

Contents

1 Hilbert space and linear operators	5
1.1 Hilbert space	5
1.2 Bounded linear operators	9
1.3 Special classes of bounded linear operators	11
1.4 Unbounded, closed, and self-adjoint operators	15
1.5 Resolvent and spectrum	19
1.6 Positivity and polar decomposition	21
2 Normed ideals of $\mathcal{K}(\mathcal{H})$	25
2.1 Compact operators and the canonical expansion	25
2.2 Eigenvalues and singular values	27
2.3 Technical interlude	30
2.4 Normed ideals of $\mathcal{B}(\mathcal{H})$	35
2.5 The Schatten ideals \mathcal{J}_p	39
2.6 Usual trace	41
3 The Dixmier trace	47
3.1 Invariant states	47
3.2 Additional sequence spaces	49
3.3 Dixmier's construction	50
3.4 Generalizations of the Dixmier trace	53
3.4.1 Extended limits	54
3.4.2 Additional spaces on \mathbb{R}_+	56
3.4.3 Dixmier traces	57
4 Heat kernel and ζ-function	61
4.1 ζ -function residue	61
4.2 The heat kernel functional	64
5 Traces of pseudo-differential operators	67
5.1 Pseudo-differential operators on \mathbb{R}^d	67
5.2 Noncommutative residue	74
5.3 Modulated operators	75
5.4 Connes' trace theorem	83

