並行分散計算特論 (11)

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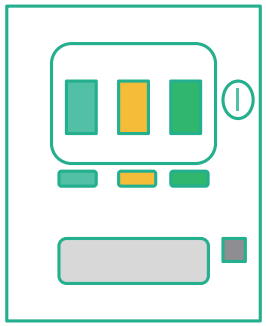
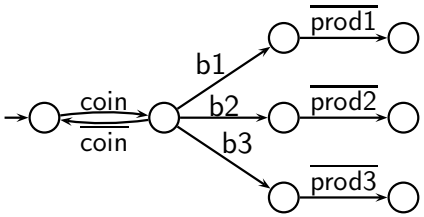
Timed behavioral model

Timed behavior:Time-out



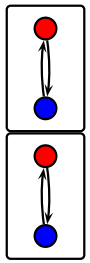


A coin is returned after certain time passed

How can such a behavior is described?

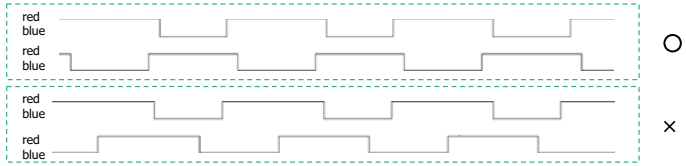
Timed behavior:Synchronization

At an intersection:



Northbound:
Blue for 60 seconds, Red for 90 seconds

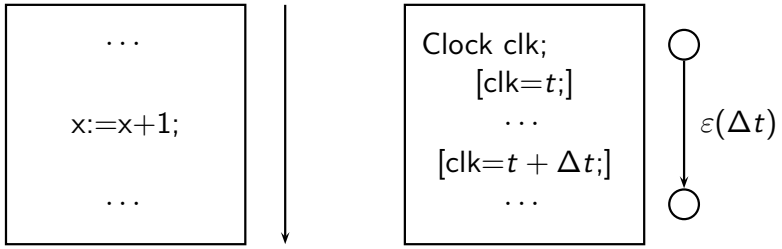
Eastbound:
Blue for 70 seconds, Red for 80 seconds



Which behavior is safe?

Time in programs

Time passes while program execution.



Clocks should be autonomously updated concurrent to execution.

Timed behavior is captured by references to clocks.

Treatment of time

- Time is an observation. (inherently, concurrent).
  Time cannot be controlled by computation.
- A clock is a self changing variable.
  Deterministic, strictly increasing.
- Concerns of critical properties
  Specification may contatin time, implicitly.

A program usually shift timing responsibility to runtime environment.

Realtime OS, Hardware clock

Interrupts by timer

Untimed vs timed specification

Untimed specification

I/O relation by functions $y = f(x)$

Temporal Logic *CTL*, CTL^*, *LTL*

Temporal I/O relation $L(I \cup O)$
I/O automata, communicating processes

Timed specification

Computation complexity $\mathcal{O}$ under same I/O relation

Real-time Temporal Logic *RTCTL*, *TCTL*, *RTL*, *MITL*

Timed I/O relation $L_T(I \cup O)$

Verification of timed behavior

$$\Gamma \models p \text{ sat } Prop$$

Judgement that a program p satisfies a property *Prop* under the environment of Γ, where *Prop* is a time sensitive predicate

Model checking

$$\Gamma, p \models F$$

Prop is expressed as a formula in some timed (temporal) logic.

Timed automata

Rajeev Alur and David Dill
A Theory of timed automata
Theoretical Computer Science, 126(2), pages 183-235

Büchi Automata

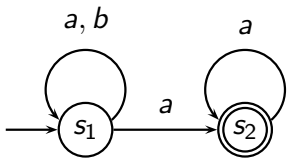
$\langle \sigma, S, S_0, E, F \rangle$

- Σ: Input alphabets
- S: Finite set of states
- $S_0 \subseteq S$: Start states
- $E \subseteq S \times \Sigma \times S$:Edges
- $F \subseteq S$: Final states

Operational Semantics:

$$r : \sigma_0 \overset{\sigma_1}{\to} \cdots \overset{\sigma_n}{\to} s_n \overset{\sigma_{n+1}}{\to}, \cdots$$

Büchi Automata example



$(a+b)^* a^\omega$

Determinstic Büchi⊂Non-deterministic Büchi
There is no deterministic Büchi automaton for the above

Timed Language

Büchi accepted word is extended by time
Each symbol is recorded by its time stamp = timed word

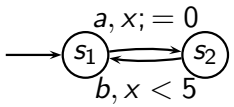
Definition
A time sequence $\tau = \tau_1 \tau_2 \cdots$ *is an infinite sequence s.t.:*

- $\tau_i > 0$ *for* $i > 1$ *and* $\tau_1 \geq 0$
- $\tau_i < \tau_{i+1}$
- *For any* $t \in \mathcal{R}^{\geq 0}$*, for some* i *such that* $\tau_i > t$
  (No convergence to a finite time)

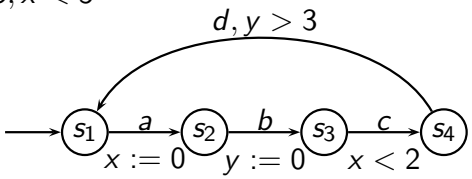
A timed word *over* Σ*:* (σ, τ) *where:*
An ininite sequence $\sigma = \sigma_1 \sigma_2 \cdots$ *and* τ *is a time sequnce.*
timed language*: set of timed words*

Timed transition diagram

x, y: clock variable (autonomously increasing as time passes)
Transition: Check time constraints over clocks, may reset clocks



$\{((ab)^\omega, \tau) | \forall i > 0. \tau_{2i} - \tau_{2i-1} < 5\}$



$\{((abcd)^\omega, \tau) | \forall i > 0. \tau_{4i-1} - \tau_{4i-3} < 2, \tau_{4i} - \tau_{4i-2} > 3\}$

Timed transition table

Definition

$\langle \Sigma, S, S_0, C, E \rangle$

- Σ : *alphabet*
- S : *set of states*
- $S_0 \subseteq S$: *start states*
- C:*set of clocks*
- $E \subseteq S \times \Sigma \times 2^C \times \phi(C) \times S$: *Edges*

where

$$\phi(X) : \delta := x \leq c \mid c \leq x \mid \neg\delta \mid \delta_1 \wedge \delta_2$$

($x \in X$, $c \in \mathcal{Q}^{\geq 0}$)

Timed word generation

Given a timed transition talbe:$\langle \Sigma, S, S_0, C, E \rangle$
A run r over a timed word

$$r : \langle s_0, \nu_0 \rangle \overset{\sigma_1}{\rightarrow}_{\tau_1} \langle s_1, \nu_1 \rangle \overset{\sigma_2}{\rightarrow}_{\tau_2} \cdots$$

$s_0 \in S_0$, $s_i \in S$ and $\nu_i \in [C \rightarrow \mathcal{R}^{\geq 0}]$
$\nu_0(x) = 0$ for all $x \in C$
$\langle s_{i-1}, \sigma_i, \lambda_i, \delta_i, s_i \rangle \in E$,
ν_{i-1} satisfies δ_i
$\nu_i = [\lambda_i \mapsto 0](\nu_{i-1} + \tau_i - \tau_{i-1})$

Timed Büchi automata

Definition

TBA(Timed Büchi Automaton):$\langle \Sigma, S, S_0, C, E, F \rangle$ *where* $\langle \Sigma, S, S_0, C, E \rangle$ *is a timed transition table with* $F \subseteq S$ *as the set of accepting states.*

A run $r = (\vec{s}, \vec{\nu})$ over (σ, τ) is accepting if $inf(r) \cap F \neq \emptyset$, where $inf(r)$ is the set of states infinitely appearing in r.

For TBA A:

$$L(A) = \{(\sigma, \tau) | A \text{ has an accepting run over } (\sigma, \tau)\}$$

Examples