Introduction to Game Theory

Studium Generale

May 31, 2018

What is Game Theory?

Game Theory is the study of strategically interdependent behavior, which means that it can be applied to any situation where people get together and affect each other’s matters.

That is, the happiness (utility) of a certain person depends not only on their actions, but also on the actions of the others.

Interdependent Behavior: Going to the Beach

https://clipartuse.com/beach-girl-clipart-13291 2018/06/05

Interdependent Behavior: Going to the Beach

https://www.behance.net/gallery/10966077/Crowded-Drawings 2018/06/05

U(beach, crowded) = -10, U(home)=0.

I should stay at home.

https://www.irasutoya.com/2017/05/blog-post_549.html 2018/06/05

U(beach, not crowded) = 15, U(home)=0.

I should go to the beach.

The Prisoner’s Dilemma

http://iq230.com/iq-testy-uznaj-uroven-svoego-intellekta 2018/06/05

Mr. Blue

http://iq230.com/iq-testy-uznaj-uroven-svoego-intellekta 2018/06/05

Mr. Pink

The Prisoner’s Dilemma

Mr. Blue and Mr. Pink have been arrested and imprisoned, each in solitary confinement.

The prosecutors have evidence to convict the pair of a lesser charge (criminal possession of a weapon), but they lack enough evidence to convict the pair of the principal charge (bank robbery).

The prosecutors offer each prisoner a bargain. Each prisoner can either betray his partner by testifying that the other committed the crime, or cooperate by remaining silent.

- If both testify, each one serves 3 years in prison.
- If both remain silent, each one serves 1 year in prison.
- If one testifies and the other remains silent, the former is set free while the latter serves 4 years in prison.

The Prisoner's Dilemma

		Mr. Pink	
		Be silent	Testify
Mr. Blue	Be silent	(-1,-1)	(-4,0)
	Testify	(0,-4)	(-3,-3)

The Prisoner's Dilemma

Mr. Blue asks himself: what should I do?

If Mr. Pink remains silent, I prefer to testify, as being free is better than serving 1 year in prison.

If Mr. Pink testifies, I prefer to testify too, as serving 3 years in prison is better than serving 4.

So, Mr. Blue is going to testify, no matter what Mr. Pink does! Then, we say that testifying is a **dominant strategy**.

Notice that, although Mr. Blue is always going to testify, he is not always equally happy: he prefers Mr. Pink to remain silent (being free is better than serving 3 years in prison), but that is not up to Mr. Blue.

The same reasoning applies to Mr. Pink, so in equilibrium they both testify and each one serves 3 years in prison.

Nobody wants to change their strategy

The Prisoner’s Dilemma

What this game teaches us is that the individual incentives can harm the social welfare, which contradicts Adam Smith and his theory of the invisible hand!

Notice that any other outcome is better from the social perspective:

- If both remained silent, only 2 years in prison would be served in the overall, against the 6 years actually served.
- If only one testified, 4 years in prison would be served in the overall, which is also less than the 6 years actually served.

But the individual incentives are so strong that Mr. Blue and Mr. Pink cannot help themselves, although as a group both would prefer to be silent.

The Prisoner's Dilemma

The Prisoner's Dilemma applies to many situations in different fields.

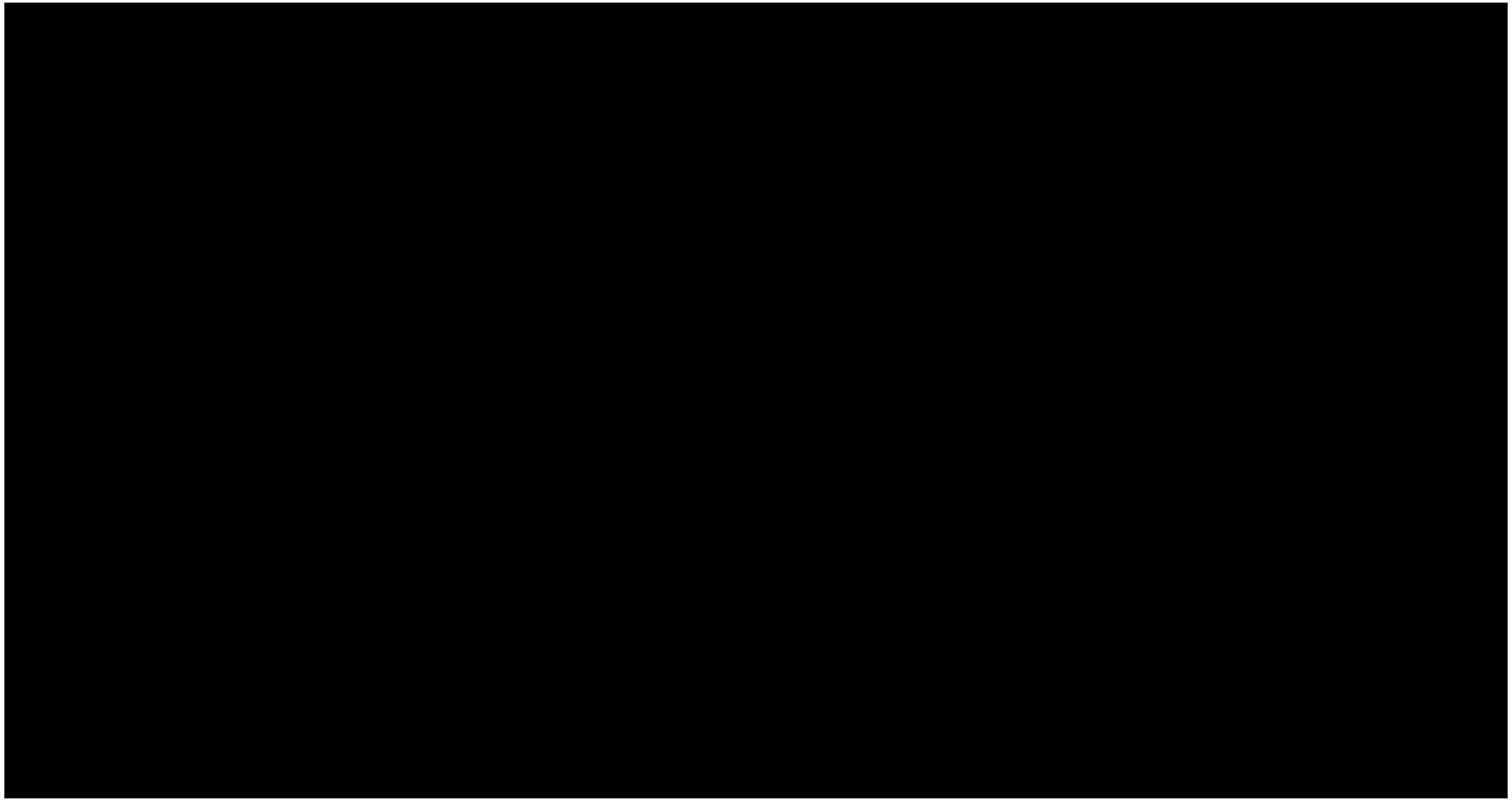
In Political Science:

Should a country invest in nuclear weapons?

In Computer Science:

Should you send messages over the Internet using correctly implemented TCP, or is it better to use defective TCP (i.e., one that does not have the backoff mechanism)?

The Prisoner's Dilemma



The Prisoner’s Dilemma

How is it possible that the prisoners are not flipping?

The reason is that the underlying assumptions of the game do not hold in this example:

1) They share the same lawyer, so the are not in isolation.
2) Someone is coordinating them by reimbursing a hazard pay, so now the benefit of being silent is much larger.

Credibility

Credibility

In the spring of 1519, Hernan Cortes left Cuba and sailed towards the Mexican coast with 508 men, 16 horses and a couple of pieces of artillery.

They disembarked at the city of Veracruz and found out about the existence of Montezuma, Aztec emperor in a splendid city in the interior who commanded a great army.

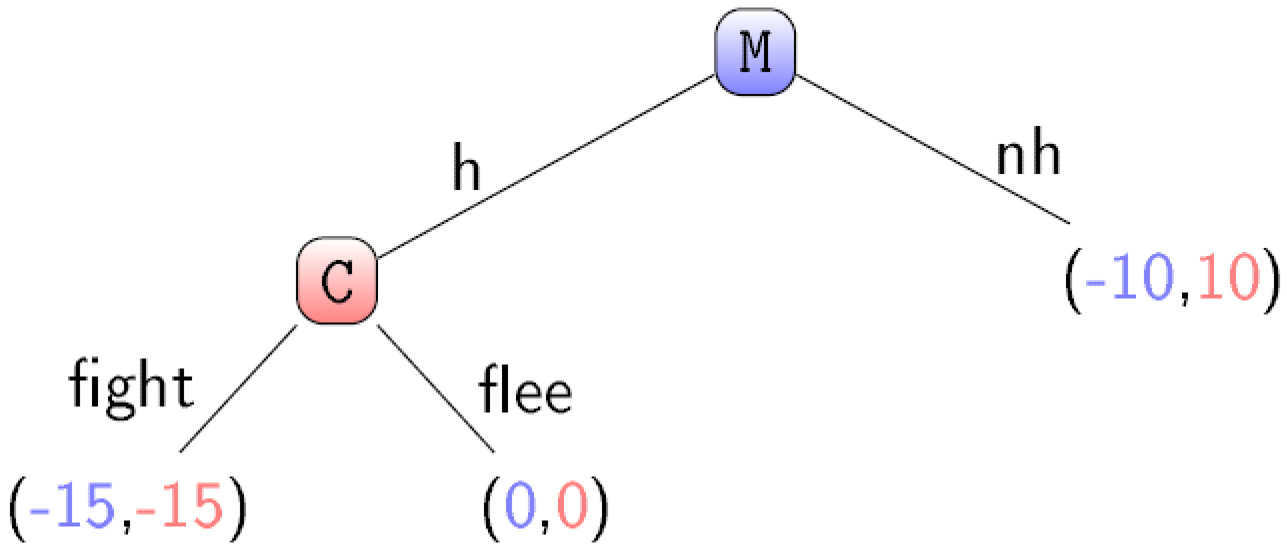
Cortes and his men also discovered that many cities under the Aztec rule actually disliked the Aztecs and that they might serve as allies. So, they decided to try and conquer the Aztecs.

Credibility

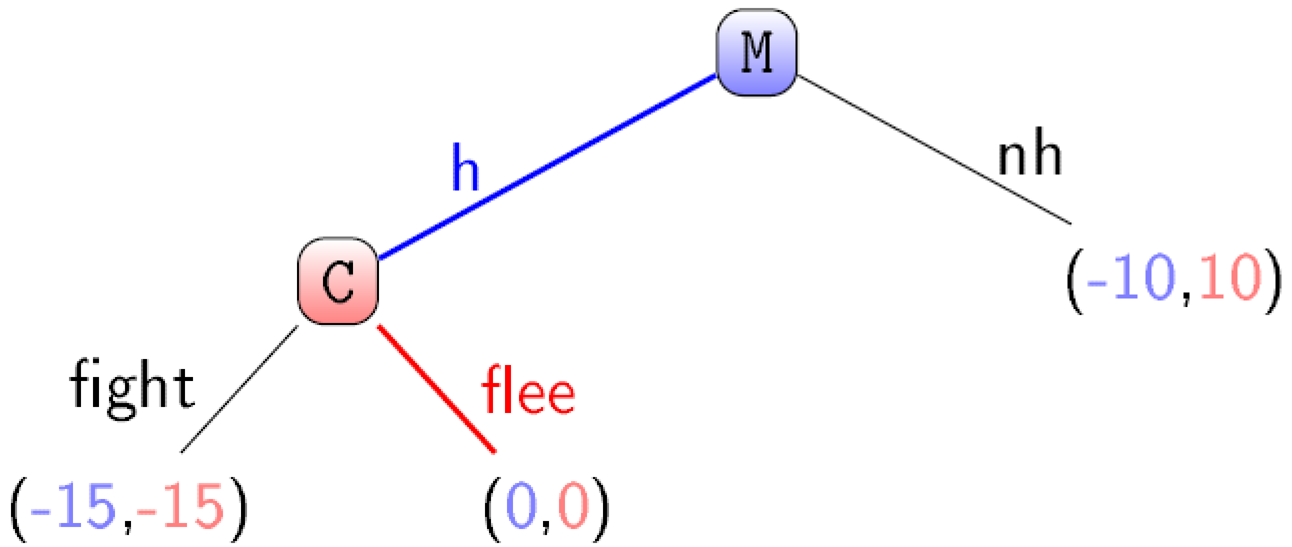
If Montezuma responded with hostility, Cortes would have the option of fighting or fleeing. In the first case, both sides would lose 15. In the second case, they would neither win nor lose anything: 0.

If Montezuma responded without hostility, Cortes and his men would gain 10, whereas Montezuma and his subjects would lose 10.

Credibility



Credibility

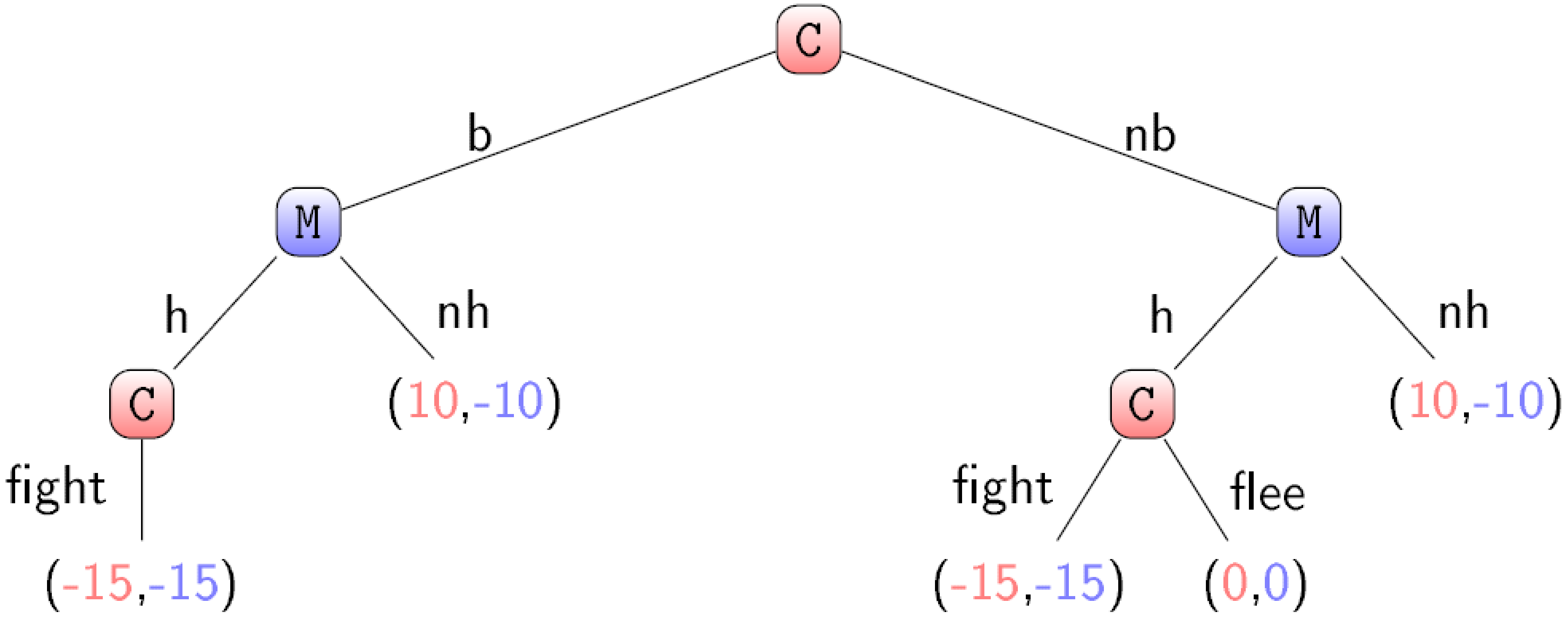


Credibility

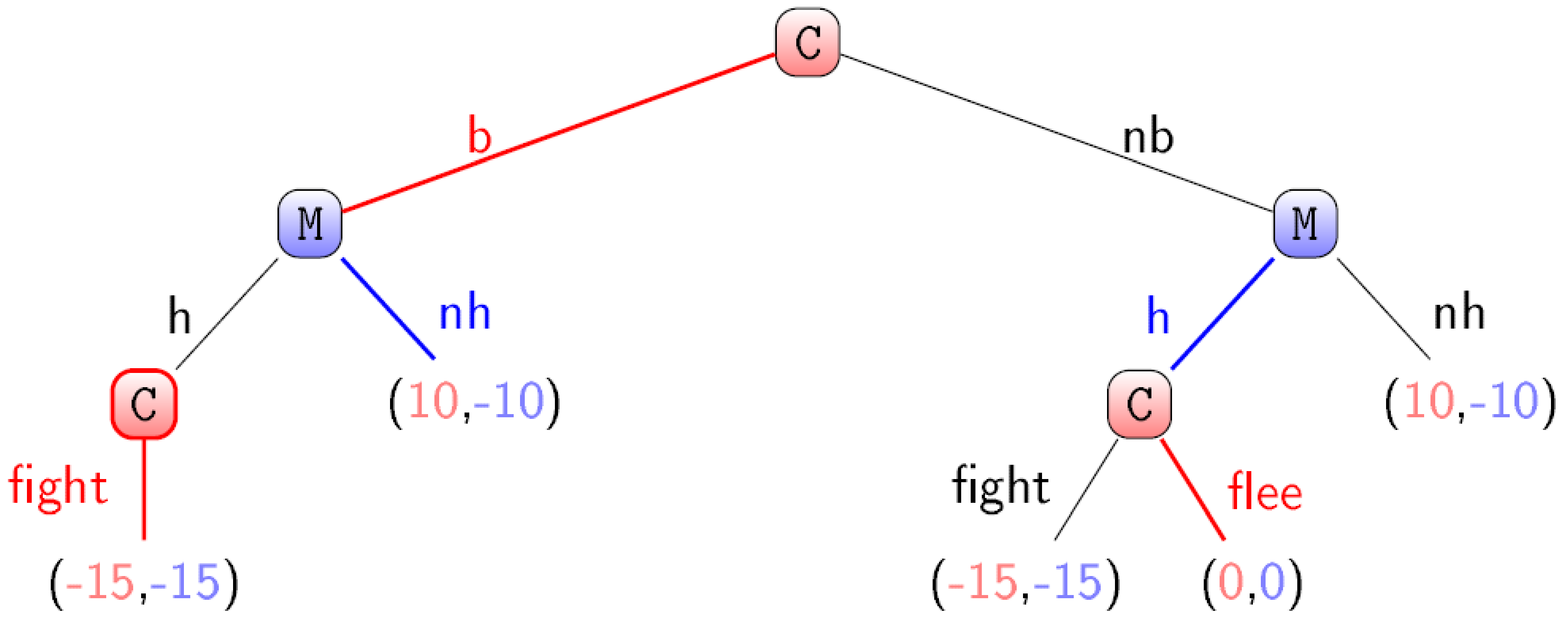
However, Cortes can change the game by burning all his boats in Veracruz before interacting with Montezuma. If so, the possibility of fleeing is no longer available.

https://www.kickstarter.com/projects/127568822/shabam-use-smart-devices-to-strengthen-family-rela?lang=ja 2018/06/05

Credibility



Credibility



Credibility

Credibility issues apply to several situations in real life.

For example, factories overinvest in capacity to tell their potential rivals that they are ready to produce more, which would decrease the market price and expulse them from the market.

Credibility

Rock, Paper, Scissors

What happens if Player A and Player B keep playing this game over and over?

Suppose that Player A always plays Rock. After few rounds, Player B figures this out and adopts a winning counterstrategy by always choosing Paper.

However, this cannot be an equilibrium: Player A observes that Player B is always playing Paper, so he switches to the strategy of always choosing Scissors.

But again, this is NOT an equilibrium for the same reason, and Player A and Player B will be chasing each other forever around the circle of strategies.

Rock, Paper, Scissors

But players could also try mixed strategies: playing each one of the shapes with certain *probability*.

Suppose that Player A plays Rock with probability *1/3*, Paper with probability *1/3*, and Scissors with probability *1/3*; and that Player B plays Rock with probability *1/2*, Paper with probability *1/4*, and Scissors with probability *1/4*. Then, the probability chart is

		Player B		
		Rock	Paper	Scissors
Player A	Rock	1/6	1/12	1/12
	Paper	1/6	1/12	1/12
	Scissors	1/6	1/12	1/12

Rock, Paper, Scissors

Considering that the winner receives 1 point, the loser loses 1 point and ties count as 0, the expected number of points received by each player when choosing the previous strategies are:

For Player A:

$$1/6(0) + 1/12(-1) + 1/12(1) + 1/6(1) + 1/12(0) + 1/12(-1) + 1/6(-1) + 1/12(1) + 1/12(0) = 0.$$

For Player B:

$$1/6(0) + 1/12(1) + 1/12(-1) + 1/6(-1) + 1/12(0) + 1/12(1) + 1/6(1) + 1/12(-1) + 1/12(0) = 0.$$

Rock, Paper, Scissors

But these strategies are NOT an equilibrium because Player A can do better!

Suppose that, given the previous strategy of Player B, Player A plays Rock with probability *1/4*, Paper with probability *1/2*, and Scissors with probability *1/4*. The new probability chart is

		Player B		
		Rock	Paper	Scissors
Player A	Rock	1/8	1/16	1/16
	Paper	1/4	1/8	1/8
	Scissors	1/8	1/16	1/16

Rock, Paper, Scissors

The expected number of points after Player A changes to the new strategy are:

For Player A:

1/8(0) + 1/16(-1) + 1/16(1) + 1/4(1) + 1/8(0) + 1/8(-1) + 1/8(-1) + 1/16(1) + 1/16(0) =

= 1/16 > 0, so Player A has incentives to deviate to the new strategy.

For Player B:

1/8(0) + 1/16(1) + 1/16(-1) + 1/4(-1) + 1/8(0) + 1/8(1) + 1/8(1) + 1/16(-1) + 1/16(0) =

= -1/16.

Rock, Paper, Scissors

Suppose now that both Player A and Player B choose each shape with probability *1/3*. The corresponding probability chart is

		Player B		
		Rock	Paper	Scissors
Player A	Rock	1/9	1/9	1/9
	Paper	1/9	1/9	1/9
	Scissors	1/9	1/9	1/9

Rock, Paper, Scissors

Now, the expected number of points are:

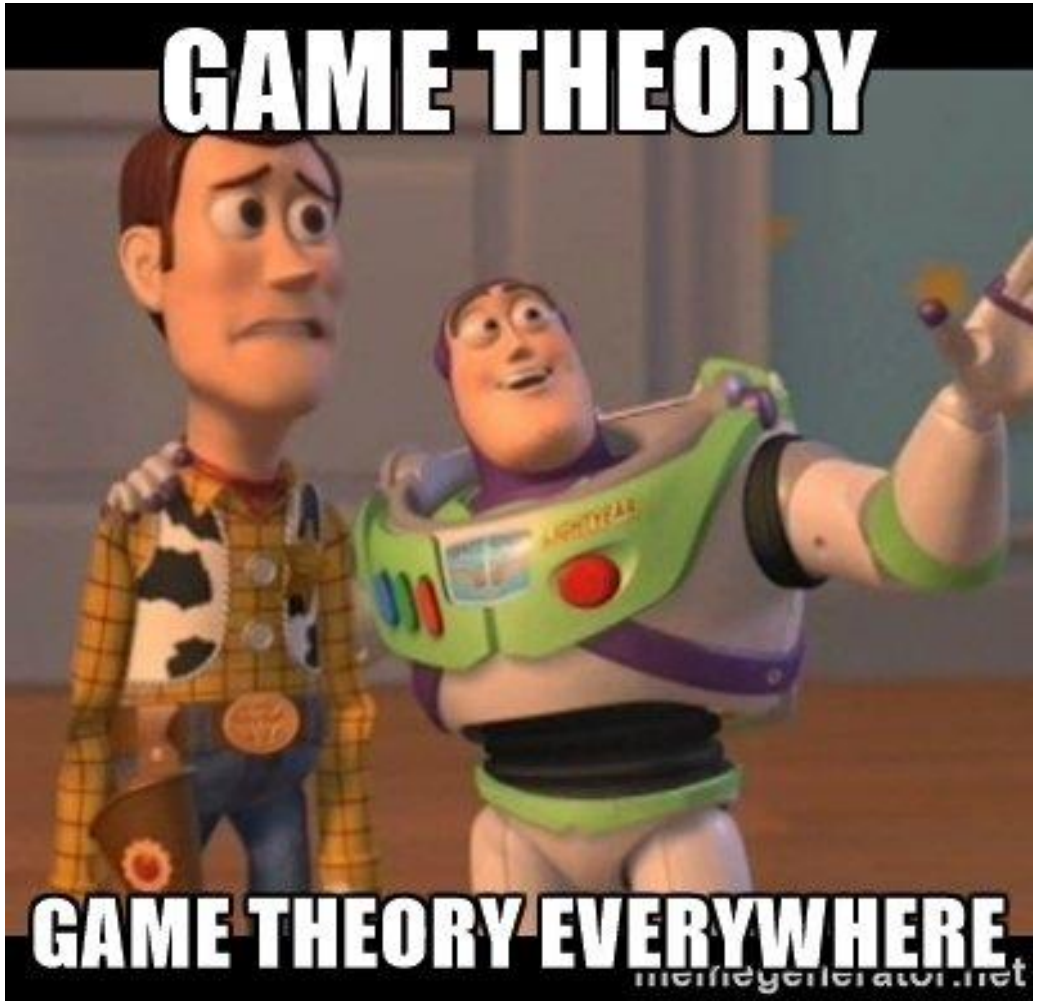
For Player A:

1/9(0) + 1/9(-1) + 1/9(1) + 1/9(1) + 1/9(0) + 1/9(-1) + 1/9(-1) + 1/9(1) + 1/9(0) = 0.

For Player B:

1/9(0) + 1/9(1) + 1/9(-1) + 1/9(-1) + 1/9(0) + 1/9(1) + 1/9(1) + 1/9(-1) + 1/9(0) = 0.

This is an equilibrium, not because the two players receive the same number of expected points, but because given the strategy of the rival, it is impossible for any player to find another strategy that improves their result.



https://www.scienceabc.com/social-science/what-is-game-theory-and-how-is-it-relevant.html 2018/06/05

https://theinterviewguys.com/exit-interview/ 2018/06/05