Equivalence of NFTA and DFTA

- **Construction of an equivalent DFTA** $A_d = (Q_d, \mathcal{F}, Q_d^f, \Delta_d)$ **to a given NFTA** $A = (Q, \mathcal{F}, Q^f, \Delta)$
  - $Q_d = 2^Q = \{S \mid S \subseteq Q\}$
  - $Q_d^f = \{S \in Q_d \mid S \cap Q^f \neq \emptyset\}$
  - Δ_d **consists of rules** $f(S_1, \ldots, S_n) \to S$ **that satisfies**

$$\forall f \in \mathcal{F},\ \forall S_1, \ldots, S_n \in Q_d$$

$$S = \{q \mid \exists q_1 \in S_1, \ldots, \exists q_n \in S_n, \\ f(q_1, \ldots, q_n) \to q \in \Delta\}$$

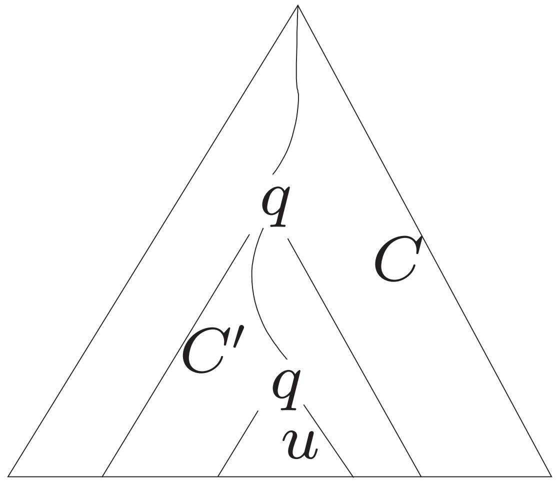
- **DFTA** $A_d = (Q_d, \mathcal{F}, Q_d^f, \Delta_d)$ **is equivalent to the Ex. in p.8 of slide 1, where** Δ **is**

  $a \to \{q\}$, $g(\{q\}) \to \{q, q_g\}$,
  $g(\{q, q_g\}) \to \{q, q_g, q_f\}$, $g(\{q, q_g, q_f\}) \to \{q, q_g, q_f\}$,
  $f(\{q\}, \{q\}) \to \{q\}$, $f(\{q, q_g\}, \{q\}) \to \{q\}$,
  $f(\{q, q_g, q_f\}, \{q\}) \to \{q\}$, $f(\{q\}, \{q, q_g\}) \to \{q\}$,
  $f(\{q, q_g\}, \{q, q_g\}) \to \{q\}$, $f(\{q, q_g, q_f\}, \{q, q_g\}) \to \{q\}$,
  $f(\{q\}, \{q, q_g, q_f\}) \to \{q\}$, $f(\{q, q_g\}, \{q, q_g, q_f\}) \to \{q\}$,
  $f(\{q, q_g, q_f\}, \{q, q_g, q_f\}) \to \{q\}$
  $Q_d^f = \{\{q, q_g, q_f\}\}$
- **Q2-1: Convert NFTA (defined in Q1-1 in p.10 of slide 1) to a DFTA.**

Pumping Lemma

- **Ex. of non-regular tree language:**
  $\{f(g^m(a), g^m(a)) \mid m \geq 0\}$
  $\{g^m(h^m(a)) \mid m \geq 0\}$**,**
  where $g^k(a) = \underbrace{g(g(\cdots g}_{k}(a)\cdots))$
- **Pumping Lemma**
  For a regular tree language L**, there exists** k **such that for any** $t \in L$ $|t| \geq k$ **implies the existence a decomposition** $C[C'[u]]$ **of** t **satisfying**
  1. $C' \notin \mathcal{X}$**, and**
  2. $\forall n \geq 0,\ C[C'^n[u]] \in L$

- **Proof Sketch: let A be NFTA that recognizes L, and k be its number of states. For a $t \in L$ satisfying $|t| \geq k$, a state occurs twice in a path**



Taking C,C', and u in above figure,

$$t = C[C'[u]] \rightarrow_A^* C[C'[q]] \rightarrow_A^* C[q] \rightarrow_A^* q_f \in Q^f$$

Thus,

$$C[C'^n[u]] \rightarrow_A^* C[C'^n[q]] \rightarrow_A^* C[C'^{n-1}[q]] \rightarrow_A^* \cdots \rightarrow_A^* C[q] \rightarrow_A^* q_f \in Q^f$$

Proof of the claim that $L = \{f(g^m(a), g^m(a)) \mid m \geq 0\}$ is non-regular

- Assuming L be regular, there exists k determined by Pumping lemma. For $t = f(g^m(a), g^m(a))$ ($m \geq k$), there exists a decomposition $t = C[C'[u]]$ saitsfying 1. and 2. of the lemma.
- In case $C \in \mathcal{X}$, $C' = f(g^\ell(x_1), g^m(a))$ ($0 \leq \ell \leq m$)、$u = g^{m-\ell}(a)$. From Pumping lemma, $C[u] \in L$ follows, which contradicts to $C[u] = u \notin L$
- In case $C = f(g^\ell(x_1), g^m(a))$ ($0 \leq \ell < m$), $C' = g^{\ell'}(x_1)$、$u = g^{m-\ell-\ell'}(a)$ ($0 < \ell' \leq m - \ell$). From Pumping lemma, $C[u] \in L$ follows, which constradicts to $C[u] = f(g^{m-\ell'}(a),\ g^m(a)) \notin L$

- **Q2-2: Let $\mathcal{F} = \{f(,), a, b\}$ and $|t|_a$ be the number of occurrences of a in t. Show that tree language $L = \{t \in \mathsf{T}(\mathcal{F}) \mid |t|_a = |t|_b\}$ is non-regular.**
  Hint: consider a member
  $f(f(f(\cdots, a), a), f(f(\cdots, b), b))$ **in** L