# Development statistics

## S05 Normal Distribution

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## Mean & Variance of Normal distribution

- PDF $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[ -\frac{(x-\mu)^2}{2\sigma^2} \right]$$

- Mean $\mu$
- Variance $\sigma^2$

# Normal Distribution in EXCEL

- General form $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[ -\frac{(x-\mu)^2}{2\sigma^2} \right]$$

- Normdist(x, mean, variance, 0): PDF
- Normdist(x, mean, variance, 1): DF



# Standardization

- General form $N(\mu, \sigma^2)$

$$z = \frac{x - \mu}{\sigma}$$

- Standard normal $N(0,1)$

# Standard Normal Distribution in EXCEL

- Standard normal $N(0,1)$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$$

- Normdist(x, mean, variance, 0): PDF
- Normdist(x, mean, variance, 1): DF
- Normsdist(x): DF of N(0,1)



# Deviation score (偏差値)

- Deviation score
- Mean 50
- SD 10

$$d \sim N(50,100)$$

$$d = 10 * z + 50$$

$$z = \frac{d - 50}{10}$$

# Important "% point" in Standard Normal

- 99% point: =normsinv(0.005)
  - Pr(-2.576 < z < 2.576) = 99%
- 95% point: =normsinv(0.025)
  - Pr(-1.960 < z < 1.960) = 95%
- 90% point: =normsinv(0.050)
  - Pr(-1.645 < z < 1.645) = 90%



# Remember "binomial distribution"

- Trial = $n$ and success = $x$
- Binomial distribution = the number times of success

$$P(x) = {}_n C_x \, p^x (1-p)^{n-x} = {}_n C_x \, p^x q^{n-x}$$

- Mean $\mu = np$
- Variance $\sigma^2 = npq$

# Distribution of the ratio

- Trial = $n$
- Success = $x$
- Average = the ratio $r = x / n$
  - Mean of $r$ $\boxed{p}$
  - Variance of $r$ $\boxed{pq / n}$



# Normal approximation of “ratio”

- If the sample size is large enough

$$\frac{r - p}{\sqrt{p(1-p)/n}}$$

$$= \frac{r - p}{\sqrt{pq/n}} \rightarrow N(0,1)$$