

Contents

1	C^*-algebras	5
1.1	Basics on C^* -algebras	5
1.2	Spectral theory	8
1.3	Matrix algebras	13
2	Projections and unitary elements	15
2.1	Homotopy classes of unitary elements	15
2.2	Equivalence of projections	19
2.3	Liftings	23
3	K_0-group for a unital C^*-algebra	25
3.1	Semigroups of projections	25
3.2	The K_0 -group	27
3.3	Functoriality of K_0	31
3.4	Examples	33
4	K_0-group for an arbitrary C^*-algebra	35
4.1	Definition and functoriality of K_0	35
4.2	The standard picture of the group K_0	37
4.3	Half and split exactness and stability of K_0	40
5	The functor K_1	43
5.1	Definition of the K_1 -group	43
5.2	Functoriality of K_1	47
6	The index map	51
6.1	Definition of the index map	51
6.2	The index map and partial isometries	55
6.3	An exact sequence of K -groups	58
7	Higher K-functors, Bott periodicity	61
7.1	The isomorphism between $K_1(\mathcal{C})$ and $K_0(S(\mathcal{C}))$	61
7.2	The long exact sequence in K -theory	63
7.3	The Bott map	66

7.4 Applications of Bott periodicity	69
8 The six-term exact sequence	71
8.1 The exponential map and the six-term exact sequence	71
8.2 An explicit description of the exponential map	72
9 Cyclic cohomology	77
9.1 Basic definitions	77
9.2 Cup product in cyclic cohomology	81
9.3 Unbounded derivations	84
9.4 Higher traces	88
9.5 Pairing of cyclic cohomology with K -theory	95
10 Application: Levinson's theorem	101
10.1 The \square -anisotropic algebra	101
10.2 Elementary scattering system	104
10.3 The abstract topological Levinson's theorem	106