Course I

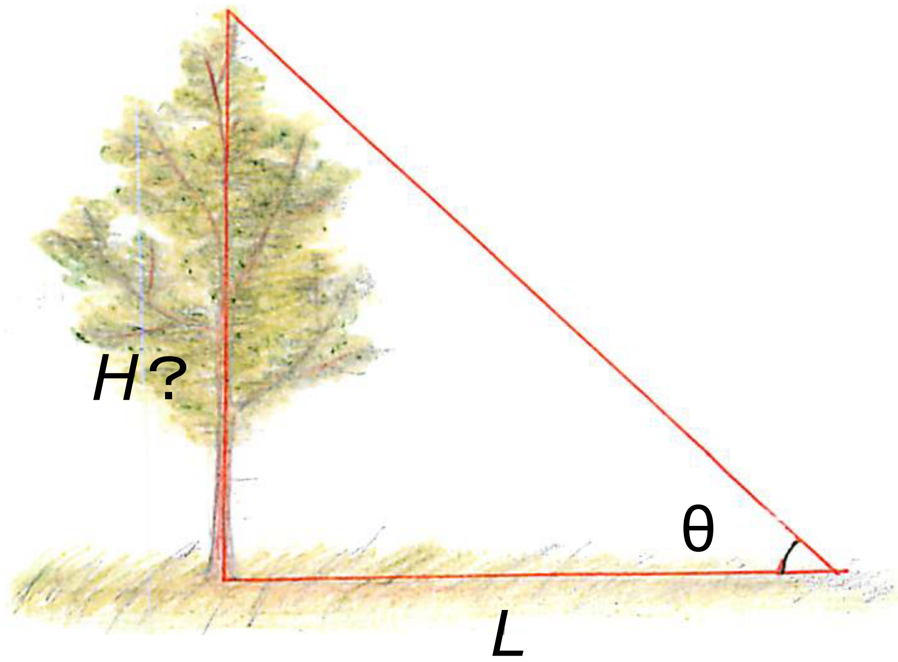
NAGOYA UNIVERSITY

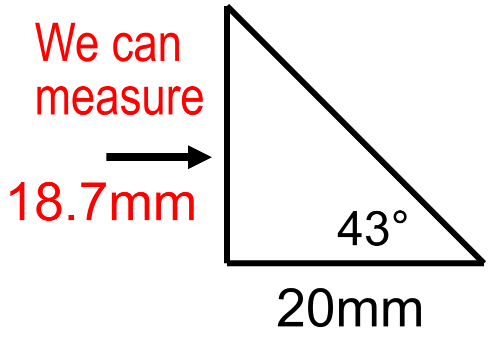
Lesson 4
Trigonometric Functions (I)

4A

- Trigonometric Functions of Acute Angles
- Some Relations Among Trigonometric Functions
- Reciprocal Form of $\sin\theta$, $\cos\theta$, $\tan\theta$

Measurement of Height





Problem

What is the height H of the tree ?
(We can measure the distance L and the angle of elevation θ.)

Answer

Draw a similar right triangle withθ and determine *H* by proportionality.

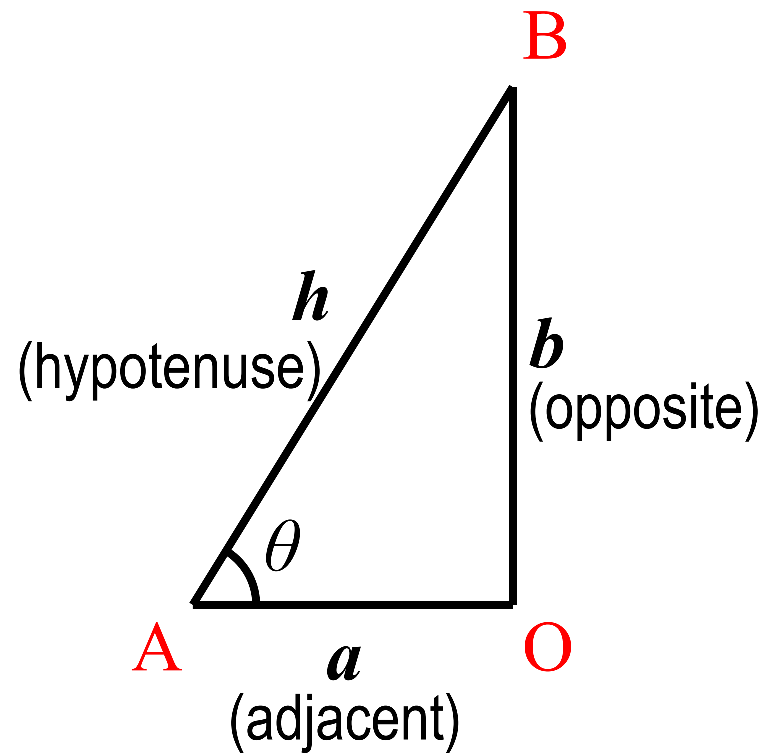
Trigonometric Function of an Acute Angle

Trigonometric Functions

The sine function $\sin\theta = \dfrac{b}{h}$

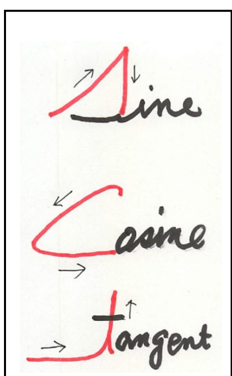
The cosine function $\cos\theta = \dfrac{a}{h}$

The tangent function $\tan\theta = \dfrac{b}{a}$



[Note] How to memorize the definition.

Japan

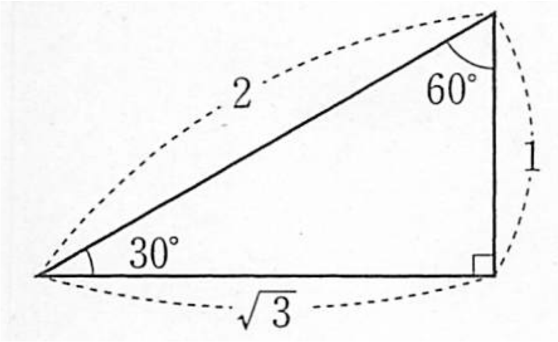


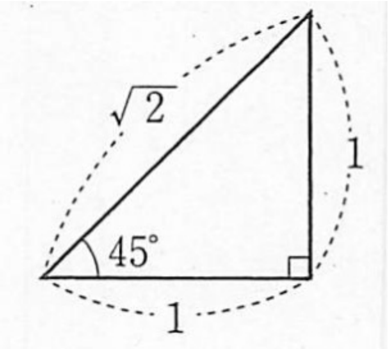
Overseas

SOH-CAH-TOA

Special Trigonometric Ratios

Special Cases to Memorize





$$\sin 30° = \frac{1}{2} \qquad \sin 45° = \frac{1}{\sqrt{2}} \qquad \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \cos 45° = \frac{1}{\sqrt{2}} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} \qquad \tan 45° = 1 \qquad \tan 60° = \sqrt{3}$$

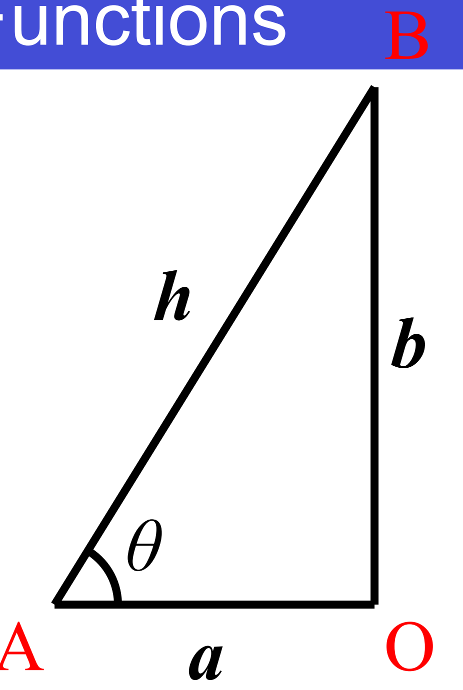
Some Relationship Among Tri. Functions

From the definition

$$\sin\theta = \frac{b}{h}$$

$$\cos\theta = \frac{a}{h} \quad \Longrightarrow \quad \boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}} \quad (1)$$

$$\tan\theta = \frac{b}{a}$$



From Pythagorean theorem

$$a^2 + b^2 = h^2 \quad \Longrightarrow \quad \boxed{\sin^2\theta + \cos^2\theta = 1} \quad (2)$$

From these two relationship

$$(1), (2) \quad \Longrightarrow \quad \boxed{1 + \tan^2\theta = \frac{1}{\cos^2\theta}}$$

[Note] $(\sin\theta)^2$, etc. is often written as $\sin^2\theta$, etc.

Reciprocal Form of sinθ, cosθ, tanθ

The following trigonometric functions are also used.

The cosecant function of angle θ is a reciprocal of sinθ and written as

$$\csc\theta = \frac{1}{\sin\theta}$$

The secant function of angle θ is a reciprocal of cosθ and written as

$$\sec\theta = \frac{1}{\cos\theta}$$

The cotangent function of angle θ is a reciprocal of tanθ and written as

$$\cot\theta = \frac{1}{\tan\theta}$$

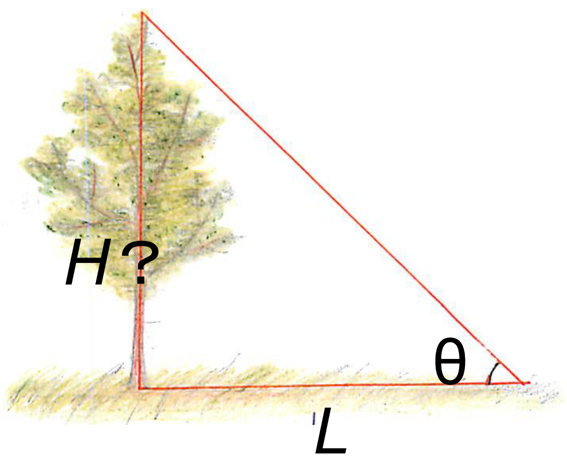
Example 1. In the first slide, answer the following.

(1) We measured the angle of elevation at the distance 20 m and obtained 43 degree. Then, we depicted a similar small right triangle with **θ**=43 degrees and the adjacent of 20mm. By measuring the opposite of this small angle and obtained that it was 18.7mm. What is the height of the tree ?

(2) We searched the position where the angle of elevation is 60 degrees. Then, we measured the distance between this position and the tree and knew that it is 10.8m. What is he height of this tree ?

Ans. (1)

$$20 \times \frac{18.7}{20} = 18.7 \quad \text{m}$$

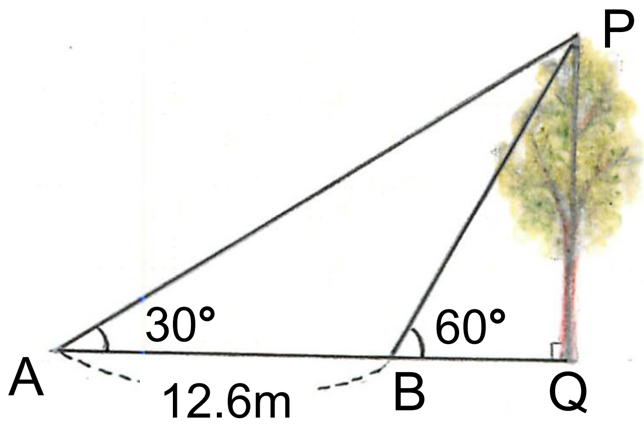


(2)

$$\tan\theta = \frac{H}{L}$$

$$\therefore\ H = L\tan\theta = 10.8 \times \tan 60^\circ = 10.8 \times \sqrt{3} = 10.8 \times 1.732 = 18.7 \quad \text{m}$$

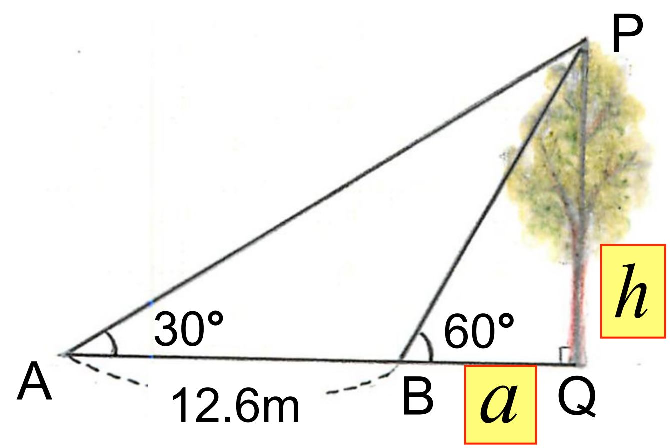
Exercise 1. We searched two positions A and B where the angles of elevation are 30 degrees and 60 degrees. The distance between A and B was 12.6m. What is the height of this tree ?



Ans.

Pause the video and solve the problem.

Exercise 1. We searched two positions A and B where the angles of elevation are 30 degrees and 60 degrees. The distance between A and B was 12.6m. What is the height of this tree ?



Ans. Let BQ and PQ be a and h, respectively.

$$\tan 30° = \frac{h}{a+12.6} \qquad \therefore\ a+12.6 = \frac{h}{\tan 30°} = \sqrt{3}h$$

and

$$\tan 60° = \frac{h}{a} \qquad \therefore\ a = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}$$

$$\therefore\ \frac{h}{\sqrt{3}} + 12.6 = \sqrt{3}h \qquad h = 12.6 \times \frac{\sqrt{3}}{2} \approx 12.6 \times \frac{1.73}{2} \approx 11.0 \text{ m}$$

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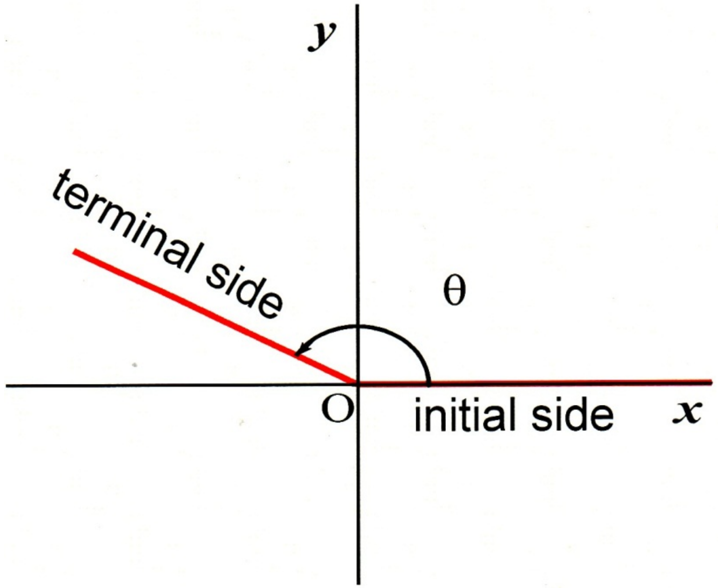
Lesson 4
Trigonometric Functions **(I)**

4B

- **General Angle**
- **Trigonometric Functions with a General Angle**

General Angle

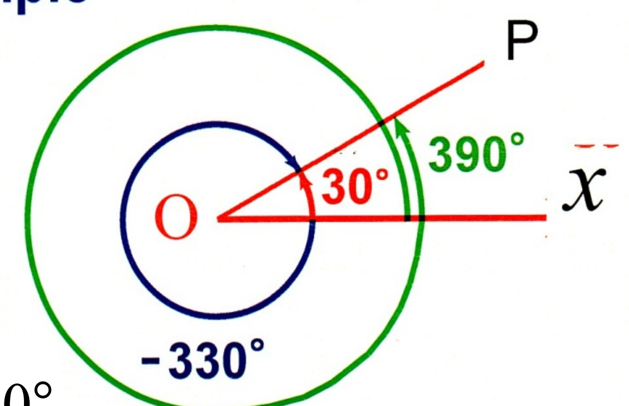


Fixed ray (initial side)
Rotating ray (terminal side)

$\theta > 0$: Counterclockwise direction

$\theta < 0$: Clockwise direction

Example



$\alpha = 30^\circ$

$\theta = 390^\circ = 30^\circ + 360^\circ$

Coterminal angles α and θ

$$\theta = \alpha + 360^\circ \times n$$
$$(n = 0,\ \pm 1,\ \pm 2,\ \cdots)$$

Trigonometric Functions with a General Angle

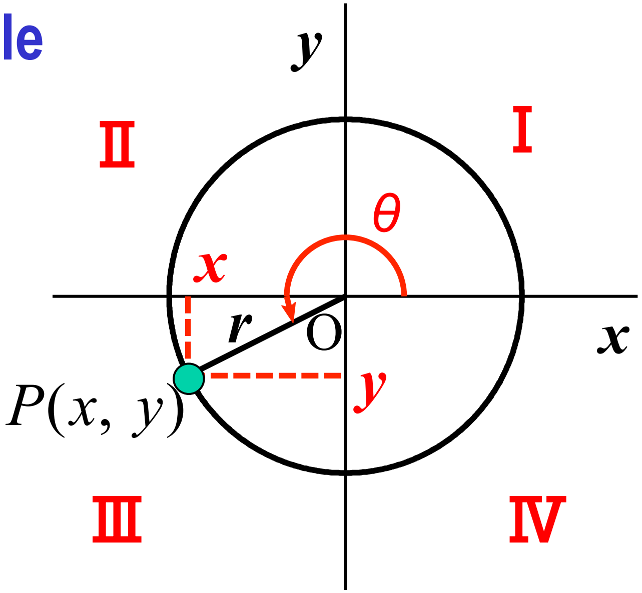
Let $P(x, y)$ be a point on a terminal side of the angle theta. The length OP is $r = \sqrt{x^2 + y^2}$

Trigonometric Function with a General Angle

$$\sin\theta = \frac{y}{r},$$

$$\cos\theta = \frac{x}{r},$$

$$\tan\theta = \frac{y}{x},$$

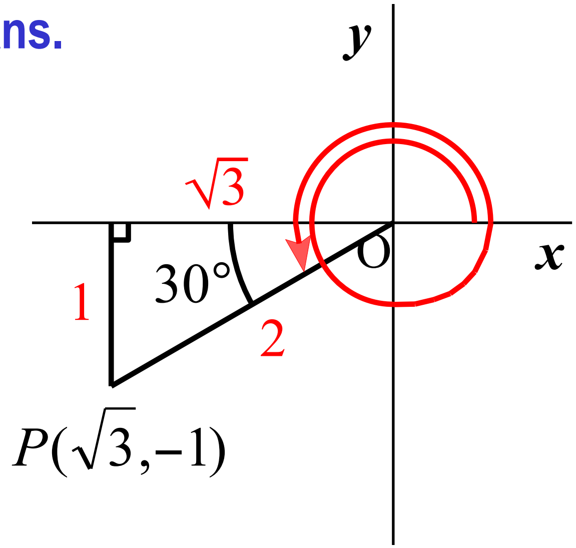


Signs of Trigonometric Functions

Quadrant	I	II	III	IV
	$x>0,\ y>0$	$x<0,\ y>0$	$x<0,\ y<0$	$x>0,\ y<0$
$\sin\theta$	+	+	−	−
$\cos\theta$	+	−	−	+
$\tan\theta$	+	−	+	−

Example 2. Find the values of trigonometric functions for (1) 570°and (2) -405°

Ans.

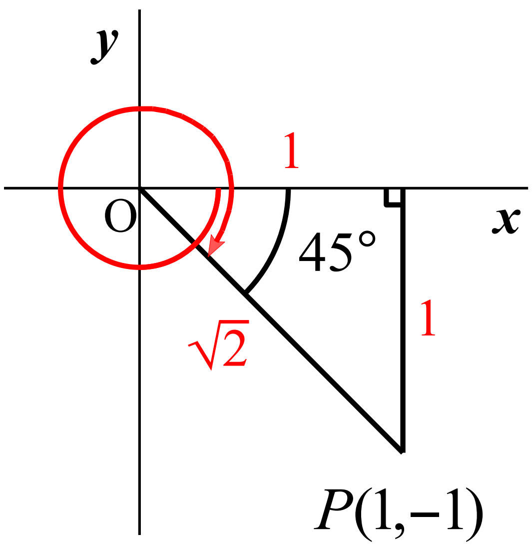


$$570° = 360° + 210°$$

$$\sin 570° = \sin(180° + 30°) = -\sin 30° = -\frac{1}{2}$$

$$\cos 570° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\tan 570° = \tan(180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$$



$$-405° = -360° - 45°$$

$$\sin(-405°) = \sin(-45°) = -\frac{1}{\sqrt{2}}$$

$$\cos(-405°) = \cos(-45°) = \frac{1}{\sqrt{2}}$$

$$\tan(-405°) = \tan(-45°) = -1$$

Exercise

Exercise 3. Suppose that the angle θ is located in Quadrant IV and $\cos\theta = \frac{3}{5}$. Find the values of $\sin\theta$ and $\tan\theta$.

Ans.

Pause the video and solve the problem.

Exercise 3. Suppose that the angle θ is located in Quadrant IV and $\cos\theta = \frac{3}{5}$. Find the values of $\sin\theta$ and $\tan\theta$.

Ans.

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$$

Since θ is located in Quadrant IV,

$$\sin\theta < 0 \quad \therefore \quad \sin\theta = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{4}{3}$$