Lecture 12

Chapter 5: General Equilibrium
Part II: Monetary Economy

7/6, 2023

Goals

- Last week, we introduced prices and market equilibrium. However, <u>we ignored nominal variables</u> and focused on the real economy.
- Today, we finally introduce money and the price level.
- However, I will only scratch the surface of monetary economics as it requires at least one semester to go over a few important topics and models.

Modelling Money

Money is What Money Does

- Definition: Money is an object that serves the following 3 functions:

1. Store of Value
2. Unit of Account
3. Medium of Exchange

- Good idea to review any undergrad macro textbook such as Mankiw.

Money as a Medium of Exchange

- Let us formally study why "money" is valued.
- The model is called the Kiyotaki-Wright model.
  - This is a nice application of dynamic programming.
- Consider a primitive economy in which people exchange their commodities to consume a variety of goods. (No supermarket or Amazon.com)
- The key question is whether (as a seller) you should accept a piece of paper called yen in exchange of an apple you own.

Money as a Medium of Exchange

- Products are differentiated and you cannot consume everything.
- The probability that you can consume a commodity is $x < 1$.
  - Likewise, the probability that someone can consume your commodity is $x < 1$.
- **Barter exchange**: Even without money, you can exchange commodities if you want what I have and I want what you have.
  - Barter exchange requires a **double-coincidence of wants**.

Money as a Medium of Exchange

- There are two state variables:
  - $s \in \{c, m\}$
  - $\Pi \in [0,1]$
- Total population is normalized to 1 (unit mass). Only a $M < 1$ of people are endowed with money.
- People can hold at most one unit of money.
  - Accumulation of money is ruled out.
- When you have no money, you are a seller and your state is $s = c$. When you have money, then you are a buyer and $s = m$.

Money as a Medium of Exchange

- As a buyer, whether you can consume or not depends on whether the seller you will meet in the future will accept money as payment.
- Π is the **aggregate acceptability of money**. If $\Pi = 0.5$, then the probability that your payment in yen is accepted is 0.5.
- If accepted, you consume and obtain u units of utility, and becomes a seller once again.
- Let $\beta < 1$ denote the discount factor.

Money as a Medium of Exchange

- The value of holding a unit of money is
$$V(m,\Pi) = (1-M)x\Pi[u+\beta V(c,\Pi)] \\ +[1-(1-M)x\Pi]\beta V(m,\Pi)$$
- It is assumed that the probability that a money holder meets a seller is $1-M$, which is the proportion of sellers.
- Upon meeting, with probability x you like the commodity and you offer a unit of yen.
- With probability Π the seller accepts money, you consume, and becomes a seller in the next period.

Money as a Medium of Exchange

- The value of a seller is given by
$$V(c,\Pi) = (1-M)x^2[u + \beta V(c,\Pi)]$$
$$+ Mx\beta \max\{V(m,\Pi), V(c,\Pi)\}$$
$$+ [1-(1-M)x^2 - Mx]\beta V(c,\Pi)$$
- With probability $1-M$, the seller meets another seller. With probability x^2, a double-coincidence of wants occurs.
- With probability M, the seller meets a money holder, and chooses whether to accept money as payment to become a money holder or not.

Money as a Medium of Exchange

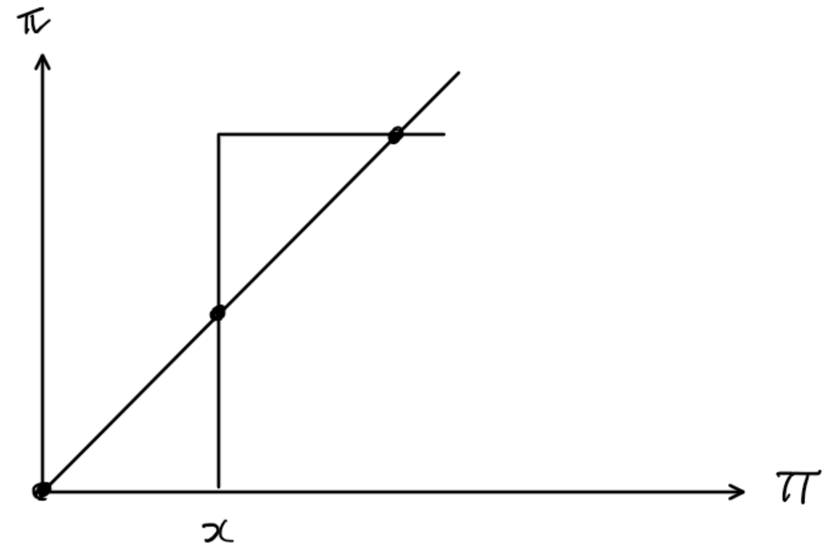
- Consider
$$\max\{V(m,\Pi), V(c,\Pi)\}$$
- It is convenient to transform this binary problem into the following:
$$\max_{\pi}[\pi V(m,\Pi) + (1-\pi)V(c,\Pi)]$$
- π can take any value in $[0,1]$
- For brevity, let $V(m,\Pi) = V_m$ and $V(c,\Pi) = V_c$.

Money as a Medium of Exchange

- We rewrite the Bellman equations as
$$(1-\beta)V_m = (1-M)x\Pi[u+\beta(V_c - V_m)]$$
$$(1-\beta)V_c = (1-M)x^2u + Mx\beta\pi(V_m - V_c)$$
- Subtract one equation from the other to obtain
$$(1-\beta)(V_m - V_c) = (1-M)x\Pi[u+\beta(V_c - V_m)] - (1-M)x^2u - Mx\beta\pi(V_m - V_c)$$
- Thus,
$$V_m - V_c = \frac{(1-M)(\Pi - x)xu}{1-\beta+(1-M)x\Pi + Mx\beta\pi}$$
- Evidently, $V_m > V_c$ if and only if $\Pi > x$.

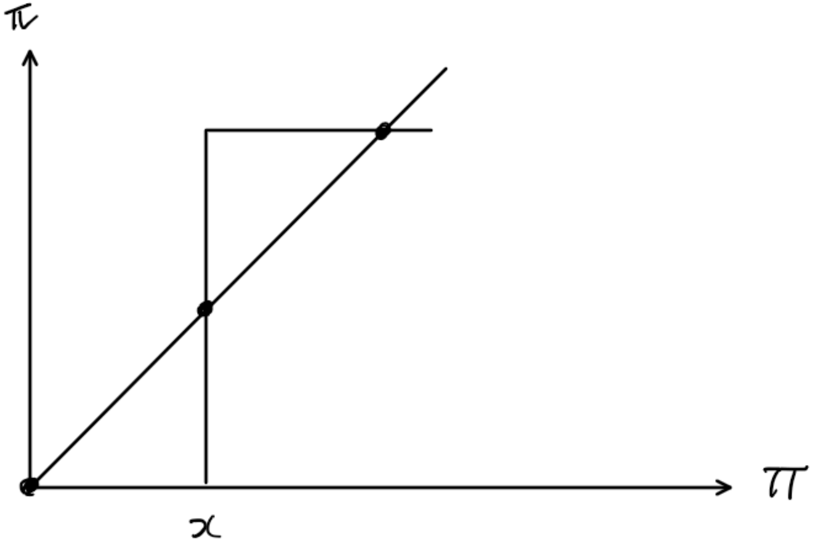
Money as a Medium of Exchange

- Thus, for each agent, the optimal decision is
  $\pi = 0 \text{ if } \Pi < x$
  $\pi \in (0,1) \text{ if } \Pi = x$
  $\pi = 1 \text{ if } \Pi > x$
- This is the **policy function**, which depends on the aggregate state Π.



Money as a Medium of Exchange

- You are willing to accept money if and only if you expect that all other people on average tend to accept money ($\Pi > x$).
- The value of money is backed by your expectations about other peoples' actions.

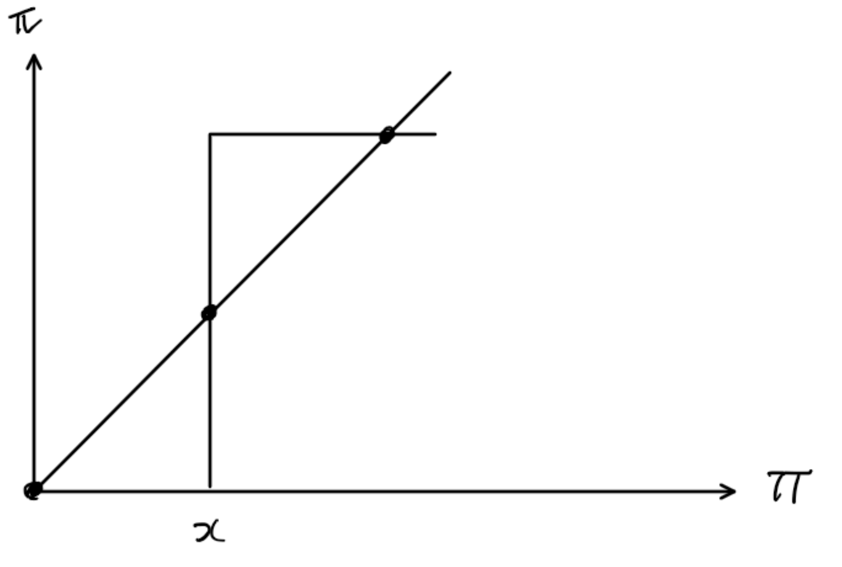


Money as a Medium of Exchange

- We look for a symmetric **Nash equilibrium**, in which a player's action is the best response against the other players actions.
- Perhaps you are not familiar with a game with a continuum of players.
- In this economy, <u>each agent plays against the aggregate variable Π</u>. Such a game is referred to as a **social game**.
  - Social games and macroeconomics have many common mathematical structures.

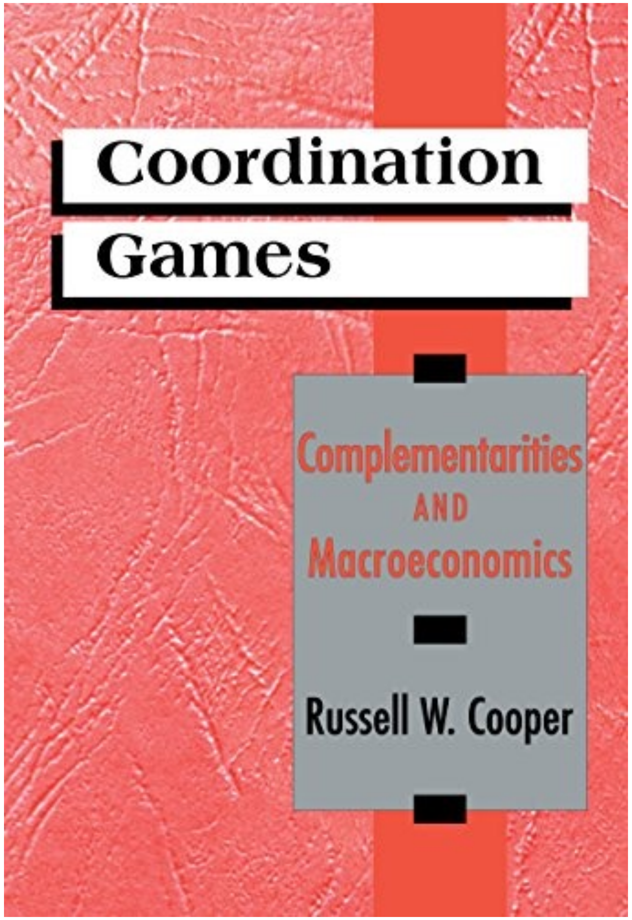
Money as a Medium of Exchange

- There are 3 equilibria: $\Pi = 0, \Pi = x, \Pi = 1$
- People have incentives to coordinate their actions.
  - Referred to as a **coordination game**.
  - Many macroeconomic examples, including financial crises and currency crises.



Coordination Games and Macro

- Cooper (1999) *Coordination Games: Complementarities and Macroeconomics*.
- I highly recommend this textbook.
- Good reading material for the summer.



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General Equilibrium with Money

Household's Budget

- Consider the competitive economy we studied last week.
- M_t: stock of money at the beginning of period t.
- The budget constraint in **nominal terms**:

$$A_t + W_t h_t + M_t = P_t c_t + \frac{A_{t+1}}{1 + i_{t+1}} + M_{t+1}$$

- A_t is nominal asset, W_t is the nominal wage rate, h_t is hours of work, P_t is the price of the consumption good, i_t is the nominal interest rate.

Household's Budget

- Divide both sides of the nominal budget constraint by P_t to obtain

$$a_t + w_t h_t + m_t$$
$$= c_t + \frac{a_{t+1}}{1 + r_{t+1}} + (1 + \pi_{t+1}) m_{t+1}$$

- This is the budget constraint <u>in real terms</u>
  - $m_t = M_t / P_t$: real money balance
  - $\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t}$: inflation rete
  - Fisher equation: $1 + i_t = (1 + r_t)(1 + \pi_t)$
  - Fisher equation can be approximated as $i_t = r_t + \pi_t$.

Models of Money Demand

- The tricky part of monetary economics is that there are too many models of money demand, and you must choose a model from many alternatives.
- If you consider monetary economics as your primary field of research, then you should read Carl Walsh's graduate textbook, *Monetary Theory and Policy*, for a guided tour.
- On the next page I give you a very short list of models. In one way or another, we need to derive a money demand function.

Models of Money Demand

- Money-in-the-Utility function (MIUF) approach:
  - m_t provides transactions service $\Rightarrow u(c_t, h_t, m_t)$
  - Short-cut for medium of exchange
- Cash-in-Advance (CIA) approach
  - $c_t \leq m_t$: m_t is needed to consume
  - Short-cut for medium of exchange
- Overlapping Generations (OLG) approach
  - Infinite sequence of finite-horizon agents
  - Money changes hands across generations
- Search Theory approach
  - Explicit trading frictions
  - Money changes hands within a generation

MIUF Model

- It is also known as the Sidrauski model.
- There are two means of saving.

$$\max_{\{c_t\},\{a_{t+1}\},\{m_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)]$$

subject to:

$$a_t + w_t + m_t = c_t + \frac{a_{t+1}}{1 + r_{t+1}} + (1 + \pi_{t+1}) m_{t+1}$$

- w_t is the exogenous wage income.
- $v(m_t)$ is captures the benefit of money.

MIUF Model

- The FOCs are:

$$u'(c_t) = \lambda_t$$
$$\lambda_t \frac{1}{1 + r_{t+1}} = \beta \lambda_{t+1}$$
$$\lambda_t (1 + \pi_{t+1}) = \beta v'(m_{t+1}) + \beta \lambda_{t+1}$$

- Thus,

$$u'(c_t) = (1 + r_{t+1}) \beta u'(c_{t+1})$$
$$(1 + \pi_{t+1}) u'(c_t) = \beta v'(m_{t+1}) + \beta u'(c_{t+1})$$
$$\Leftrightarrow$$
$$(1 + i_{t+1}) u'(c_{t+1}) = v'(m_{t+1}) + u'(c_{t+1})$$

MIUF Model

- Rewrite them as

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = R_t$$

$$v'(m_{t+1}) = i_{t+1} u'(c_{t+1})$$

- The first one is the standard Euler equation.
- The second equation determines the demand for real money balance:

$$v'(m_t) = i_t u'(c_t)$$

MIUF Model

- Functional forms:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, v(m) = \frac{m^{1-\gamma}}{1-\gamma}$$

- Then, $v'(m_t) = i_t u'(c_t)$ implies

$$m_t^{-\gamma} = i_t c_t^{-\sigma}$$

- Thus,

$$m_t = i_t^{\frac{-1}{\gamma}} c_t^{\frac{\sigma}{\gamma}}$$

- Money demand is decreasing in the nominal interest rate, as in the undergraduate textbook.

Note on Notations

- We defined $\frac{P_{t+1}-P_t}{P_t} = \pi_{t+1}$ is the inflation rete from period t and $t+1$.
- However, it is OK to define $\frac{P_{t+1}-P_t}{P_t} = \pi_t$ to mean the same object.
- Likewise, it is your choice to denote the interest rate between t and $t+1$ as i_t or i_{t+1}.
- Similarly, M_t is defined as the stock of money at the beginning of the period (= before transactions), but it is OK to denote M_{t-1} as the current stock.

Fiscal Implications of Money and Inflation

Inflation as a Tax

- Suppose you have M units of money.
- The price level in this period is P_t
- The value (= purchasing power) of money in this period is

$$\frac{M}{P_t}$$

- Suppose you did not spend it. The purchasing power in the next period becomes

$$\frac{M}{P_{t+1}} = \frac{P_t}{P_{t+1}} \frac{M}{P_t}$$

Inflation as a Tax

- Now suppose there is a direct tax on M
- The tax rate is x.
- Then the quantity of goods you can buy is

$$\frac{(1-x)M}{P_t}$$

- Under inflation, the purchasing power is

$$\frac{P_t}{P_{t+1}}\frac{M}{P_t}$$

- These scenarios are essentially the same thing.

Inflation as a Tax

- Solve the following equation:

$$1 - x = \frac{P_t}{P_{t+1}} = \frac{1}{1+\pi}$$

$$\Leftrightarrow$$

$$x = \frac{\pi}{1+\pi} \approx \pi$$

- This is the **inflation tax rate.**
- **Observation:** We can study monetary policy from a viewpoint of the theory of **public finance**.
  - **Optimal inflation rate ⇔ Optimal inflation tax rate**

Household's Budget

- The budget constraint in nominal terms:

$$A_t + W_t h_t + M_t - T_t = P_t c_t + \frac{A_{t+1}}{1+i_t} + M_{t+1}$$

- Divide both sides by P_t

$$a_t + w_t h_t + \frac{P_{t-1}}{P_t}\frac{M_t}{P_{t-1}} - \tau_t = c_t + \frac{a_{t+1}}{1+r_t} + \frac{M_{t+1}}{P_t}$$

- We <u>redefine</u> m_t as

$$m_t \equiv \frac{M_t}{P_{t-1}}$$

- We do it only for exposition.

Household's Budget

- Then,

$$a_t + w_t h_t + \frac{1}{1+\pi_t} m_t - \tau_t = c_t + \frac{a_{t+1}}{1+r_t} + m_{t+1}$$

$$\Leftrightarrow$$

$$a_t + w_t h_t + \left(1 - \frac{\pi_t}{1+\pi_t}\right) m_t - \tau_t = c_t + \frac{a_{t+1}}{1+r_t} + m_{t+1}$$

$$\Leftrightarrow$$

$$a_t + w_t h_t + (1 - x_t) m_t - \tau_t = c_t + \frac{a_{t+1}}{1+r_t} + m_{t+1}$$

- Our definition of m_t allows us to see the inflation tax rate on the left-hand side.

Household's Budget

- Thus,

$$a_t + w_t h_t + m_t - \boldsymbol{x_t m_t} - \tau_t = c_t + \frac{a_{t+1}}{1 + r_t} + m_{t+1}$$

- Is now clear that the household pays the inflation tax ($x_t m_t$ in each period). But, to whom?
- Our next question is whether the government receives the inflation tax from us.
  - The government's revenue from inflation tax is referred to as **seigniorage**.

Open Market Operations

- Government's budget in nominal terms:
$$P_t G_t + i_t^b B_t = P_t T_t + B_{t+1} - B_t$$
- Money is supplied through **open market purchases of government bonds from the public**:
$$B_t = B_t^p + B_t^c$$
- B_t^p is the nominal bonds held by the public.
- B_t^c is the nominal bonds held by the central bank.
- The central bank's balance sheet:
$$B_{t+1}^c - B_t^c = M_{t+1} - M_t + i_t^b B_t^c$$

Consolidated Budget Constraint

- Combine the two balance sheets:

$$P_t G_t + i_t^b B_t^p = P_t T_t + B_{t+1}^p - B_t^p + \boldsymbol{M_{t+1}} - \boldsymbol{M_t}$$

- Divide both sides by P_t

$$D_t = \frac{B_{t+1}^p}{P_t} - \left(1 + i_t^b\right)\frac{B_t^p}{P_t} + \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t}$$

- Redefine $b_t \equiv B_t^p / P_t$ and $m_t \equiv M_t / P_{t-1}$ to get

$$D_t = b_{t+1} - \left(1 + r_t^b\right) b_t + \underbrace{\left(m_{t+1} - \frac{1}{1 + \pi_t} m_t\right)}_{\text{Seigniorage}}$$

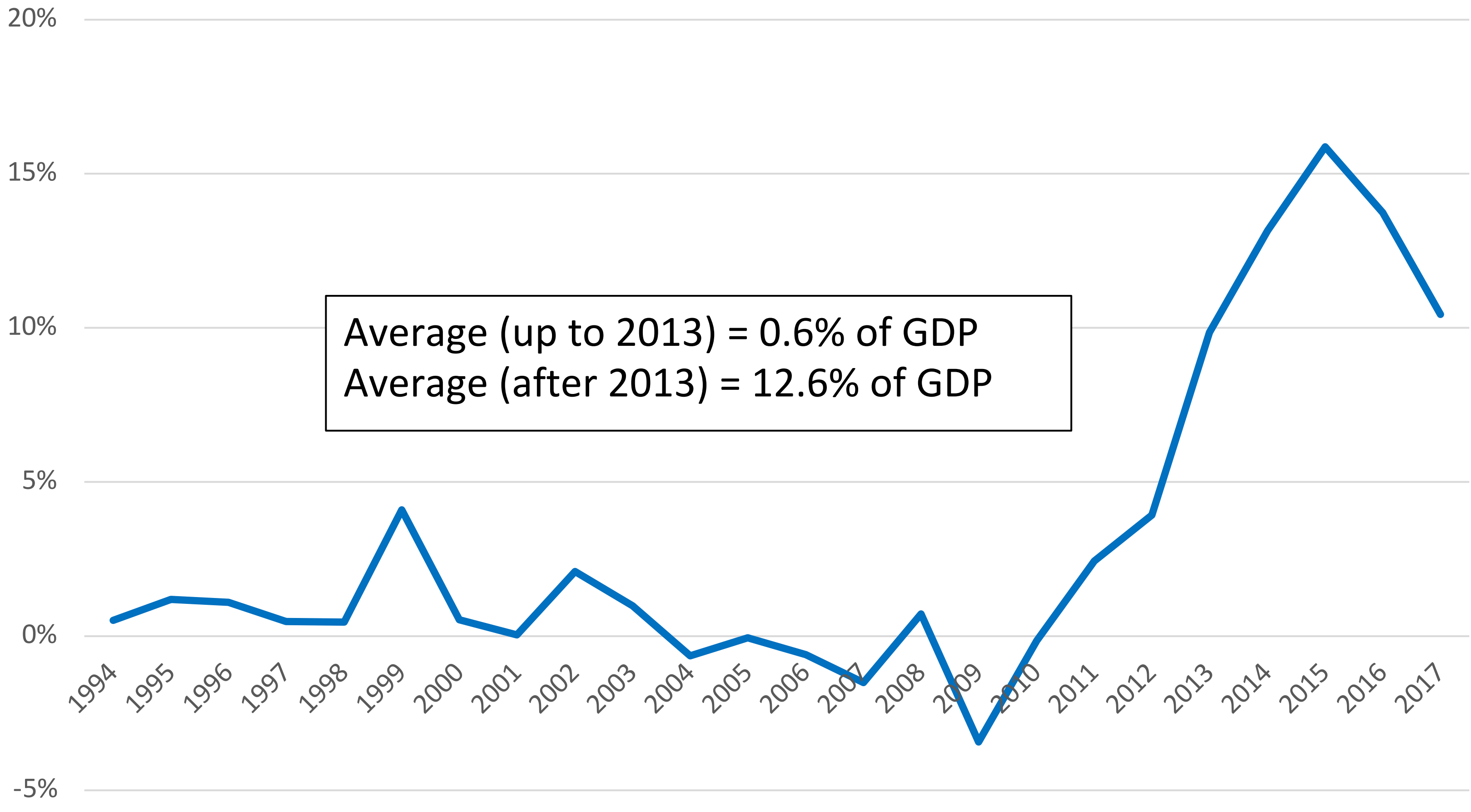
Consolidated Budget Constraint

- Rewrite it as

$$D_t + r_t^b b_t$$
$$= b_{t+1} - b_t + \underbrace{\left(m_{t+1} - m_t + m_t - \frac{1}{1+\pi_t} m_t\right)}_{\text{Seigniorage}}$$
$$= b_{t+1} - b_t + m_{t+1} - m_t + x_t m_t$$

- In any steady state, $\text{Seigniorage} = xm$.
  - This amount is the same as what we pay.
  - The government indeed receives the inflation tax.

Seigniorage in Japan



Helicopter Money

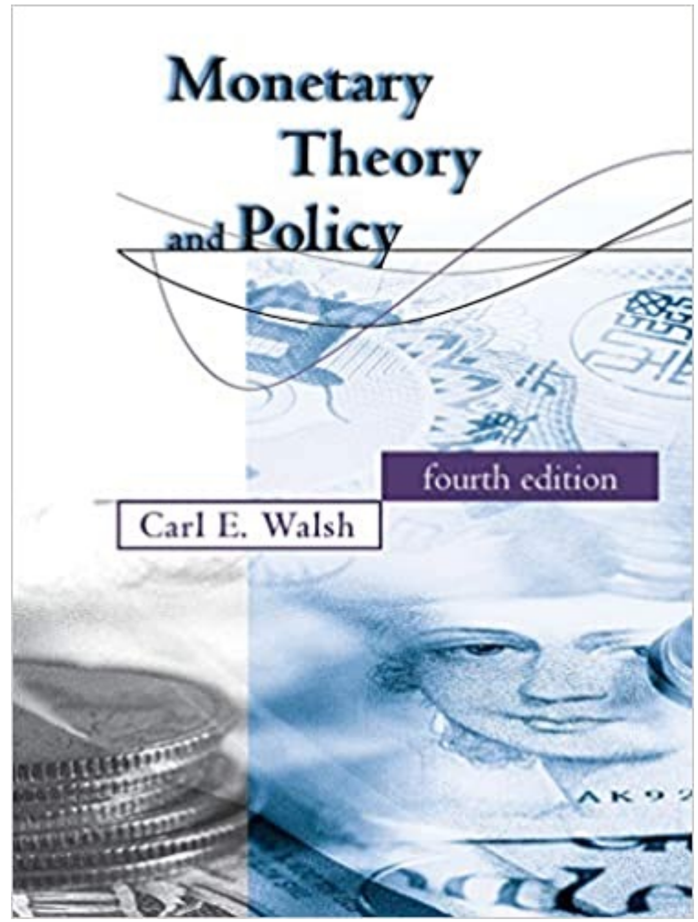
- Suppose there is no government bonds.
- When $B_t^c = 0$, there is no way to supply money.
- Let $G_t = 0$. The previous budget constraint is

$$-T_t = \underbrace{\left(m_{t+1} - \frac{1}{1+\pi_t} m_t\right)}_{\text{Seigniorage}}$$

- When open market operations are not modeled, you need to assume government transfer: $-T_t > 0$.
- Transfer takes the form of **money injection**.

Further Readings

- Walsh (2017) *Monetary Theory and Policy*, 4th edition.
- I highly recommend this textbook as your (serious) summer reading.
- You may be able to find your research topic in this book.



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