Lecture 13

Chapter 6: Imperfect Competition

7/13, 2023

Perfect Competition

- Consider a perfectly competitive firm in the product market.
- Under perfect competition, the firm is a price taker.
- Let y, p, and $c(y)$ denote output, price, and the cost function. Then the firm’s profit is

$$py - c(y)$$

- The profit maximization implies

$$p = c'(y) = MC$$

- Price equals the marginal cost.

Monopoly

- Now consider a monopoly firm.
- Let $y = D(p)$ denote the demand function.
- The monopolist's profit is
$$py - c(y) = pD(p) - c\big(D(p)\big)$$
- The first-order condition with respect to p is
$$D + pD'(p) - c'D'(p) = 0$$
- Assuming $D' \neq 0$, we solve it for p:
$$p = c' - \frac{D}{D'} = c' + \frac{1}{-\frac{p}{D}D'}p = c' + \frac{1}{\epsilon}p$$

Monopoly

- Rewrite it as

$$\left(1 - \frac{1}{\epsilon}\right) p = c'$$

- Because $c' > 0$, the demand elasticity must satisfy

$$\epsilon \geq 1$$

- We finally obtain

$$p = \left(1 + \frac{1}{\epsilon - 1}\right) c' = (1 + \mu) MC$$

- $1 + \mu$ is the **markup**. μ is the markup rate.

Markups

- The markup rate:

$$\mu = \frac{1}{demand\ elasticity - 1} \geq 0$$

- In the limit as the demand elasticity gets arbitrarily large ($\epsilon \to \infty$),

$$\mu \to 0$$
$$p \to MC$$

- This the perfectly competitive outcome.
- The markup captures the degree of market concentration (or competitiveness) of an economy.

Markups

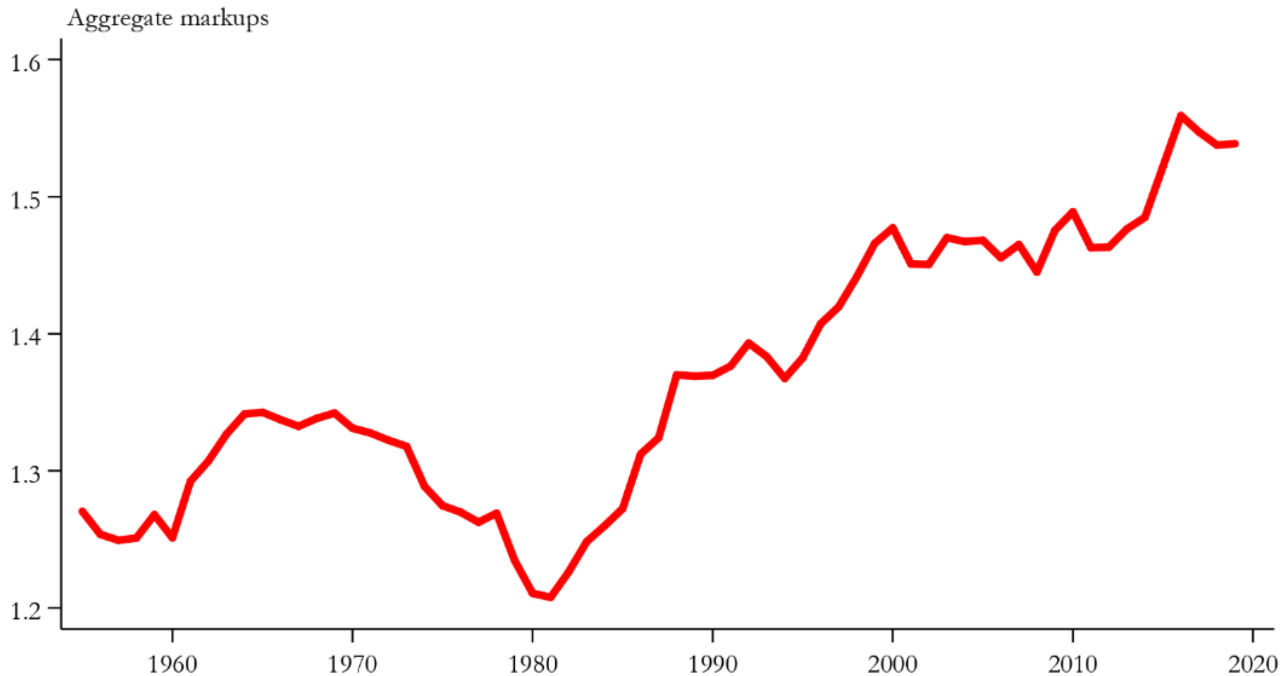

Figure 3. Aggregate markups in the United States

Revenue-weighted average markup of US publicly traded firms (source: De Loecker, Eeckhout & Unger, 2020)

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Markups

Aggregate markups

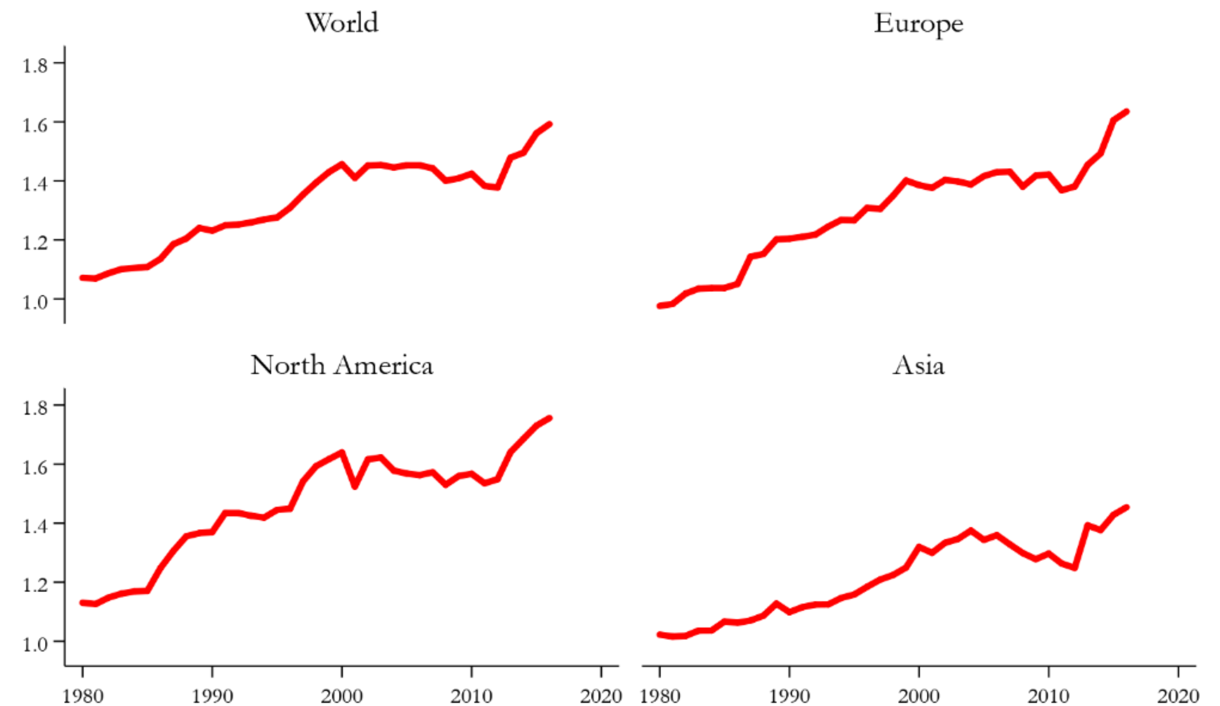

Figure 4. Aggregate global markups

Revenue-weighted average markup of publicly traded firms: World, Europe, North Amercia and Asia (source: De Loecker & Eeckhout, 2018)

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Monopolistic Competition

Differentiated Goods

- Let n denote the **number of input variety**.
- Let $y(z)$ denote the input of **variety** z.
- Production function:

$$Y = \left(\int_0^n y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 1$ is the **elasticity of substitution** between any two varieties.

Intermediate Goods

- Let $p(z)$ denote the input price of variety z.
- The input demand minimizes total expenditure:

$$\min_{y(z)} \int_0^n p(z)y(z)dz$$

$$s.t.$$

$$Y = \left(\int_0^n y(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate Goods

- The Lagrangian is

$$\int_0^n p(z)y(z)dz + \lambda\left[Y - \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}\right]$$

- FOC for variety z is

$$p(z) = \lambda\frac{\sigma}{\sigma-1}\left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}y(z)^{\frac{\sigma-1}{\sigma}-1}$$

- FOC for variety j is

$$p(j) = \lambda\frac{\sigma}{\sigma-1}\left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}y(j)^{\frac{\sigma-1}{\sigma}-1}$$

Intermediate Goods

- Taking the ratio:

$$\frac{p(z)}{p(j)} = \frac{y(z)^{\frac{\sigma-1}{\sigma}-1}}{y(j)^{\frac{\sigma-1}{\sigma}-1}} = \left(\frac{y(z)}{y(j)}\right)^{\frac{-1}{\sigma}}$$

- Thus,

$$y(j) = \left(\frac{p(z)}{p(j)}\right)^{\sigma} y(z) = p(z)^{\sigma} p(j)^{-\sigma} y(z)$$

- Thus,

$$y(j)^{\frac{\sigma-1}{\sigma}} = p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}}$$

Intermediate Goods

- Substitute it into the production function:

$$Y = \left(\int_0^n y(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\int_0^n \left[p(z)^{\sigma-1} p(j)^{1-\sigma} y(z)^{\frac{\sigma-1}{\sigma}} \right] dj \right)^{\frac{\sigma}{\sigma-1}}$$

$$= p(z)^{\sigma} y(z) \left(\int_0^n [p(j)^{1-\sigma}] dj \right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate Goods

- We obtain

$$y(z) = Yp(z)^{-\sigma}\left(\int_0^n [p(j)^{1-\sigma}]dj\right)^{\frac{\sigma}{1-\sigma}}$$

- This is the input demand function for variety z.
- We can simplify it further…

Intermediate Goods

- Now let us use $y(j) = p(z)^{\sigma} p(j)^{-\sigma} y(z)$ to rewrite the expenditures:
$$\int_0^n p(j)y(j)dj = y(z)p(z)^{\sigma} \int_0^n p(j)^{1-\sigma} dj$$
- Perfect competition in the final-goods market implies zero profit: $PY - \int_0^n p(j)y(j)dj = 0$
- Thus,
$$PY = y(z)p(z)^{\sigma} \int_0^n p(j)^{1-\sigma} dj$$

Intermediate Goods

- Solve it for $y(z)$ as

$$y(z) = PY\left[\int_0^n p(j)^{1-\sigma}dj\right]^{-1} p(z)^{-\sigma}$$

- Substitute it into the production function:

$$Y = PY\left[\int_0^n p(j)^{1-\sigma}dj\right]^{-1}\left(\int_0^n p(z)^{1-\sigma}dz\right)^{\frac{\sigma}{\sigma-1}}$$

- It simplifies to

$$1 = P\left[\int_0^n p(j)^{1-\sigma}dj\right]^{\frac{1}{\sigma-1}}$$

Intermediate Goods

- Finally,

$$P = \left[\int_0^n p(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$$

- RHS is the **price index** of the intermediate goods.

Intermediate Goods

- The input demand is therefore

$$y(z) = Yp(z)^{-\sigma}\left(\int_0^n [p(j)^{1-\sigma}]dj\right)^{\frac{\sigma}{1-\sigma}}$$
$$= Yp(z)^{-\sigma}P^{\sigma}$$

- Thus,

$$\frac{y(z)}{Y} = \left[\frac{p(z)}{P}\right]^{-\sigma}$$

- Relative demand for input z is decreasing in the relative price of the input.

Intermediate Goods

- Production requires labor input $\ell(z)$:
$$y(z) = \max\{0, \varphi[\ell(z) - f]\}$$
- φ is the marginal product of labor (MPL).
- f is the **overhead cost** of production.
  - e.g.) I spent so much time on course preparation before the semester.
- The labor units needed to produce $y(z)$ is
$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Intermediate Goods

- Let w denote the wage rate. The profit is
$$\pi(z) = p(z)y(z) - w\ell(z)$$
- We assume perfectly competitive labor market.
- Input demand function:
$$\frac{y(z)}{Y} = \left[\frac{p(z)}{P}\right]^{-\sigma} \Leftrightarrow y(z) = YP^{\sigma}p(z)^{-\sigma}$$
- Production technology:
$$\ell(z) = f + \frac{y(z)}{\varphi}$$

Markup Pricing

- Profit:

$$\pi(z) = p(z)y(z) - w\ell(z)$$
$$= p(z)y(z) - w\left[f + \frac{y(z)}{\varphi}\right]$$
$$= YP^{\sigma}p(z)^{1-\sigma} - wf - \frac{w}{\varphi}YP^{\sigma}p(z)^{-\sigma}$$

- FOC with respect to $p(z)$:

$$p(z) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$$
$$= markup \times \frac{w}{MPL}$$

Price Index

- Thus,

$$P = \left[\int_0^n \left[\frac{\sigma}{\sigma-1}\frac{w}{\varphi}\right]^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$$

$$= \left[n\left[\frac{\sigma}{\sigma-1}\frac{w}{\varphi}\right]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma-1}\frac{w}{\varphi}n^{\frac{1}{1-\sigma}}$$

- P is decreasing in n (because $\sigma > 1$).

Aggregate Output

- Because $p(z)$ is the same for all intermediate-good firms, the input $y(z)$ is the same for all z.
- Thus, the aggregate production satisfies

$$Y = \left(\int_0^n y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} = y n^{\frac{\sigma}{\sigma-1}}$$

- Because ny units of intermediate goods are used, the aggregate productivity is

$$\frac{Y}{ny} = n^{\frac{1}{\sigma-1}}$$

Revenue

- Input demand:

$$y(z) = YP^{\sigma}p(z)^{-\sigma}$$
$$= Y\left[\frac{\sigma}{\sigma-1}\frac{w}{\varphi}n^{\frac{1}{1-\sigma}}\right]^{\sigma}\left[\frac{\sigma}{\sigma-1}\frac{w}{\varphi}\right]^{-\sigma}$$
$$= Yn^{\frac{\sigma}{1-\sigma}}$$

- The firm's revenue is

$$r(z) = p(z)y(z) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}y(z)$$

Profit

- The profit is

$$\begin{aligned}\pi(z) &= p(z)y(z) - w\left[f + \frac{y(z)}{\varphi}\right] \\ &= \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}y(z) - \frac{w}{\varphi}y(z) - wf \\ &= \frac{1}{\sigma - 1}\frac{w}{\varphi}y(z) - wf \\ &= \frac{r(z)}{\sigma} - wf\end{aligned}$$

Closing the Model

- Households supply L units of labor.
- There are n firms, and each employs ℓ workers.
- Thus,

$$L = n\ell = n\left[f + \frac{y}{\varphi}\right]$$

- Thus,

$$\frac{y}{\varphi} = \frac{L}{n} - f$$

Firm Entry

- The profit is

$$\pi(z) = \frac{1}{\sigma - 1}\frac{w}{\varphi}y(z) - wf$$
$$= \frac{1}{\sigma - 1}w\left[\frac{L}{n} - f\right] - wf$$

- Thus, $\pi(z) \geq 0$ if and only if

$$\frac{1}{\sigma - 1}\left[\frac{L}{n} - f\right] \geq f \Leftrightarrow \frac{\mu}{1 + \mu}\frac{L}{n} \geq f$$

- This is the condition for firm entry.

Equilibrium Number of Firms

- The free entry condition:

$$\pi(z) = 0 \Leftrightarrow \frac{\mu}{1+\mu}\frac{L}{n} = f$$
$$\Leftrightarrow n = \frac{\mu}{1+\mu}\frac{L}{f}$$

- Thus, the equilibrium number of firms is
  - Increasing in the markup rate μ.
  - Decreasing in the overhead fixed cost f.
- $L = n\ell$ implies that an increasing in n decreases ℓ, which decreases y and increases p.

Externality

- The aggregate production:

$$Y = \left(\int_0^n y^{\frac{\sigma - 1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma - 1}} = yn^{\frac{\sigma}{\sigma - 1}}$$

- Because ny units of intermediate goods are used, the aggregate productivity is

$$\frac{Y}{ny} = n^{\frac{1}{\sigma - 1}}$$

- Firm entry causes **externality**, or **scale effects**.
- Models of **industry dynamics** and **international trade** tend to have endogenous n.

Macro Model with Monopolistic Competition

- Many macro models ignore changes in the number of firms by normalizing $n = 1$.
- The aggregate production:

$$Y = \left(\int_0^1 y^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- Because there is no need to pin down n, overhead cost f is not necessary.
- Thus, the firm earns a positive profit: $\pi(z) = \frac{r(z)}{\sigma}$.
- Because the household owns the firms, the household receives π in the form of dividends.

Final Remarks

You Are Ready

- I believe I gave you pretty much everything I know about the foundations of modern macroeconomic analysis.
- You are ready to read articles published in leading professional journals as well as advanced textbooks.
- For M1 students or below, it is good idea to start reading some graduate-level textbooks (a short list is found in Lecture 1). But, know this: to write a paper, you need to read papers. Find your favorite paper and try to replicate the results.

Good luck and have a nice (& productive) summer!