Propositional Linear-time Temporal Logic PLTL

- **Syntax of PLTL**
  - Atomic formula: p (in a set P of propositions)
  - Expression: constructed from atomic formulae by $\wedge$, $\vee$, $\Rightarrow$, $\neg$, U and N, where U and N are used in forms of $\phi U \psi$, $N\phi$

- **Semantics of PLTL**

  Let $w = p_1 \ldots p_n$ **be a string over** P

  w **satisfies** ϕ **(**$w \models \phi$**) is defined as**

  - $w \models p$ **iff** $p_1 = p$
  - $w \models N\phi$ **iff** $2 \leq n$ **and** $p_2 \ldots p_n \models \phi$
  - $w \models \phi U \psi$ **iff** i $(1 \leq i \leq n)$ **exists such that**

    $p_i \ldots p_n \models \psi$ **and**

    $p_j \ldots p_n \models \phi$ **for all** j **such that** $1 \leq j < i$
  - $w \models \neg\phi$ **iff** $w \models \phi$ **does not hold**

    $\vdots$

- **Semantics of PLTL (cont.)**
  - **Intuitively, $w = pp'p''$ represents a situation that p holds now, p' next moment, and p'' after that.**
  - **$N\phi$ means that ϕ holds in the next moment**
  - **$\phi U \psi$ means that ϕ holds until ψ holds**

- **Ex.:** $N(pUp')$ **defines** $L = \{w \mid w \models N(pUp')\}$**,** **which is represented as regular expression** $Rp^*p'R^*$**,** **where** R **denotes all** $p \in P$ **combined by** $+$**, i.e.,** $R = (p_1 + \cdots + p_m)$ **for** $P = \{p_1, \ldots, p_m\}$
- **Ex.: formula that defines each language**
  - R^*pR^***:** $true\,U p$ **(**p **holds at a moment)**
    ($true$ **is defined as, e.g.** $p \vee \neg p$**)**
    $true\,U\phi$ **is sometimes written as** $F\phi$
  - pp^***:** $\neg(F(\neg p))$ **(**p **holds continuously)**
    $\neg(F(\neg\phi))$ **is sometimes written as** $G\phi$

- Satisfiability problem of PLTL formulae: Problem whether there exists a string w that satisfies a given formula ϕ
- Th.: Satisfiabiltiy problem of PLTL formulae is decidable
- Proof: by transformation into WS1S
  - Prepare 2nd order variable X_p for each $p \in P$ The coding $\widehat{w}$ of a string w is shown by examples below

    For $P = \{p, p'\}$ and $w = pp'p$,

    $\widehat{w} = (X_p, X_{p'})$, where$X_p = \{\varepsilon, 11\}$、$X_{p'} = \{1\}$

- **Proof (cont.)**
  Construct WS1S formula ψ as $T(\phi, \varepsilon)$ from PLTL formula ϕ so that $w \models \phi$ iff $\widehat{w} \models \psi$

  - $T(p, x) : x \in X_p$
  - $T(N\phi, x) : T(\phi, x1)$
  - $T(\phi U \phi', x)$:

  $$\begin{aligned} &\exists y.(x \leq y \wedge T(\phi', y) \\ &\qquad \wedge \forall z.(x \leq z \wedge z1 \leq y \Rightarrow T(\phi, z))) \end{aligned}$$

  - $T(\neg\phi) : \neg T(\phi)$

  $\vdots$