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# 衝撃波中の粒子加速

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最初は1-2年ごとに新課題に取組んだ

Vlasov code: Nonlinear self-modulation of ion-acoustic waves Magnetostatic code: Plasma paramagnetism Monte Carlo code: Plasma confinement in RF-plugged cusp field Electromagnetic code: Stability of bumpy torus with hot electron rings

1984年から「衝撃波と粒子加速」の研究を開始

2012年12月にまとめの論文

Ultrarelativistic particle acceleration in collisionless shock waves

#### Ultrarelativistic Particle Acceleration in Collisionless Shock Waves

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#### Abstract

This paper describes the theory and particle simulations of ultrarelativistic particle acceleration caused by shock waves in a collisionless magnetized plasma.

Since knowledge of field strengths and structures is necessary for the analysis of particle motions, theories of magnetosonic waves are reviewed first: (1) linear and nonlinear magnetosonic waves in a single-ion-species plasma, (2) those in a two-ion-species plasma, (3) those in an electron-positron-ion (EPI) plasma, and (4) the electric field parallel to the magnetic field,  $E_{\parallel}$ . The first topic contains a general introduction to the magnetosonic wave. The second and third topics are concerned with three-component plasmas, in which the magnetosonic wave is split into two modes; the plasma behavior can thus be considerably different from that in a single-ion-species plasma. The fourth topic is the parallel electric field  $E_{\parallel}$  in a nonlinear magnetosonic wave. It is shown that  $E_{\parallel}$  can be strong even in low frequency, magnetohydrodynamic phenomena.

Next, nonstochastic particle acceleration in intense electric and magnetic fields formed in a shock wave is studied with theory and with fully kinetic, fully relativistic, electromagnetic, particle simulations. The subjects include (1) electron trapping and acceleration, (2) energization of thermal and relativistic ions, (3) heavy-ion acceleration and resultant damping of nonlinear pulses in a multi-ion-species plasma, and (4) positron acceleration due to  $E_{\parallel}$  in the shock transition region in an EPI plasma. In addition to these processes near a shock front, (5) the evolution of large-amplitude Alfvén waves generated behind a shock front and acceleration of electrons in the Alfvén

wave region are examined.

Simulations demonstrate particle acceleration caused by these nonlinear magnetohydrodynamic waves to ultrarelativistic energies much higher than those of solar energetic particles. The acceleration theory based on the investigation of nonlinear waves quantitatively accounts for these simulation results.

Keywords:

particle acceleration, collisionless shock wave, KdV equation,

single-ion-species plasma, multi-ion-species plasma, electron-positron-ion plasma

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December 11, 2012

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#### 1. Introduction

Cosmic rays have been investigated for nearly a century and are still attracting increasing attention from plasma, particle, and astrophysics communities [1]-[14]. Their acceleration mechanism, however, remains unresolved. Unlike the studies of plasma-based accelerators initiated by John Dawson *et al.* in the late 1970's [15, 16], in which detailed comparisons between the experiments, theories, and simulations are possible, it is quite difficult to directly observe the acceleration processes of cosmic rays produced in the distance, although we have a huge amount of experimental data, such as time variations of x-ray and gamma-ray emission associated with solar flares [3].

Because of the rapid increase in the power of computers, however, we can now perform simulations that solve large-scale plasma behavior and individual relativistic particle motions in a self-consistent manner. Their precise information about particle motions and electromagnetic fields would enable us to create new theories for particle acceleration and to test existing theories. With use of relativistic particle simulations, indeed, several distinct nonstochastic particle acceleration mechanisms caused by shock waves in a magnetized collisionless plasma have been found and analyzed in the past few decades [17]-[35]. Furthermore, to account for the field structures that lead to energization of particles, nonlinear wave theory has been developed [36]-[44]: A coherent theory for nonlinear waves and particle acceleration mechanisms has thus been constructed. This paper reviews these studies.

Before looking at detailed theories, however, we briefly describe in this section some fundamental properties of cosmic rays for the readers who are not familiar with them and then outline the structure of this paper.

#### 1.1. Cosmic rays

The origin of the research of cosmic rays may date far back to 1912, when Hess revealed with balloon flight experiments that radiation causing ionization in the atmosphere comes mainly from the sky, not from the ground [45].

#### Ultrarelativistic Particle Acceleration in Collisionless Shock Waves

#### 1 Introduction

#### Wave

2 Structure of nonlinear magnetosonic waves in a single-ion-species plasma

- 3 Waves in a multi-ion-species plasma
- 4 Waves in an EPI plasma
- 5 Parallel electric field

#### Acceleration

- 6 Trapping and ultrarelativistic acceleration of electrons
- 7 Ion acceleration
- 8 Heavy-ion acceleration
- 9 Positron acceleration
- 10 Wave evolution and particle acceleration behind a shock front

Appendices A ---- J

特徴

#### 大振幅波の中に形成される強力な電磁場によって引き起こされる nonstochasticな粒子加速を探求 (Fermi加速や乱流のようなstochasticなモデルではない)

粒子加速と大振幅波を並行して研究 第一原理から

超相対論的加速 (γ>100) を粒子シミュレーションで実証

(太陽高エネルギー電子の γ~100、 陽子 γ<10)

注:様々なエネルギー

地球大気	0.03 eV
太陽表面	0.6 eV
太陽コロナ	100 eV
太陽の中心部	1.5 keV
核融合プラズマ	10 keV
太陽宇宙線陽子	1~10 GeV
電子	数十MeV (γ~100)

## **Observations of high-energy particles**

Cosmic Rays ~ 10<sup>20</sup> eV

Phys. Rev. Lett. 100, 101101 (2008)

EHECR: GZK cutoff, Anisotropic arrival directions

SN1006, Crab Nebula ~ 10<sup>14</sup> eV (electrons)

Nature **378**, 255 (1995) , ApJ **539**, 317 (2000) **Supernova Remnant RS J1713.7-3946** ~ 10<sup>12</sup> eV (protons) PASJ **55**, L61 (2003), Nature **432**, 75 (2004) **Solar Energetic Particles** 10<sup>9</sup>~ 10<sup>10</sup> eV (protons), 10<sup>7</sup> ~ 10<sup>8</sup> eV (electrons)

< a few seconds

```
ApJ 318, 913 (1987)
```

Elemental Composition of cosmic rays similar to that of the universe

ApJ. Suppl. 57, 173 (1985)

## 無衝突衝撃波とは

#### 二体衝突の非常に少ない高温プラズマにおける衝撃波 例:バウショック、超新星爆発による星間空間の衝撃波

#### 無衝突でなぜ衝撃波ができるか?

Morawetz イオン反射で衝撃波の構造 多数の著者 不安定性が乱流をつくり、散逸を生じる →プラズマ加熱



## Three waves in one-fluid MHD



## Nonlinear magnetosonic waves in a single-ion-species plasma Small-amplitude waves propagate as solitary waves or wavetrains governed by the KdV Equation

#### Large-amplitude waves evolve into shock waves

$$\frac{B_{lzm}}{B_{lz0}} = 1 + \gamma_{sh}^2 \left[ \left( 1 + \frac{2v_{sh}^2}{v_A^2 \sin^2 \theta} \right)^{1/2} - 1 \right],$$
  
$$\frac{e\phi_{lm}}{m_i v_A^2} = \left( \sin^2 \theta + \frac{\sin \theta \cos \theta}{\gamma_{sh} (1 + \gamma_{sh}^2 \tan^2 \theta)^{1/2}} \right) \left[ \left( 1 + \frac{2v_{sh}^2}{v_A^2 \sin^2 \theta} \right)^{1/2} - 1 \right].$$

KdV equation

Finite-amplitude, stationary, perpendicular wave Shock wave

field strengths

Nakazawa & Ohsawa, J. Phys. Soc. Jpn. 66, 2044 (1997)

Miyahara, Kawashima, & Ohsawa, Phys. Plasmas 10, 98 (2003)



## Multi-ion-species plasma

In a two-ion-species plasma, the magnetosonic wave is split into two modes: High- and low-frequency modes

Although the high-frequency mode has a finite cutoff frequency, we have derived the KdV equation for each mode

The pulse width of the high-frequency mode,  $\sim c/\omega_{\rm pe}$ , Is much shorter than that of the low-frequency mode,  $\sim c/\omega_{\rm pi}$ 

Even a perpendicular pulse is damped in a multi-ion-species plasma due to the heavy-ion acceleration 0.

Toida, Ohsawa, & T. Jyounouchi, Phys. Plasmas **2**, 3329 (1995) Dogen, Toida, & Y. Ohsawa, Phys. Plasmas **5**, 1298 (1998) Irie & Ohsawa, Phys. Plasmas **10**, 1253 (2003)





High-frequency-mode solitons are generated from a low-frequencymode pulse



## **Electron-positron-ion (EPI) plasma**

The theory for two-ion-species plasmas has been extended to EPI plasmas

Linear dispersion relations are obtained Nonlinear evolution equations for two magnetosonic modes (M & H) are derived Their field structures are analyzed



The electric potential decreases with increasing positron density:  $\phi = 0$  in a pure electron-positron plasma

> Hasegawa, Irie, Usami, & Y. Ohsawa, Phys. Plasmas **9**, 2549 (2002) Hasegawa & Ohsawa, J. Phys. Soc. Jpn. **73**, 1764 (2004)

## **Parallel Electric field**

In the Ideal MHD,

$$\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} = 0$$

$$E_{\parallel} = \frac{\mathbf{E} \cdot \mathbf{B}}{B} = 0$$

$$F = -\int E_{\parallel} ds = 0$$
Parallel pseudo potential

It was thought that  $E_{||}$  was weak in MHD phenomena

However, some simulations show that *F* >>*T* 

F becomes large in shock waves  $E_{11}$  can cause strong particle acceleration  $\theta = 60^{\circ}, m_i/m_e = 400, v_{Te}/c = 0.2$ Small-amplitude pulses  $\varepsilon <<1$ 10<sup>1</sup>  $eF_{T} \sim \mathcal{E} I_{e} I_{e} \qquad \text{warring} \qquad \int_{V_{A}}^{+} \int_{V_{A}}^{+} \int_{U_{A}}^{+} \int$  $eF_T \sim \varepsilon \Gamma_\rho T_\rho$  $(m_i v_A^2 + \Gamma_e T_e) \cdot (B_{z1}/B_0)$  $|\mathsf{F}_{\mathsf{R}}|$ Shock waves  $\varepsilon^{-1}$ • F,  $\bigcirc \phi$  for  $|\Omega_{e}|/\omega_{pe} = 0.5$ • F,  $\triangle \phi$  for  $|\Omega_{e}|/\omega_{pe} = 0.2$ 10<sup>-2</sup>  $eF \sim \mathcal{E}(m_i v_A^2 + \Gamma_e T_e)$ 2 10 5  $B_{71}/B_{0}$ 

Takahashi & Ohsawa, Phys. Plasmas **14**, 112305 (2007): electron-ion plasma Takahashi, Sato, & Ohsawa, Phys. Plasmas **15**, 082309 (2008) : EPI plasma

## Simulations of particle acceleration

#### Stochastic models

Fermi acceleration model (1949)

No evidence has been shown by particle simulations

#### Turbulence due to instabilities

Many simulations have been performed: For instance, Dieckmann *et al.*, Instabilities, v~20 v<sub>Te</sub> Astron. Astrophys. **356**, 377 (2000)

#### Non-stochastic model

Acceleration caused by strong electric and magnetic fields formed in shock waves

Several different acceleration mechanisms to ultrarelativistic energies,  $\gamma$ >100, have been demonstrated with particle simulations

Theory and particle simulations have shown ultrarelativistic particle acceleration in collisionless shock waves

1. protons

Phys. Fluids 28, 2130 (1985), Phys. Plasmas 9, 1069 (2002)

2. heavy ions

Solar Phys. 171, 161 (1997), Phys. Plasmas 2, 3329 (1995); 5,1298 (1998)

#### 3. electrons

Phys. Plasmas **6**, 3076 (1999); **9**, 979 (2002)

Phys. Plasmas 12, 052308 (2005); 13, 063110 (2006); 18, 092307 (2011)

### 4. positrons

Phys. Plasmas 10, 3455 (2003); 12, 082306 (2005); 19, 022302 (2012)

## Relativistic, electromagnetic particle simulation

$$\frac{d\mathbf{p}_{j}}{dt} = q_{j}\mathbf{E}(\mathbf{x}_{j}) + \frac{q_{j}}{c}\mathbf{v}_{j} \times \mathbf{B}_{j}(\mathbf{x}_{j})$$
$$j = 1, 2, 3, \dots N$$
$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{J}$$
$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$
$$\nabla \cdot \mathbf{E} = 4\pi\rho, \qquad \nabla \cdot \mathbf{B} = 0$$

## **Geometry and Simulation Code**



One dimensional (three velocities), Fully kinetic, Relativistic, Electromagnetic, Particle code

## Relativistic ions are promptly produced in a shock wave



Ohsawa, Phys. Fluids 28, 2130 (1985); Ohsawa, J. Phys. Soc. Jpn. 59, 2782 (1990)



Relativistic ions can stay near the shock front for long periods of time, if  $v_{sh} \sim c \cos \theta$ 

#### Their energies rise stepwise to ultrarelativistic energies



Usami & Ohsawa, Phys. Plasmas 9, 1069 (2002); ibid. 11, 918 (2004)



## All the heavy ions that enter a shock wave are accelerated

Their maximum speeds are independent of particle species

$$v \approx \frac{B_m - B_0}{B_m + B_0} v_{sh}$$

Energetic heavy ions thus have an elemental composition similar to that of the background plasma



θ=90

Toida & Ohsawa, Solar Phys. 171, 161 (1997)

B & O 1



## Oblique shock waves can accelerate electrons to ultrarelativistic energies

Some electrons are reflected and then trapped in a shock wave

Bessho & Ohsawa, Phys. Plasmas **6**, 3076 (1999); **9**, 979 (2002) Zindo *et al.*, Phys. Plasmas **12**, 052321 (2005)

$$\Omega_{e} \mid / \omega_{pe} = 3, \ \theta = 45^{\circ}, v_{sh} = 2.2 v_{A}$$

## Maximum $\gamma$ vs shock speed v<sub>sh</sub>

![](_page_25_Figure_1.jpeg)

## Strong particle acceleration has **not** been observed in shock waves in an **electron-positron** plasma;

for instance, Langdon, Arons, Max, PRL 61, 779 (1988)

# However, in an electron-positron-ion (EPI) plasma, intense $E_{||}$ persistently accelerates positrons

Hasegawa, Usami, & Ohsawa, Phys. Plasmas **10**, 3455 (2003); Hasegawa, Kato, & Ohsawa, *ibid*. **12**, 082306 (2005)

## $E_{||}$ =0 in an electron-positron plasma, while $E_{||}$ can be strong in an EPI plasma

Takahashi & Ohsawa, Phys. Plasmas 14, 112305 (2007); Takahashi, Sato, & Ohsawa, ibid. 15, 082309 (2008)

## Positron acceleration to $\gamma \sim 10^4$ in an EPI plasma

![](_page_27_Figure_1.jpeg)

## Acceleration of positrons and electrons to $\gamma \sim 10^4$

![](_page_28_Figure_1.jpeg)

Wave evolution and particle acceleration behind a shock front

Strong disturbances produce shock waves and, behind their fronts, large-amplitude Alfven waves

Three types of ultrarelativistic electron acceleration are found in the Alfven waves

> Sato, Miyahara, & Ohsawa, Phys. Plasmas **12**, 052308 (2005) Sato & Ohsawa, Phys. Plasmas **13**, 063110 (2006) Yamauchi & Ohsawa, Phys. Plasmas **14**, 053110 (2007) Takeyama, Nakayama, & Ohsawa, Phys. Plasmas **18**, 092307 (2011)

### Strong disturbance produces two shock waves

![](_page_30_Figure_1.jpeg)

![](_page_31_Figure_0.jpeg)

![](_page_32_Figure_0.jpeg)

Three types of acceleration are found in the Alfven wave region

$$\gamma_C \sim \frac{1 + E_{II}/B_{II}}{1 - E_{II}/B_{II}} \gamma_B,$$

Acceleration of electron gyrating along the strongmagnetic-field pulse

 $E_{I}$ 

BI

D

 $\mathbf{p}_{\mathbf{v}}$ 

А

Gyration in the

Π

EII

 $oldsymbol{igo}$ 

 $B_{II}$ 

configuration space

![](_page_33_Figure_1.jpeg)

Acceleration of electron meandering along a moving neutral sheet

I

EI

 $\otimes$ 

BI

DB

Π

EII

⊙ B<sub>II</sub>

![](_page_34_Figure_1.jpeg)

Acceleration of electron traversing the alternating magnetic field region

Ш

lacksquare

 $\otimes$ 

E

E<sub>I</sub>

 $\bigcirc B_I$ 

IV

Ò

F

 $E_{\mathbf{V}}$ 

 $B_V \bigcirc$ 

![](_page_35_Figure_1.jpeg)

まとめ

# 「衝撃波中の粒子加速」を30年間研究しました

# 永い間のご支援、有難うございます

Multi-dimensional codes are being developed by Toida et al.

![](_page_37_Figure_1.jpeg)

Toida, Ueno & Ohsawa, J. Phys. Soc. Jpn. 77, 084501 (2008)