Contents

1 C*-algebras ................................. 5
   1.1 Basics on C*-algebras .................. 5
   1.2 Spectral theory ........................ 8
   1.3 Matrix algebras ......................... 13

2 Projections and unitary elements .................. 15
   2.1 Homotopy classes of unitary elements .... 15
   2.2 Equivalence of projections ............... 19
   2.3 Liftings .............................. 23

3 K0-group for a unital C*-algebra ................. 25
   3.1 Semigroups of projections ................ 25
   3.2 The K0-group .......................... 27
   3.3 Functoriality of K0 .................... 31
   3.4 Examples ............................. 33

4 K0-group for an arbitrary C*-algebra ............. 35
   4.1 Definition and functoriality of K0 ......... 35
   4.2 The standard picture of the group K0 ....... 37
   4.3 Half and split exactness and stability of K0 .. 40

5 The functor K1 ............................ 43
   5.1 Definition of the K1-group ............... 43
   5.2 Functoriality of K1 .................... 47

6 The index map ............................. 51
   6.1 Definition of the index map ............... 51
   6.2 The index map and partial isometries ..... 55
   6.3 An exact sequence of K-groups .......... 58

7 Higher K-functors, Bott periodicity .......... 61
   7.1 The isomorphism between K1(C) and K0(S(C)) .. 61
   7.2 The long exact sequence in K-theory ........ 63
   7.3 The Bott map .......................... 66