Development statistics

S07 Estimation

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Small sample theorem

- If population = normal $x_i \sim N(\mu, \sigma^2)$

- Even if σ^2 is unknown
- Even if we have only small sample
- We can use t-distribution to estimate μ

$$t = \frac{\bar{X} - \mu}{\sqrt{\frac{s^2}{n}}} \sim t(n-1)$$

Estimation of μ (1)

1. known σ^2

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \quad \Rightarrow \quad z = \frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$$

$$\Pr(-1.96 < z < 1.96) = 95\%$$

$$\Pr(\bar{X} - 1.96\sqrt{\frac{\sigma^2}{n}} < \mu < \bar{X} + 1.96\sqrt{\frac{\sigma^2}{n}}) = 95\%$$

Estimation of μ (2)

2. Large sample & unknown σ^2

$$\bar{X} \sim N(\mu, \frac{s^2}{n}) \quad \Rightarrow \quad z = \frac{\bar{X} - \mu}{\sqrt{\frac{s^2}{n}}} \sim N(0,1)$$

$$\Pr(-1.96 < z < 1.96) = 95\%$$

$$\Pr(\bar{X} - 1.96\sqrt{\frac{s^2}{n}} < \mu < \bar{X} + 1.96\sqrt{\frac{s^2}{n}}) = 95\%$$

Estimation of μ (3)

3. Small sample & unknown σ^2

$$T = \frac{\bar{X} - \mu}{\sqrt{\frac{s^2}{n}}} \sim t(n-1)$$

$$\Pr(-t(n-1,0.05) < T < t(n-1,0.05)) = 95\%$$

$$\Pr(\bar{X} - t(n-1,0.05)\sqrt{\frac{s^2}{n}} < \mu < \bar{X} + t(n-1,0.05)\sqrt{\frac{s^2}{n}}) = 95\%$$

Distribution of proportion

- Distribution of ratio

$$\frac{x}{n} = \hat{p} \sim N(p, p(1-p)/n)$$

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0,1)$$

- If the sample size is large enough

$$z = \frac{\hat{p} - p}{\sqrt{\hat{p}(1-\hat{p})/n}} \sim N(0,1)$$

Estimation of population proportion

- If the sample size is large enough

$$z = \frac{\hat{p} - p}{\sqrt{\hat{p}(1-\hat{p})/n}} \sim N(0,1)$$

- Confidence interval of ratio "p"

$$\Pr\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 95\%$$