Power Electronics
No.3 Smoothing Circuit

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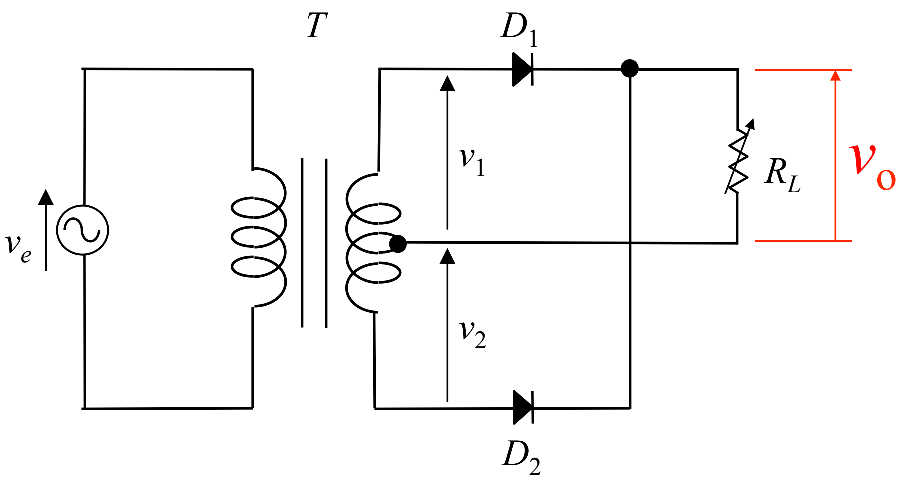
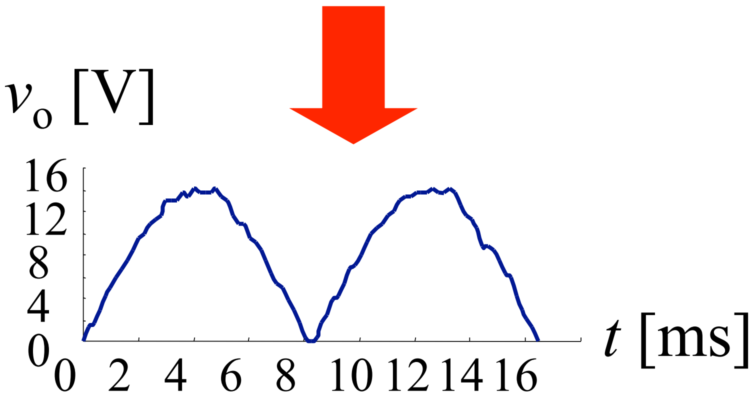
Smoothing circuit

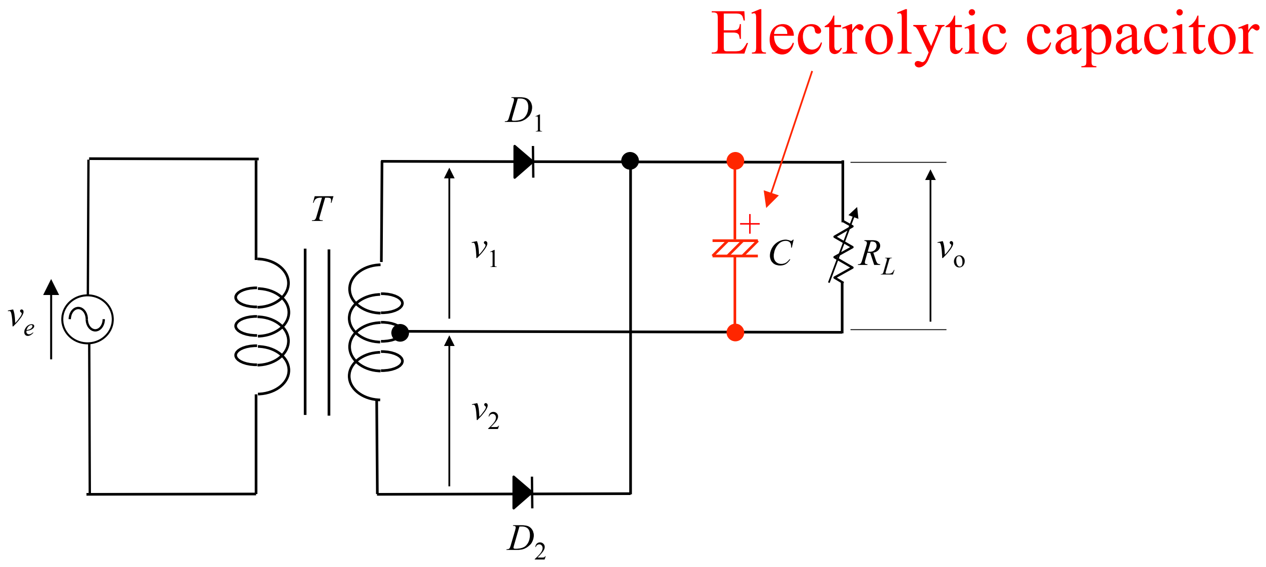


Full-wave rectifier

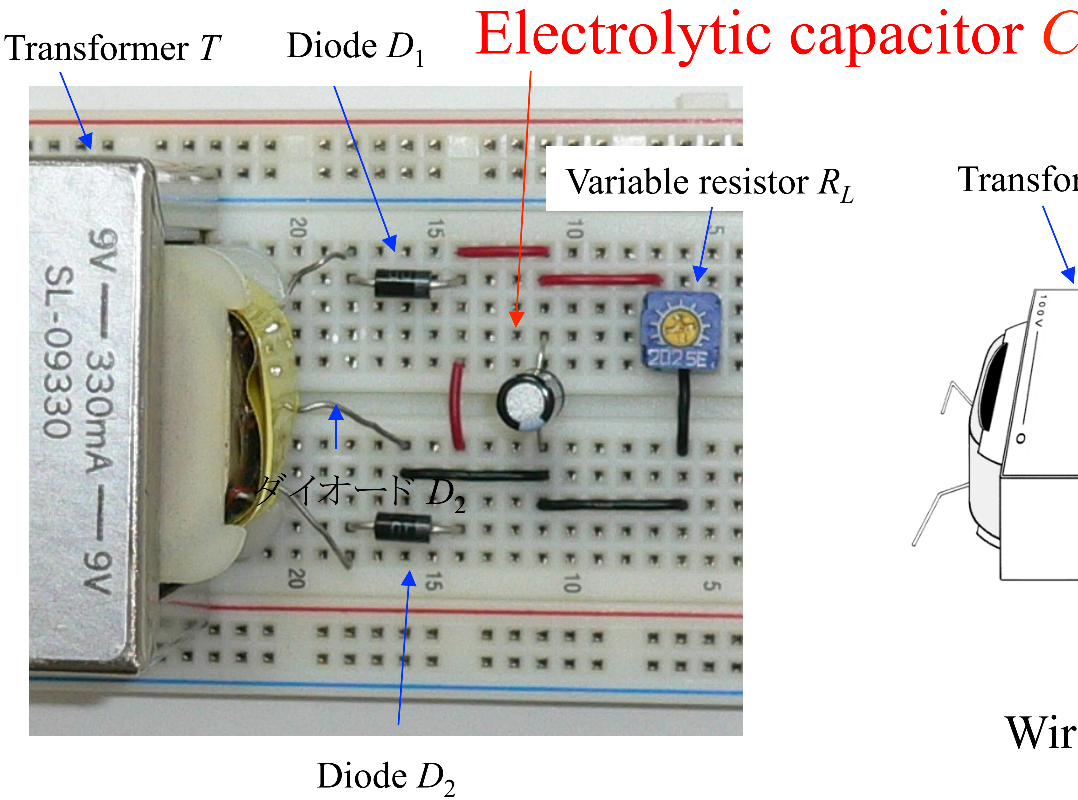
This voltage cannot be applied to electronic circuits



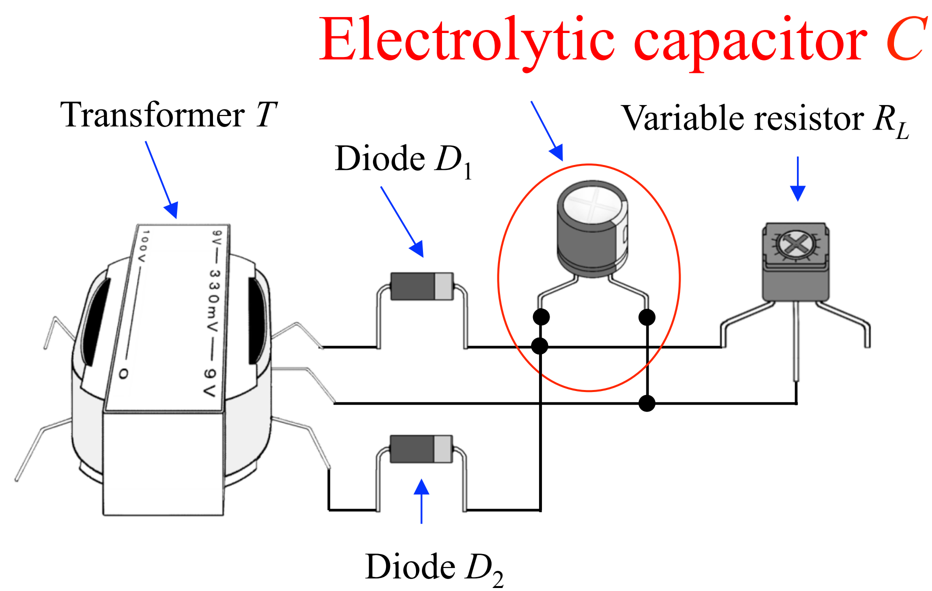
Wave form of output voltage v_o



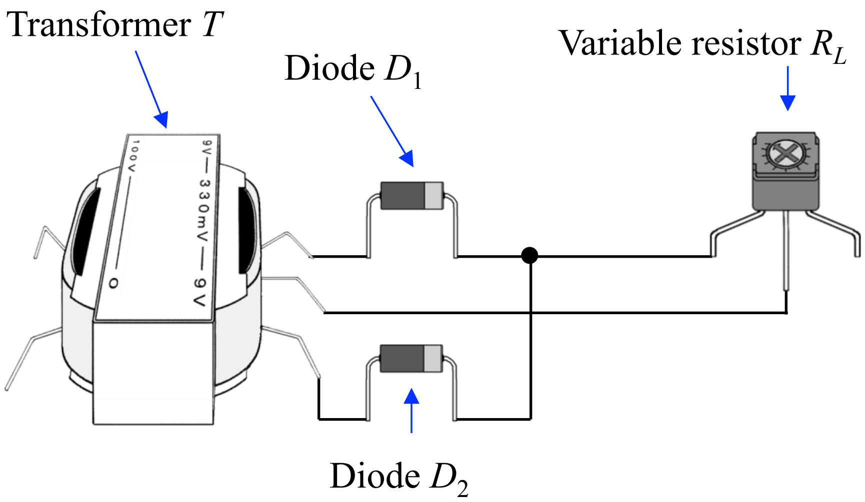
Full-wave rectifier with smoothing circuit using a capacitor

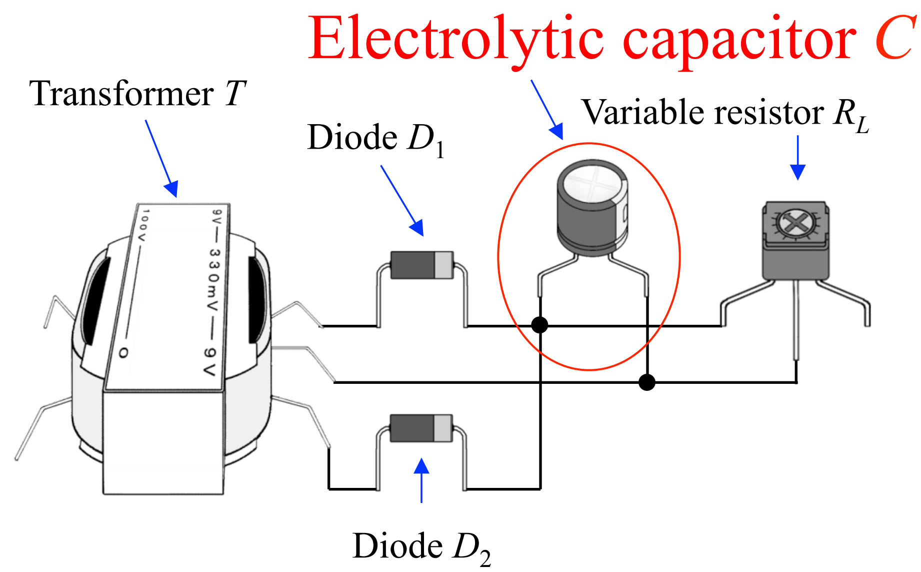


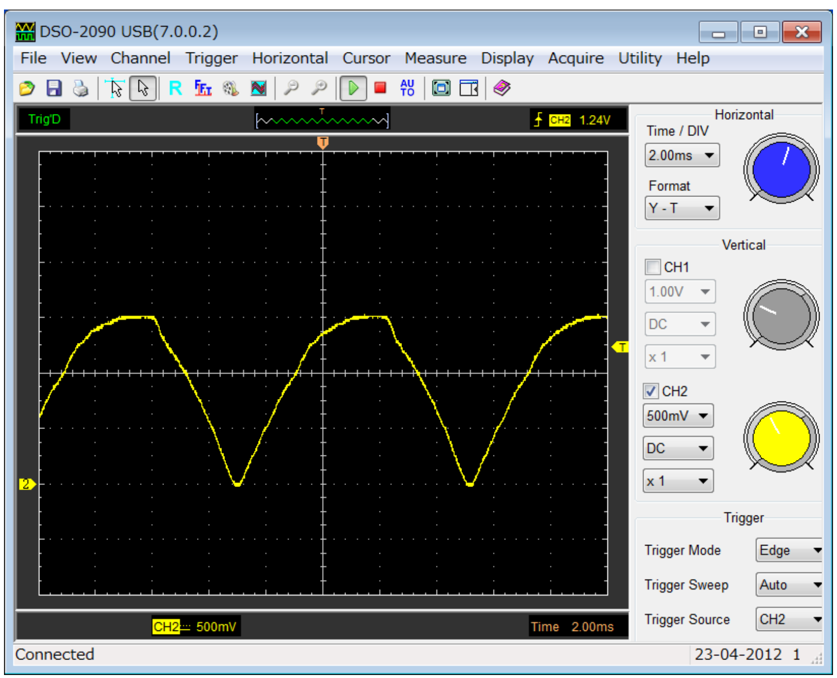
Example of a smoothing circuit

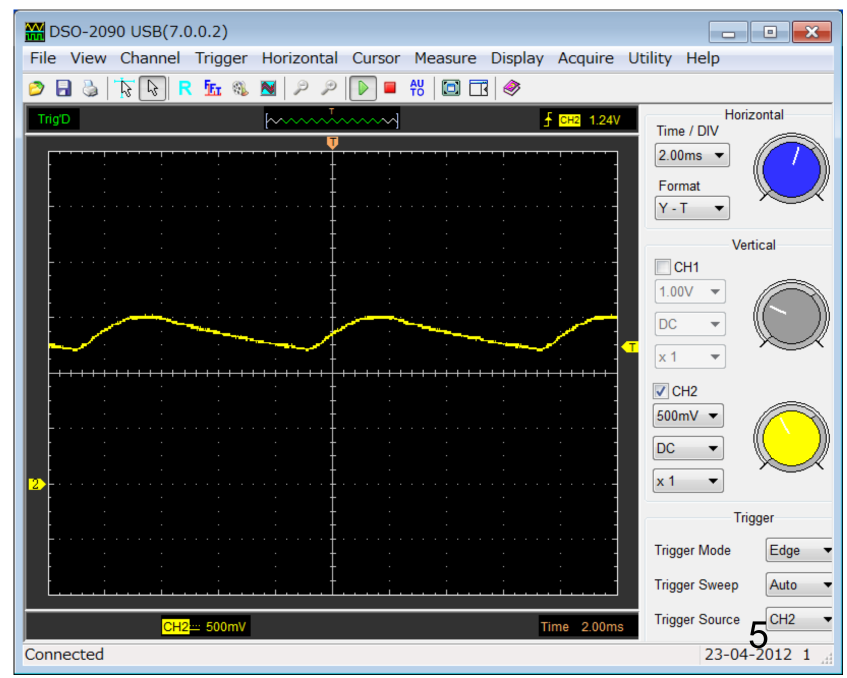


Wiring diagram of a smoothing circuit


Transformer T
Diode D_1
Variable resistor R_L
Diode D_2


Electrolytic capacitor C
Transformer T
Diode D_1
Variable resistor R_L
Diode D_2


DSO-2090 USB(7.0.0.2)
File View Channel Trigger Horizontal Cursor Measure Display Acquire Utility Help
Trig'D
CH2 1.24V
Horizontal
Time / DIV
2.00ms
Format
Y - T
Vertical
CH1
1.00V
DC
x 1
CH2
500mV
DC
x 1
Trigger
Trigger Mode Edge
Trigger Sweep Auto
Trigger Source CH2
CH2 500mV
Time 2.00ms
Connected
23-04-2012 1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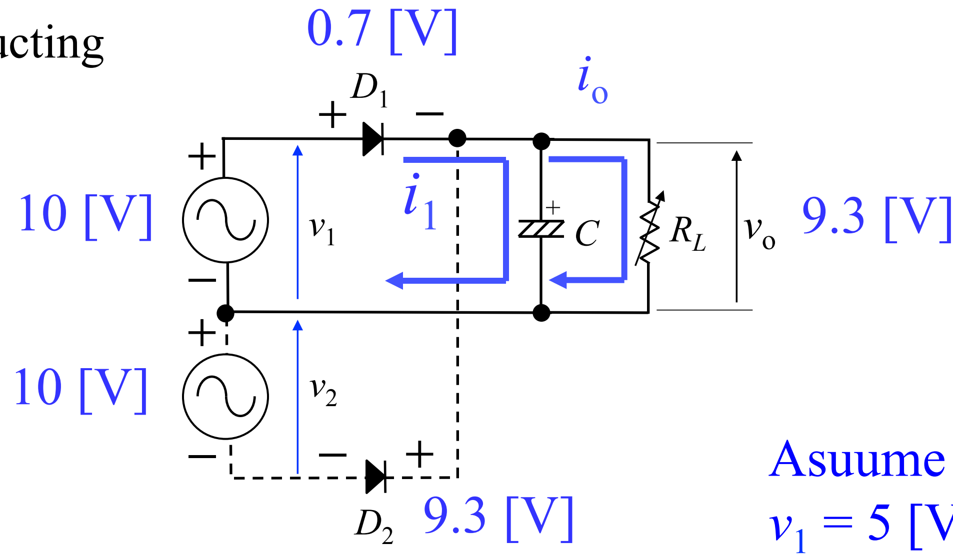
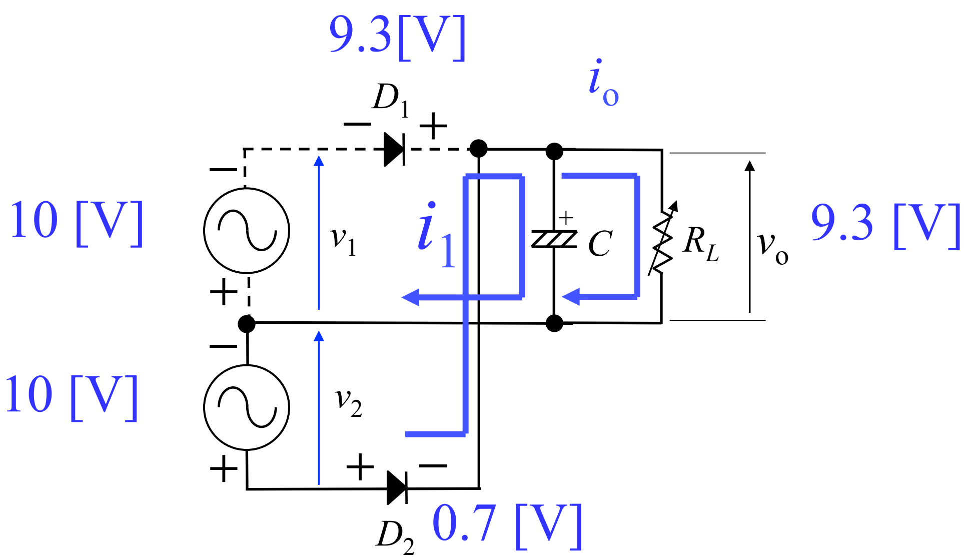
x 1
CH2
500mV
DC
x 1
Trigger
Trigger Mode Edge
Trigger Sweep Auto
Trigger Source CH2
CH2 500mV
Time 2.00ms
Connected
23-04-2012 1

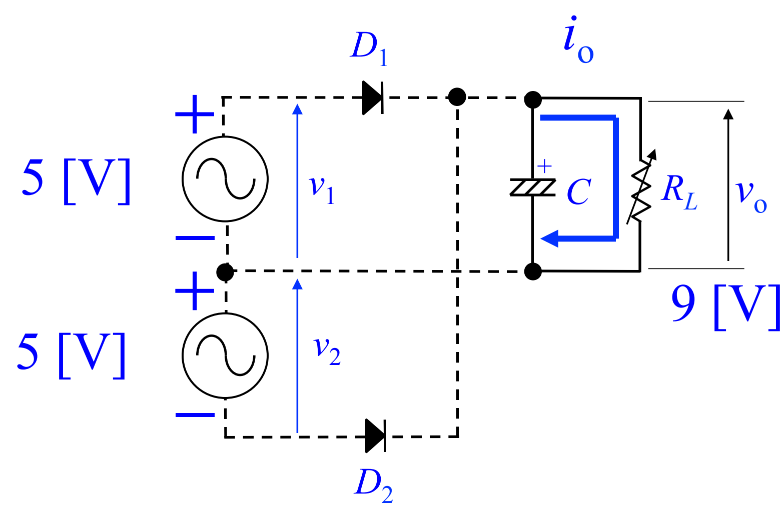
(a) Diode D_1 is conducting
($v_1 > v_o$, $v_1 > -v_2$)



Asuume that
$v_1 = 5$ [V], $v_2 = 5$ [V]
$v_o = 9$ [V]

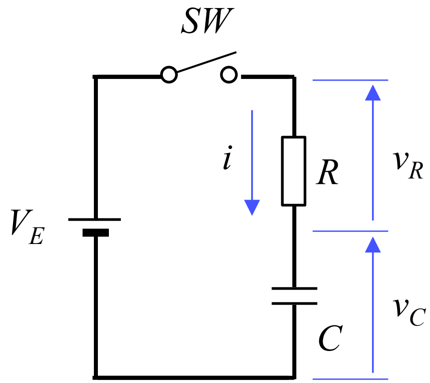


(b) Diode D_2 is conducting
($-v_2 > v_o$, $-v_2 > v_1$)



(c) Diodes are not conducting
($v_1 < v_o$, $-v_2 < v_o$)

Operating modes of a full-wave rectifier with a smoothing circuit



At $t = 0$, SW is turned on and the initial voltage of C : $v_C = V_{C0}$

$$V_E = Ri + \frac{1}{C}\int_0^t i\,dt + V_{C0}$$

$$i = \frac{dq}{dt}, \quad q = \int_0^t i\,dt + CV_{C0}$$

Then

$$V_E = R\frac{dq}{dt} + \frac{q}{C} \qquad (1)$$

Assuming that the charge q is in the form given by

$$q = Ae^{-\frac{t}{RC}} + B \qquad (2)$$

where A and B are constants. By substituing (2) into (1),

$$V_E = -\frac{A}{C}e^{-\frac{t}{RC}} + \frac{1}{C}(Ae^{-\frac{t}{RC}} + B) \qquad (3)$$

The initial condition is that at $t = 0, q = CV_{C0}$. Then, from (2)

$$CV_{C0} = A + B \,, \qquad (4)$$

and from (3),

$$B = CV_E \,. \qquad (5)$$

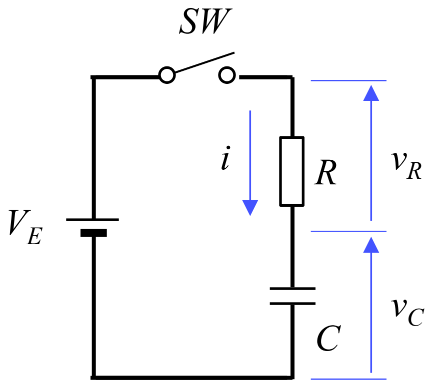
By substituing (5) into (4),

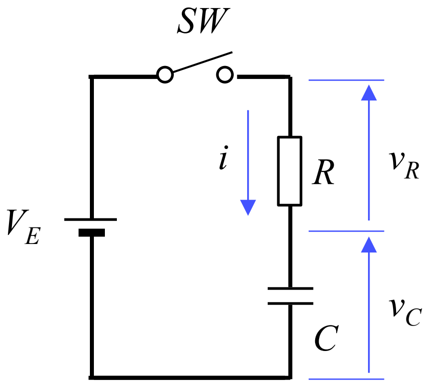
$$A = C(V_{C0} - V_E).$$

Thus,

$$q = CV_E(1 - e^{-\frac{t}{RC}}) + CV_{C0}e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = \frac{V_E - V_{C0}}{R} e^{-\frac{t}{RC}}$$





At $t = 0$, SW is turned on and the initial voltage of C : $v_C = V_{C0}$

$$V_E = Ri + \frac{1}{C}\int_0^t i\, dt + V_{C0}$$

By applying the Laplace transform to this eq.,

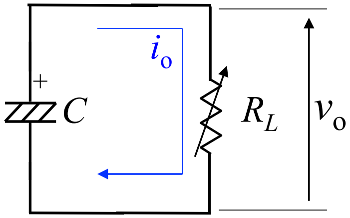
$$\frac{V_E}{s} = RI(s) + \frac{I(s)}{sC} + \frac{V_{C0}}{s}$$

$$\left(R + \frac{1}{sC}\right) I(s) = \frac{V_E - V_{C0}}{s}$$

$$I(s) = \frac{V_E - V_{C0}}{R} \frac{1}{s + \frac{1}{RC}}$$

By applying the Inverse Laplace transform to the above eq.,

$$i(t) = \frac{V_E - V_{C0}}{R} e^{-\frac{1}{RC}t}$$



(b) Diode is not conducting

Equivalent circuit of the rectifier

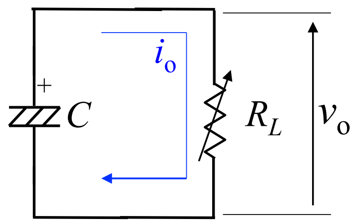
$$V_E = Ri + \frac{1}{C}\int_0^t i\,dt + V_{C0}$$

At $t = 0$,

$$i = i_o$$

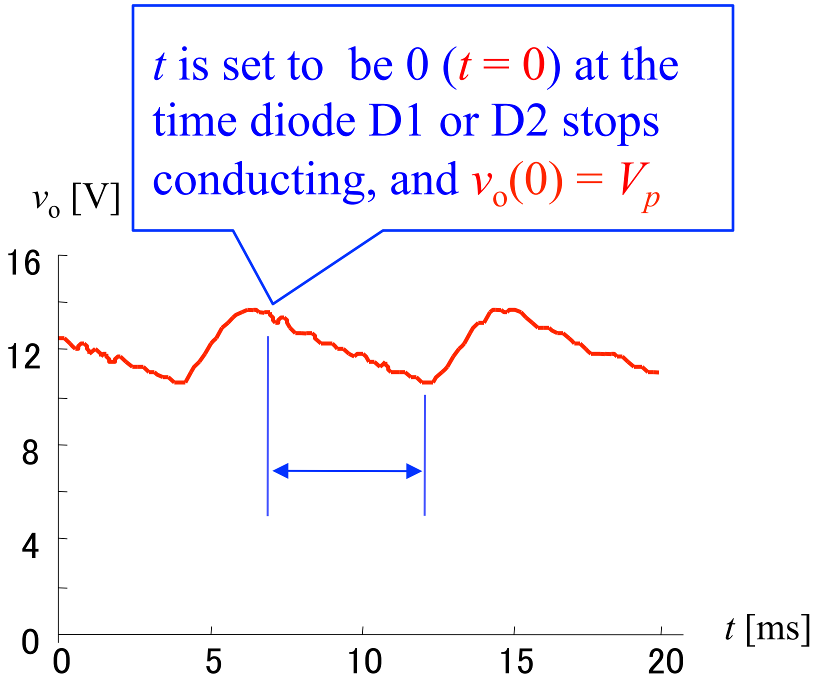
$$V_E = 0, V_{C0} = -v_0(0)$$

$$R_L i_o + \frac{1}{C}\int_0^t i_o dt = v_o(0)$$



(b) Diode is not conducting

Equivalent circuit of the rectifier



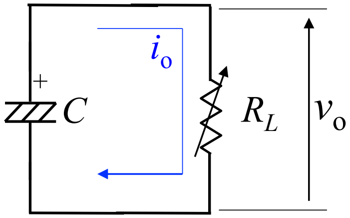
Voltage waveform of v_o

$$R_L i_o + \frac{1}{C}\int_0^t i_o dt = v_o(0)$$

$$R_L I_o + \frac{1}{sC} I_o = \frac{V_p}{s}$$

$$I_o = \frac{1}{s + \frac{1}{R_L C}} \frac{V_p}{R_L}$$

$$i_o = \frac{V_p}{R_L} e^{-\frac{t}{R_L C}}$$

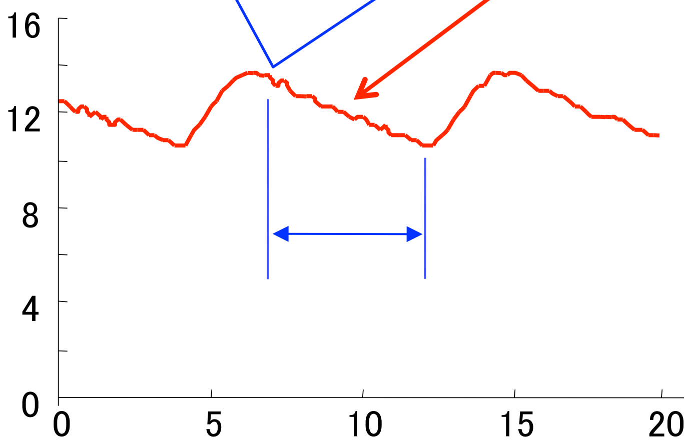


(b) Diode is not conducting

Equivalent circuit of the rectifier

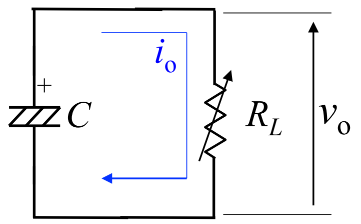
$$v_o = R_L\, i_o = V_p\, e^{-\frac{t}{R_L C}}$$

t is set to be 0 ($t = 0$) at the time diode D1 or D2 stops conducting, and $v_o(0) = V_p$



$R_L\,C$: time constant [sec]

Voltage waveform of v_o



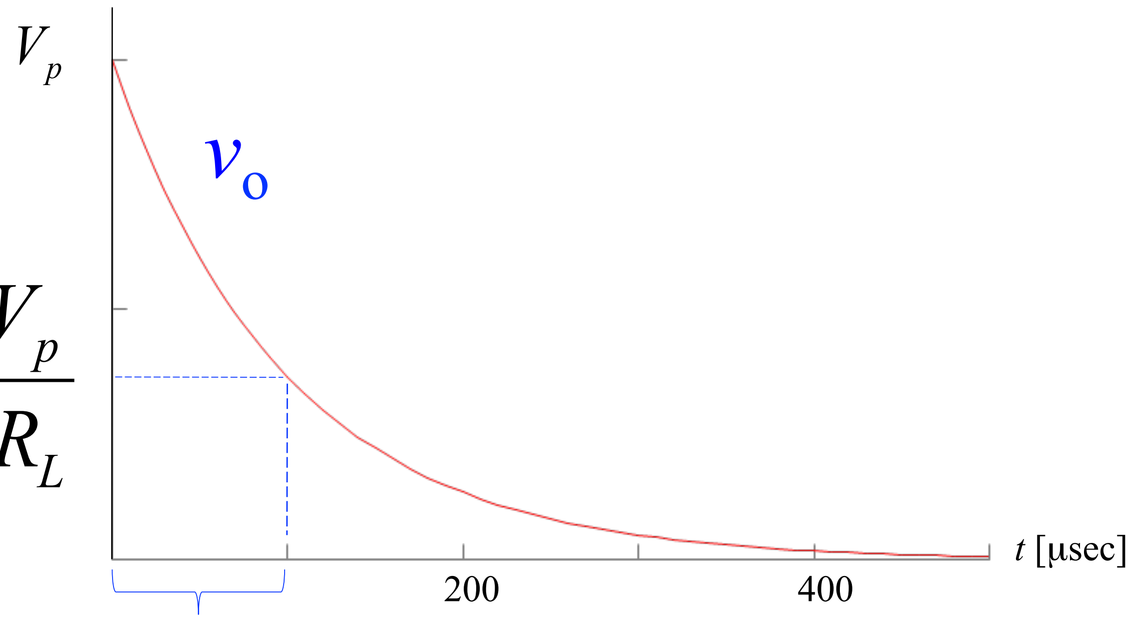
$$v_o = V_p \, e^{-\frac{t}{R_L C}}$$

(b) Diode is not conducting

$R_L = 100[\Omega], C = 1\,[\mu\mathrm{F}]$のとき

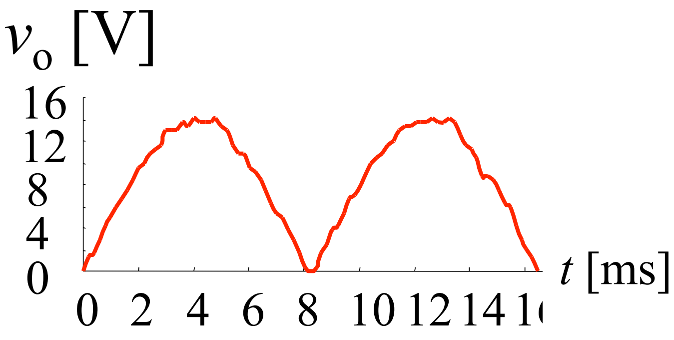
Equivalent circuit of the rectifier

$$e^{-1}\frac{V_p}{R_L} = 0.37\frac{V_p}{R_L}$$

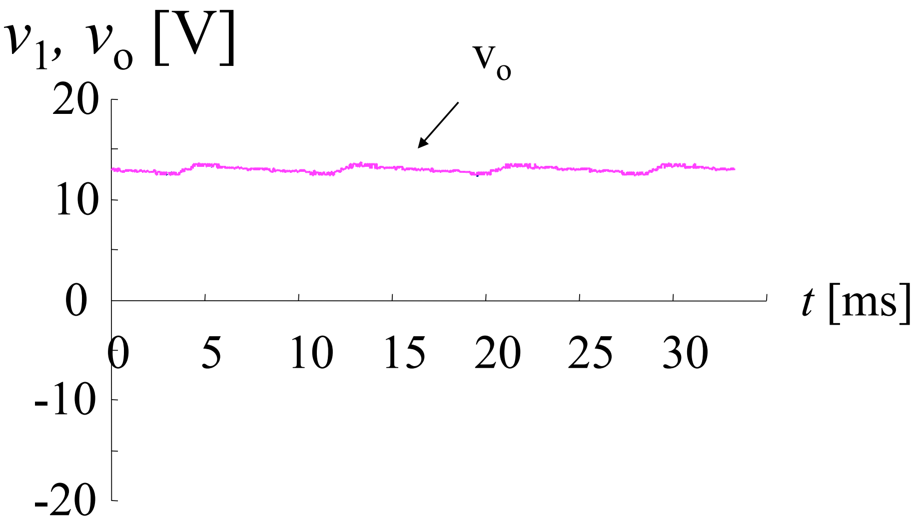


$$\tau = R_L C = 100 \times 10^{-6} = 100\,[\mu\mathrm{sec}]$$

Time constant



Not smoothed

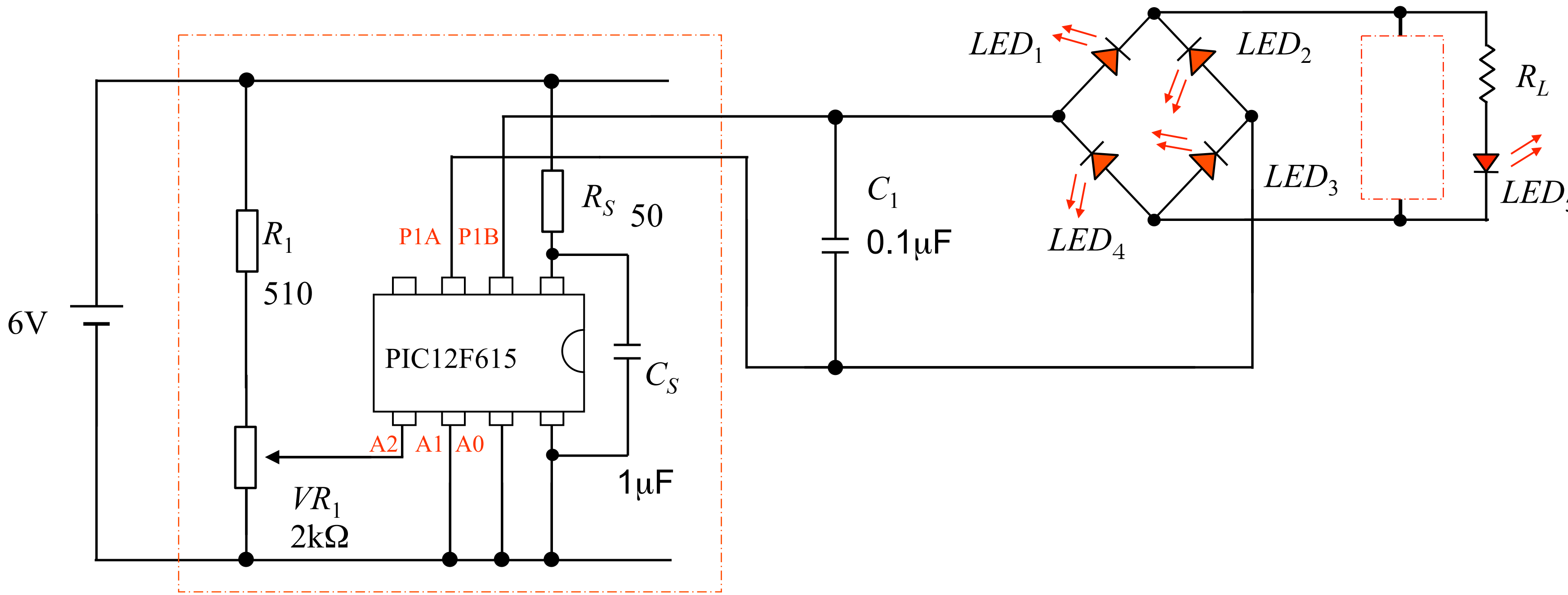


Smoothed

Example of the measured output voltage waveform v_o

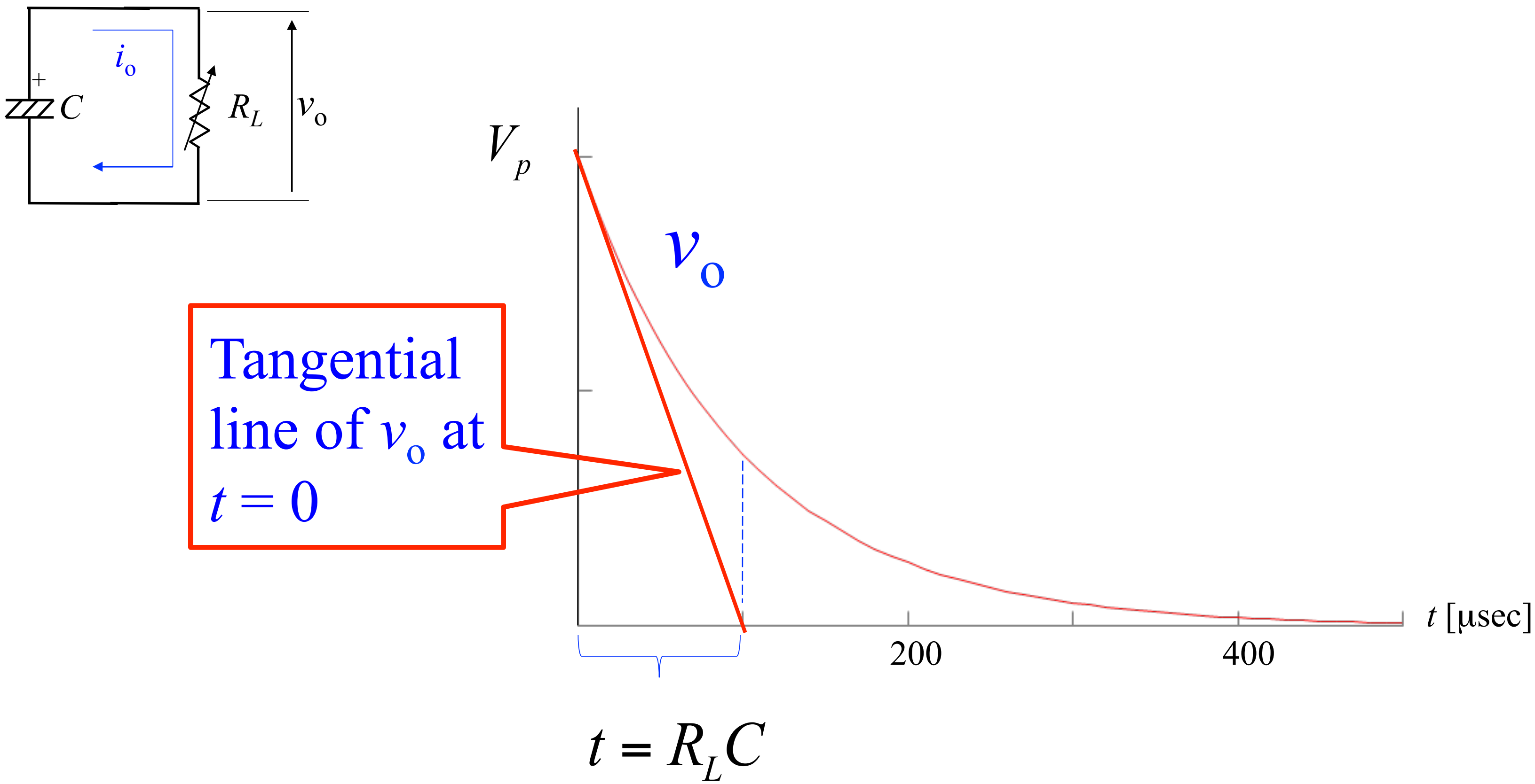
STEP 2. Circuit construction practice (smoothing circuit)

Design a smoothing circuit. Capacitor C should be designed so that the capacitor can be connected and disconnected to the rectifier by a push switch. Determine the capacitance of capacitor C and the resistance of resistor R_L so that the time constant of the smoothing circuit is approximately 0.24[sec].



STEP 2. Problem 1

The figure below shows a waveform of output voltage v_o in the non-conducting mode of diodes. At $t = 0$, the diode is assumed to stop conducting. Show that the tangential line of v_o at $t = 0$ crosses the horizontal axis at $t = R_L C$.



STEP 2. Problem 2

The figure below shows a series circuit of resistor R and reactor L. Answer the following questions:

(a)Assume that switches SW_1 and SW_2 are OFF. At $t = 0$, switch SW_1 is turned on. Write the differential equation of this circuit at $t \geq 0$.

(b)At $t = 0$, current $i = 0$. Solve the differential equation at $t \geq 0$, and obtain the equation of current i.

(c) At $t = 10\ L/R$, switch SW_1 is turned off, and switch SW_2 is turned on. Write the differential equation of this circuit at $t \geq 10\ L/R$.

(d) At $t = 10\ L/R$, current $i = I_0$. Solve the differential equation at $t \geq 10\ L/R$, and obtain the equation of current i.

(e) Draw the waveform of current i.