Development Statistics

S02 Probability

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Key wards for probability

- Experiment (試行)
  - For example, throwing a dice
- Sample space (標本空間)
  - Outcome of an experiment
- Event (事象)
  - A certain set in an outcome
- Probability (確率)
  - Ratio of “event / sample space”

Basic rule of probability

- Probability is between 0 and 1

$$0 \le \mathrm{P(A)} \le 1$$

- Compliment event rule (余事象確率)

$$\mathrm{P}(\overline{\mathrm{A}}) = 1 - \mathrm{P(A)}$$

- Joint probability (和事象確率)

$$\mathrm{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Mutually exclusive events

- Definition of “exclusive”

If A and B are exclusive $\quad \mathrm{P(A \cap B)} = 0$

- Joint probability of Mutually exclusive events

If A and B are exclusive

$$\mathrm{P(A} \cup B) = \mathrm{P(A) + P(B)}$$

Conditional Probability

- Conditional Probability

$$P(A \mid B) = P(A \cap B)/P(B)$$

- Example: throwing a dice
  - Under the condition of B: “<=3” , the probability of A: “even number”

$$P(A \mid B) = (1/6)/(1/2) = 1/3$$

Independent event

- Definition of independence

$$P(A \mid B) = P(A),\ \ P(B \mid A) = P(B)$$

- Product rule

$$P(A \cap B) = P(A) \cdot P(B)$$

- Ex: throwing 2 dices A:1 & B:1

$$P(A \cap B) = (1/6) \cdot (1/6) = 1/36$$

Union rule

- There are *n* independent event

$$P(A_1 \cup A_2 \cup \cdots \cup A_n)$$
$$= 1 - P(\overline{A}_1) \cdot P(\overline{A}_2) \cdots P(\overline{A}_n)$$

- Ex: throwing 2 dices A:1 or B:1
  in other words, at least one time 1

$$P(A \cup B) = 1 - (5/6) \cdot (5/6) = 9/36 = 1/4$$

Marginal Probability

	Event B1	Event B2	Event B3	Total B
Event A1	P11	P12	P13	P1t
Event A1	P21	P22	P23	P2t
Total A	Pt1	Pt2	Pt2	1.0

- Joint Probability P11, P12, P13
- Marginal Probability P1t or Pt1
- Conditional Probability P11/P1t, P11/Pt1

Quiz 1
number of the students

	Japanese	Other Asian	African	Others
Male	26	28	30	52
Female	24	20	14	44
Secret	0	2	6	4

1. Probability of “African”
2. Probability of “Asian”
3. Under “African”, Probability of “male”
4. Probability of “African” and “male”

Permutation (順列)

- The number of the arrangements
  - Total: *n*
  - Selection: *r*

$$ {}_{n}\mathrm{P}_{r} = \frac{n!}{(n-r)!} = \frac{n\cdot(n-1)\cdots 2\cdot 1}{(n-r)\cdot(n-r-1)\cdots 2\cdot 1} $$

Combination (組合わせ)

- The number of the selection
  - Total: n
  - Selection: r

$$
{}_{n}\mathrm{C}_{r} = \frac{n!}{r!(n-r)!}
$$

$$
= \frac{n\cdot(n-1)\cdots2\cdot1}{[r\cdot(r-1)\cdots2\cdot1][(n-r)\cdot(n-r-1)\cdots2\cdot1]}
$$

Quiz 2a

- You have 6 books
  1. If you arrange all of these books, how many arrangement manners ?
  2. If you choose 3 books, how many arrangement manners ?
  3. If you choose 3 books, how many combination manners ?

Quiz 2b

- You have 6 books:
  3 Micro economics & 3 Macro economics
  1. If you choose 3 books,
     probability that 3 Micro books are chosen?
  2. If you choose 3,
     probability that 2 Micro & 1 Macro is chosen?
  3. If you choose 1 and return this & repeat 3 times, probability that 3 Micros are chosen?

Quiz 3

- There are 30 PCs in a class room &
  you know 3 are broken.
- 3 students choose PC at random,
  1. probability that 3 students choose all broken?
  2. probability that 3 students never choose broken?
- 10 students choose PC at random,
  1. probability that all 3 broken PCs are chosen?
  2. probability that 3 broken PCs are never chosen?