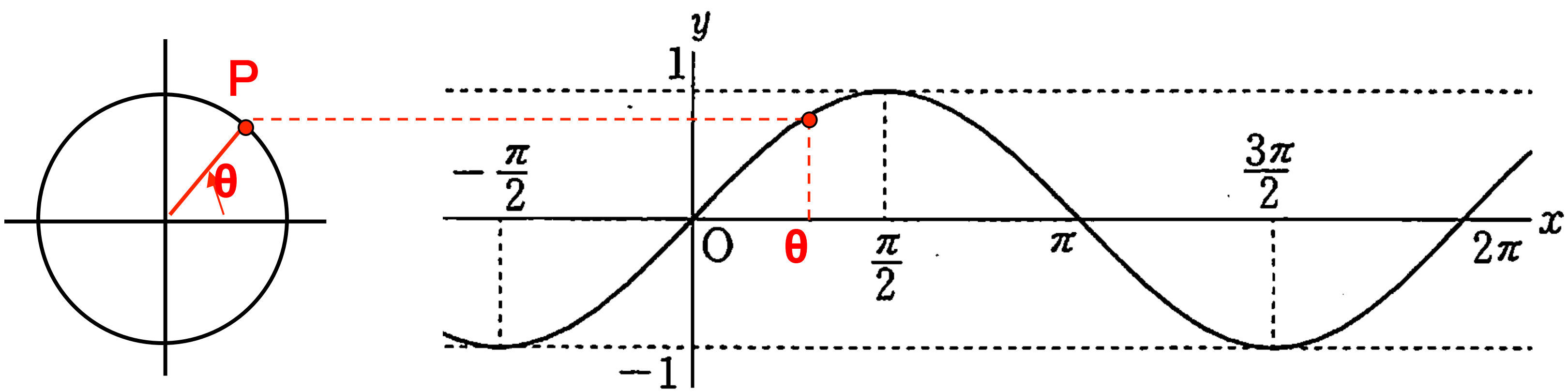


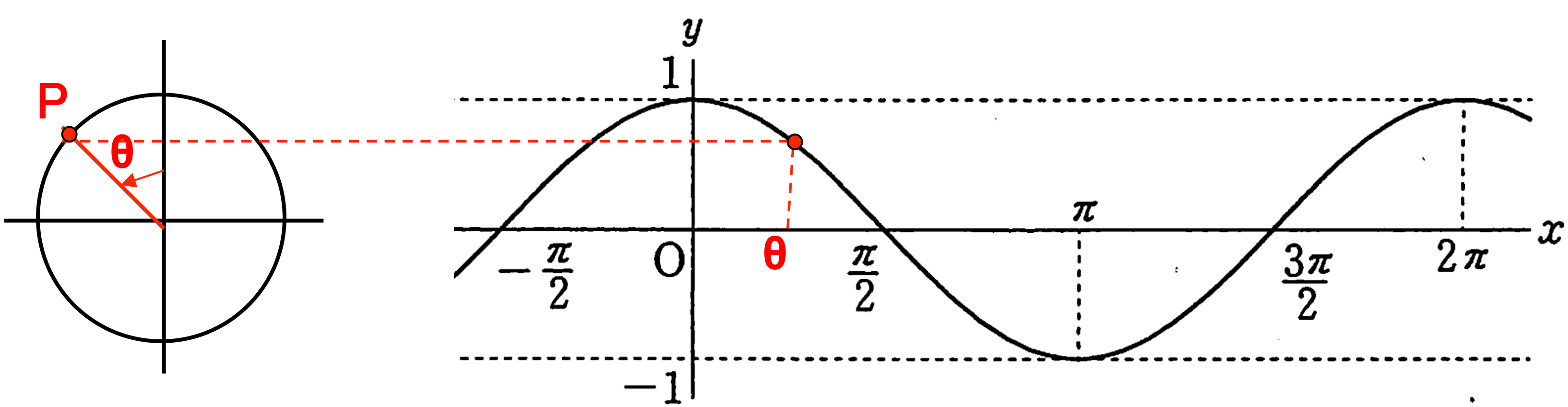
$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

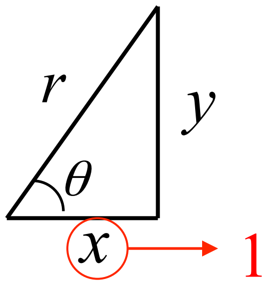
$$y = r\sin\theta \;\xrightarrow{r=1}\; y = \sin\theta$$

$$x = r\cos\theta \;\longrightarrow\; x = \cos\theta$$





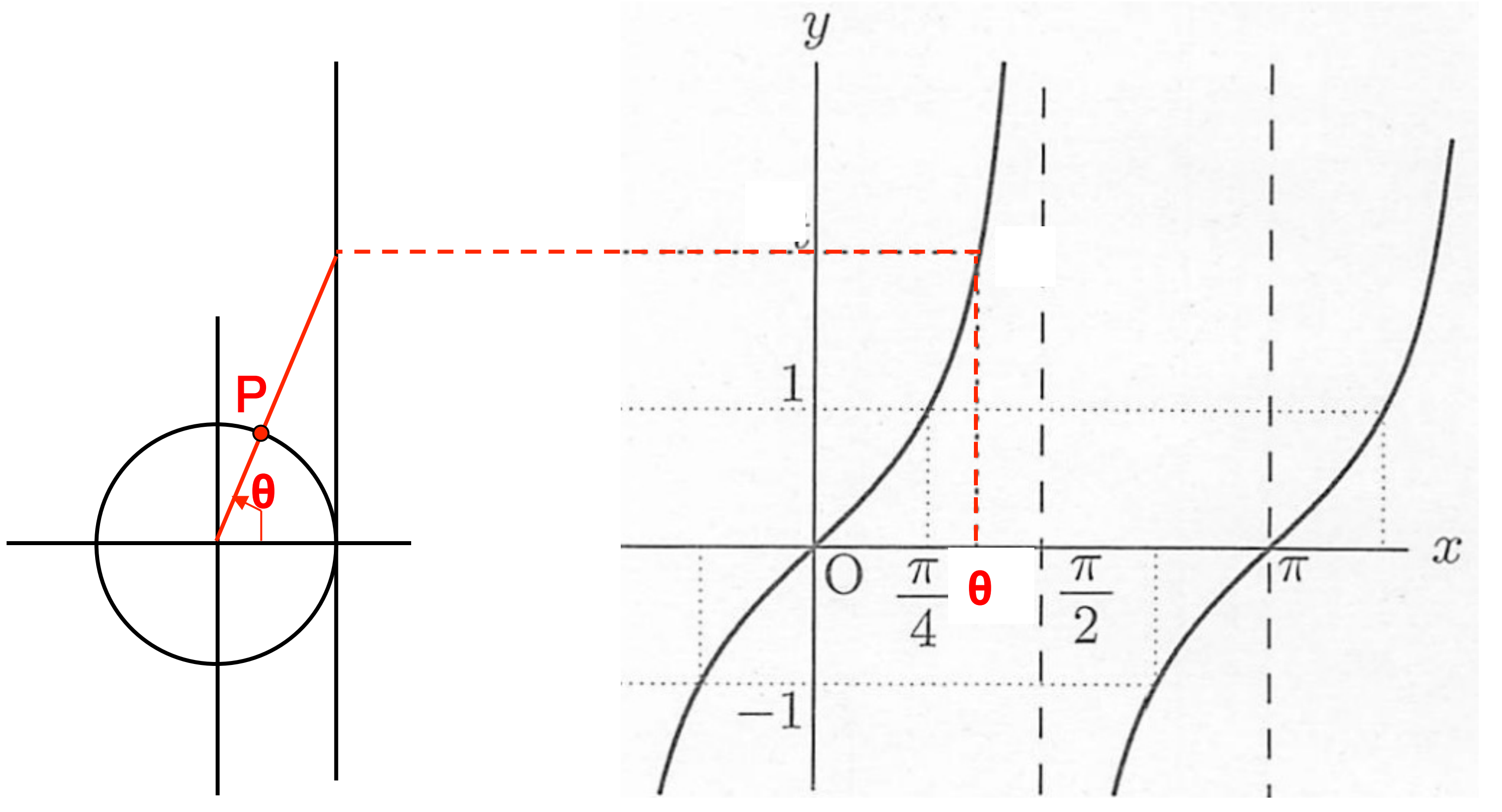
Graph of the Tangent Function



$$\tan\theta = \frac{y}{x}$$

$x = 1$

$$y = \tan\theta$$



Periodic function

A function $f(x)$ is said to be periodic with period p if we have

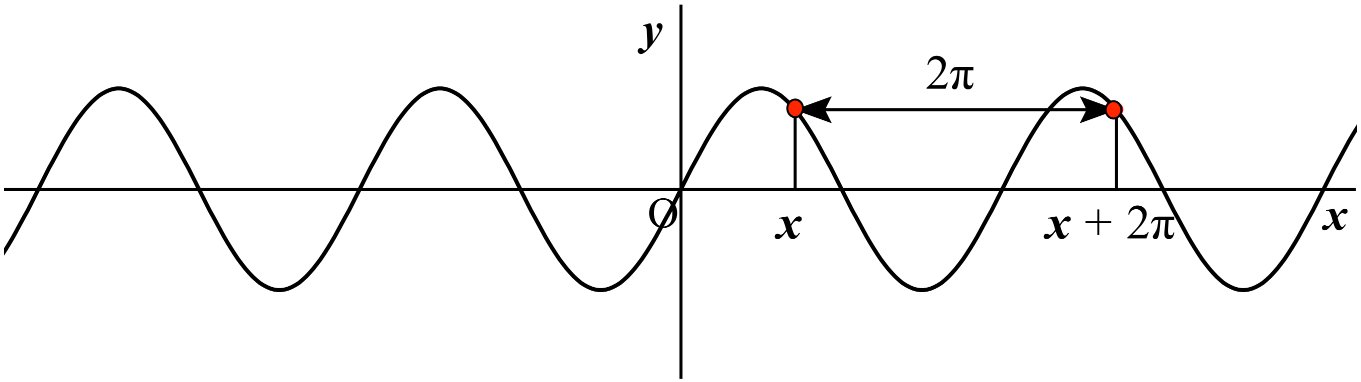
$$f(x+p)=f(x)$$

Namely, the values of a function repeat themselves regularly.

Examples 1 Find the period of the sine functions $y=\sin x$

$$\sin(x+2\pi)=\sin x$$

Period =2π



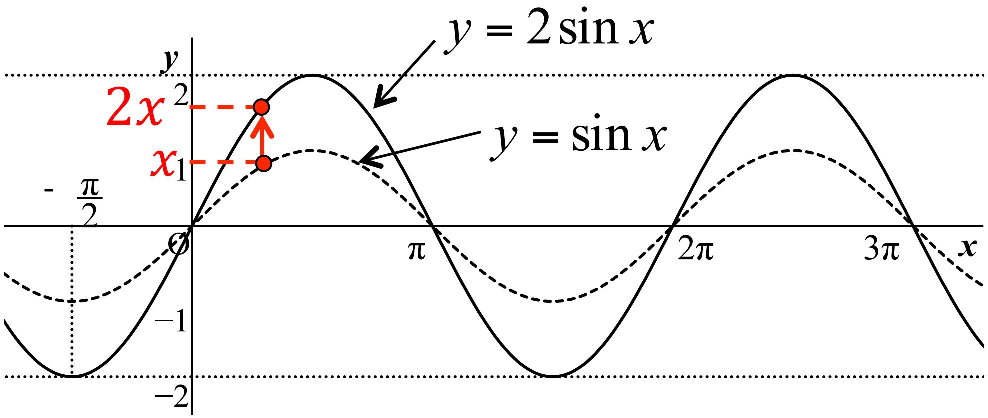
Example

Example 2. Illustrate the following functions and show their periods

(1) $y = 2\sin x$ (2) $y = \sin 2x$ (3) $y = \sin\left(x - \dfrac{\pi}{3}\right)$

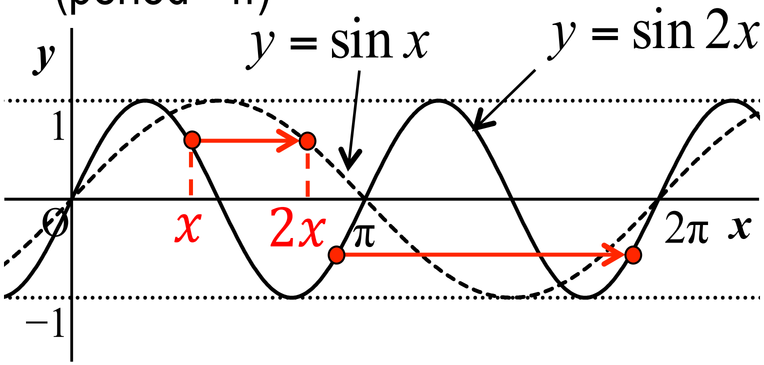
Ans. (1) Expansion in the y-direction

(period =2π)

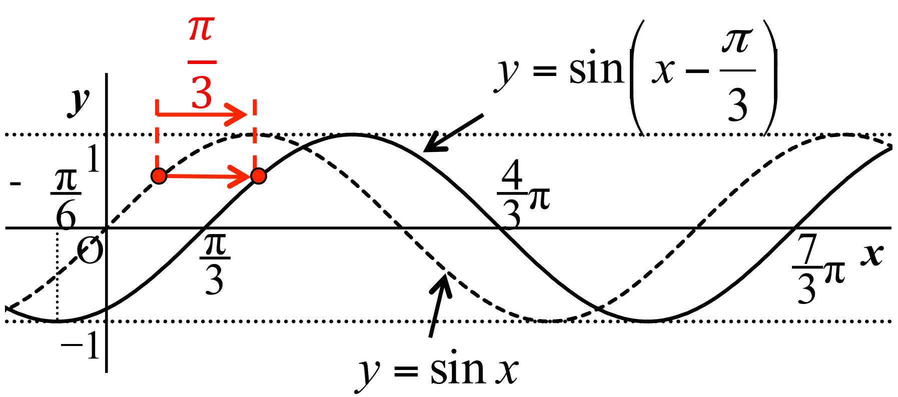


(2) Expansion in the x -direction

(period =π)



(3) Shift in the x-direction (period =2π)



Exercise

Exercise 1. Answer about the following function

$$y = 2\sin(2x - \frac{\pi}{3}) \qquad (0 \le x \le 2\pi)$$

(1) When dos this function becomes zero ? (2) What are the values of this function at $x=0, 2\pi$ (3) Illustrate this function.

Ans.

Pause the video and solve the problem.

Exercise

Exercise 1. Answer about the following function

$$y = 2\sin(2x - \frac{\pi}{3}) \qquad (\,0 \le x \le 2\pi)$$

(1) When does this function become zero ? (2) What are the values of this function at x=0, 2π (3) Illustrate this function.

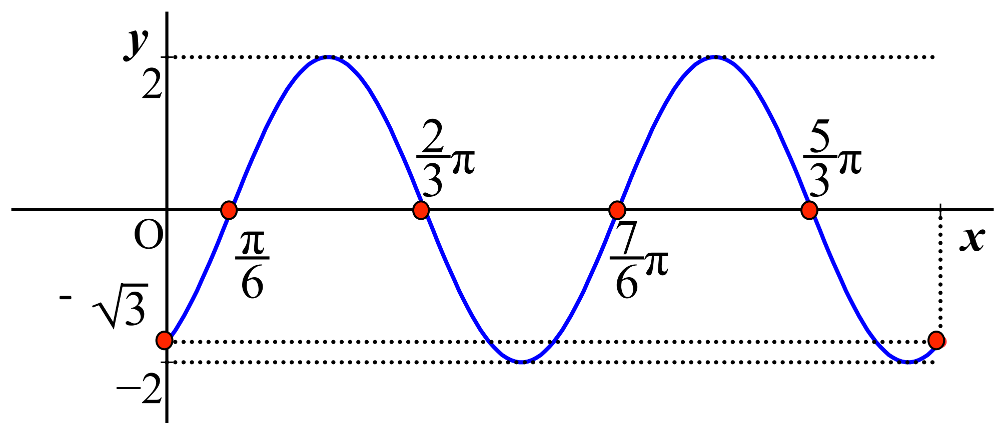
Ans. (1) $0 \le x \le 2\pi, \quad \therefore\ 0 \le 2x \le 4\pi, \quad \therefore\ -\frac{\pi}{3} \le 2x - \frac{\pi}{3} \le 4\pi - \frac{\pi}{3}$

Therefore y becomes zero at $2x - \frac{\pi}{3} = 0,\ \pi,\ 2\pi,\ 3\pi \quad \therefore x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

(2) At $x = 0: \quad y = 2\sin(-\frac{\pi}{3}) = -\sqrt{3}$

At $x = 2\pi: \quad y = 2\sin(4\pi - \frac{\pi}{3}) = 2\sin(-\frac{\pi}{3}) = -\sqrt{3}$

(3)



Ahh! That's so easy!

Course I

NAGOYA UNIVERSITY

Lesson 5
Trigonometric Functions **(II)**

5B

- **Trigonometric Equation**
- **Trigonometric Inequality**

Trigonometric Equation

A trigonometric equation is any equation that contains unknown trigonometric function.

Ex. $2\sin^2 x + 3\cos x - 3 = 0$

- This kind of equation is true for certain angles.

  [Note] A trigonometric equation that holds true for any angle is called a trigonometric identity, which we will study next lesson.

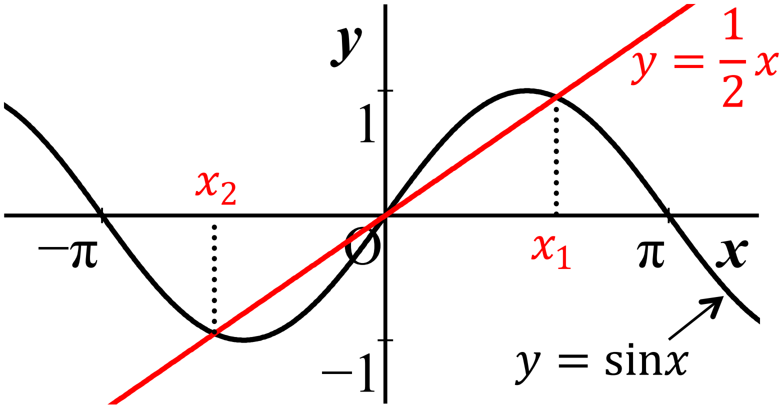
- Some trigonometric equation can be solved *easily* by using algebra ideas, while others may not be solved exactly but *approximately*.

Example 1 $2\sin x - 1 = 0$

This can be easily solved.
See next slide.

Example 2 $2\sin x - x = 0$

Roots x_1 and x_2 can be found numerically (See the figure).



Example

Example 1. Solve the following trigonometric equation. $2\sin x - 1 = 0$

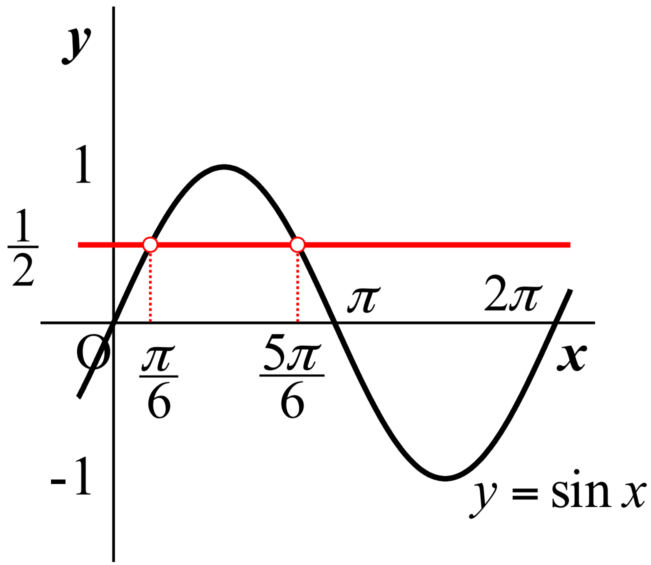
Ans.

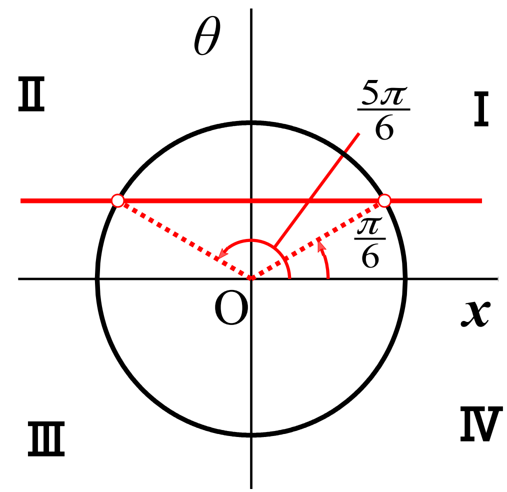
Step 1 We first look at $\sin x$ as being the variable of the equation a

solve as we did before. $\therefore \sin x = \frac{1}{2}$

Step 2 Recall the graph of $y = \sin x$ from 0 to 2π or a unit circle, and ob

the value of x which satisfy this expression. $\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$





Step 3 Considering the periodicity, add $2n\pi$ $\therefore x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

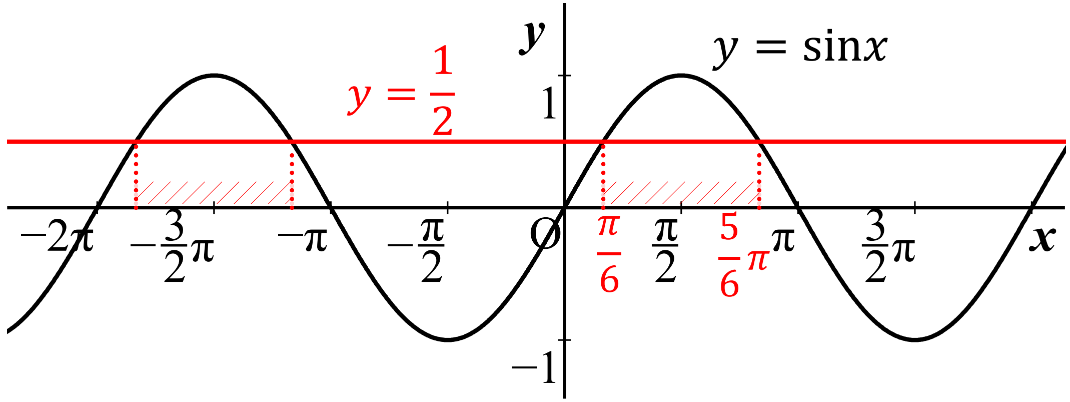
Trigonometric Inequality

A trigonometric inequality is any inequality that contains unknown trigonometric function. It can be solved based on a trigonometric e

Example 2. Solve the following trigonometric equation. $2\sin x - 1 > 0$

Ans. **Step 1** Convert the given inequality to a trigonometric equation by replacin sign to equality sign. $2\sin x - 1 = 0$

Step 2 Solve the resulting equation in the interval $[0, 2\pi]$: $\pi/6, \quad 5\pi/6$



Step 3 Among intervals divided by the obtained roots, find the intervals wh satisfy the trigonometric inequality. $\frac{\pi}{6} < x < \frac{5\pi}{6}$

Step 4 Extend the soltion to the whole domain. $\frac{\pi}{6} + 2n\pi < x < \frac{5\pi}{6} + 2n\pi$

Exercise

Exercise 1. Solve the following trigonometric equation.

$$2\sin^2 x + 3\cos x - 3 = 0$$

Ans.

Pause the video and solve the problem.

Exercise

Exercise 1. Solve the following trigonometric equation.

$$2\sin^2 x + 3\cos x - 3 = 0 \qquad 0 \le x \le 2\pi$$

Ans.

$$2\sin^2 x + 3\cos x - 3 = 0$$

$$\sin^2 x + \cos^2 x = 1$$

Put $X = \cos x$

$\therefore\ 2(1 - X^2) + 3X - 3 = 0 \quad \therefore\ 2X^2 - 3X + 1 = 0 \quad \therefore\ (X-1)(2X-1) = 0$

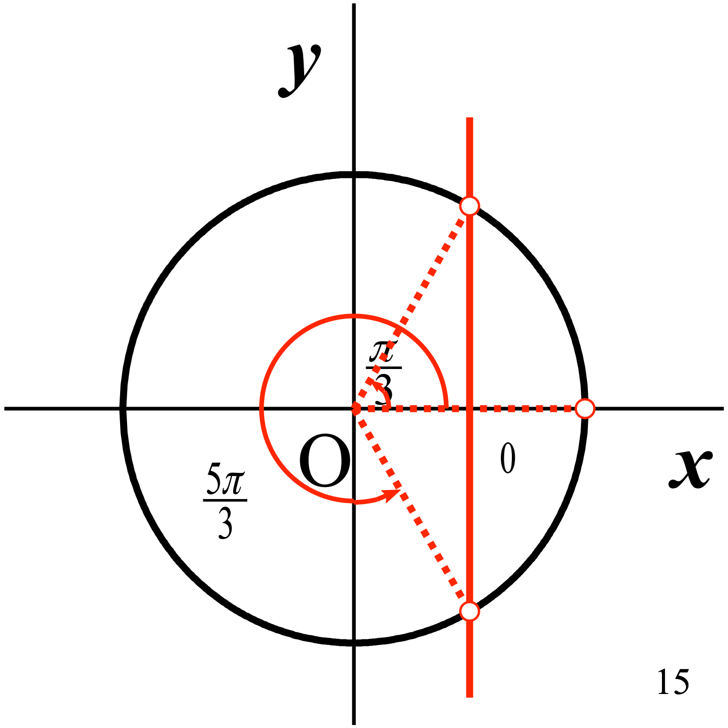
$\therefore\ X = 1,\quad X = \dfrac{1}{2}$

From $\cos x = 1$

$\therefore\ x = 0,\ 2\pi$

From $\cos x = \dfrac{1}{2}$

$\therefore\ x = \dfrac{\pi}{3},\quad \dfrac{5\pi}{3}$



Exercise

Exercise 2. Solve the following trigonometric inequality $\tan x \geq -\sqrt{3}$

Ans.

Pause the video and solve the problem.

Answer to the Exercise

Exercise 2. Solve the following trigonometric inequality $\tan x \geq -\sqrt{3}$

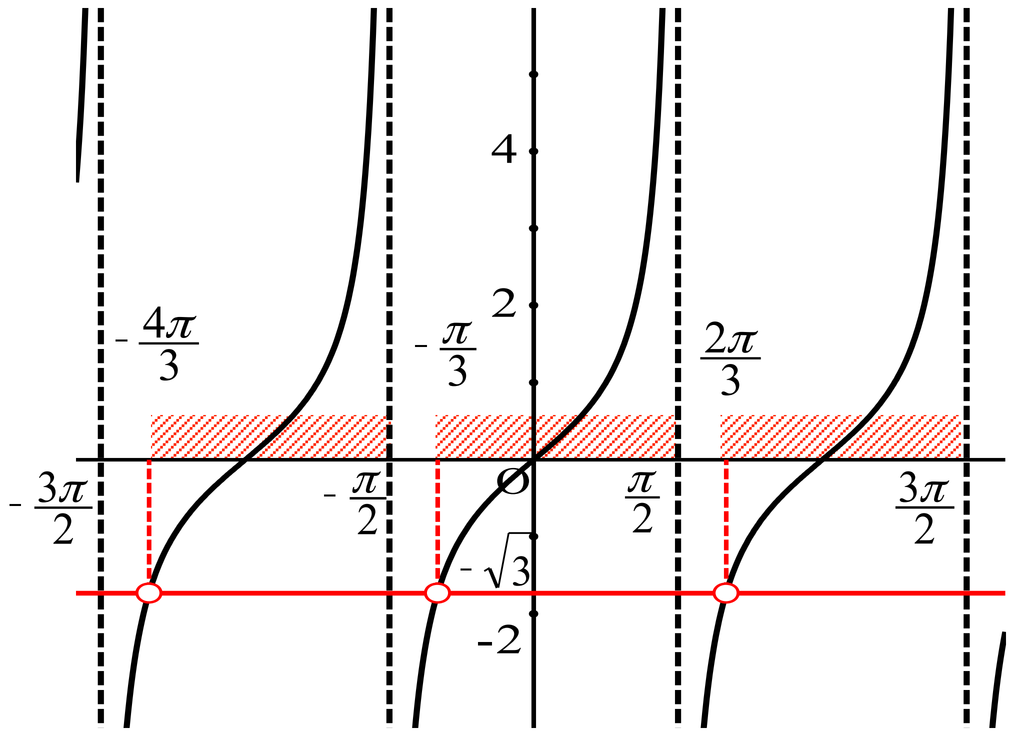
Ans.

The corresponding trigonometric equation is

$$\tan x = -\sqrt{3}$$

Tangent has the period π as shown in the figure. In the interval $[-\pi/2, \pi/2]$, the root is

$$x = -\frac{\pi}{3}$$



From the graph and considering
the periodicity, the solution
is $-\frac{\pi}{3} + n\pi \leq x < \frac{\pi}{2} + n\pi$